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What will my income be in retirement?

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The **'Minimising Longevity and Investment Risk while Optimising Future Pension Plans'** research programme is being funded by the Actuarial Research Centre.

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Overview

- What should people want in retirement?
- Improving communication by product design
- A closer look at the mathematics and wealth distribution
- Outlook

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Planning for retirement

- Plan today…

- …for the future, but which one?

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What people want

An inflation-indexed retirement income that lasts for their lifetime.

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Robert C. Merton (2014) The Crisis in Retirement Planning. HBR.

- Goal= inflation-increasing income for life.
- Risk = failure to meet goal.
- Align investment strategy with goal.

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Improving communication by product design

- How much income in retirement?
- Target: The income you'd like to live on.
- Minimum: The minimum income that you are happy to live on.

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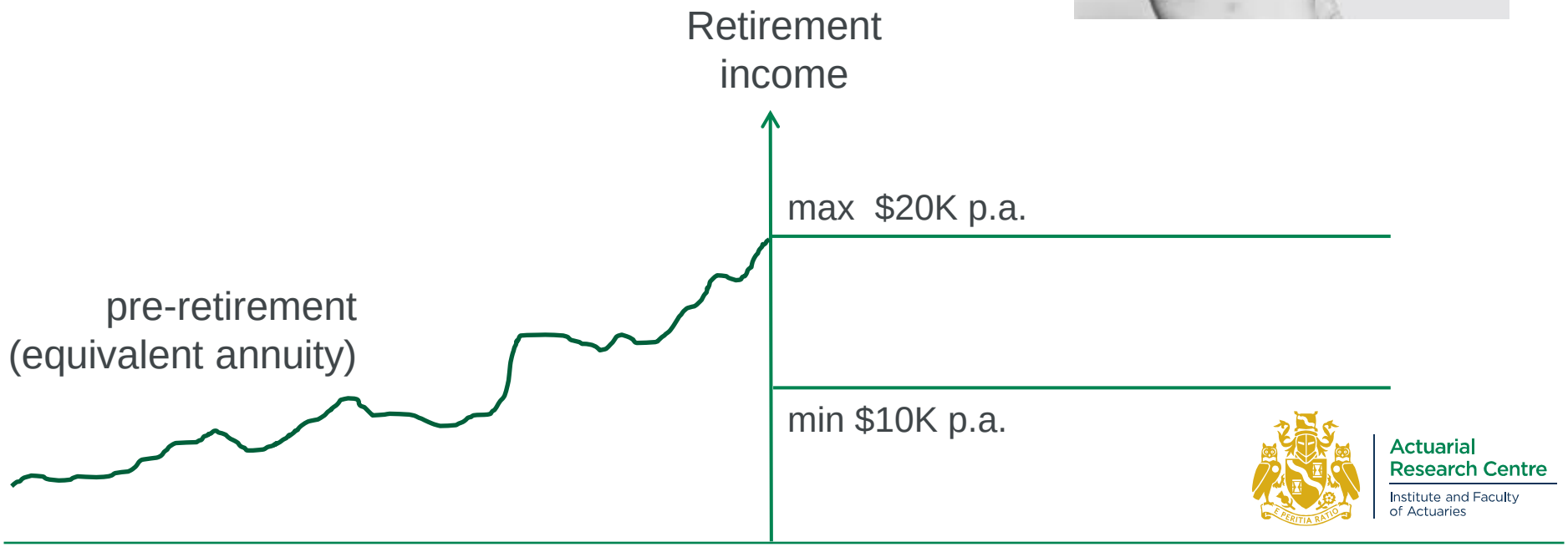
Improving communication by product design

- Target income $20,000
- Minimum income $10,000
- Expressed in today's money.



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Customer communication

- Current contribution rate 12% of salary.
- Target income $20K p.a.
- Minimum income $10K p.a.

0% chance of <$10K → 72% chance of $10K-$20K → 28% chance of $20K → 0% chance of >$20K

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Customer communication

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Customer communication

- Current **contribution rate 17%** of salary.
- Target income $20K p.a.
- Minimum income $10K p.a.

0% chance of <$10K → 22% chance of $10K-$20K → **78%** chance of $20K → 0% chance of >$20K

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Securing a life-long income

- In first run-through, buy index-linked life annuities at retirement.
  - $20K p.a. ≡ $600K lump sum at age 70,
  - $10K p.a. ≡ $300K lump sum at age 70.
- Investment strategy
  - baseline strategy,
  - + long synthetic put to have wealth ≥ $300K,
  - + short synthetic call to have wealth ≤ $600K.

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Securing a life-long income

	Chance of getting at least $20K p.a.		
Contribution rate	No Target, No Minimum		**Target $20K p.a, Minimum $10K p.a. (customer feedback)**
12%	55%		**28%**
15%	68%		**65%**
17%	74%		**78%**

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Securing a life-long income

	Chance of getting at least $20K p.a.		
Contribution rate	No Target, No Minimum	Target $20K p.a, but no Minimum	**Target $20K p.a, Minimum $10K p.a. (customer feedback)**
12%	55%	59%	**28%**
15%	68%	74%	**65%**
17%	74%	83%	**78%**

- A hard income target:
  - Increases the chance to hit the target,

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Securing a life-long income

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Securing a life-long income

	Chance of getting at least $20K p.a.		
Contribution rate	No Target, No Minimum	No Target, Minimum $10K p.a.	**Target $20K p.a, Minimum $10K p.a. (customer feedback)**
12%	55%	26%	**28%**
15%	68%	58%	**65%**
17%	74%	69%	**78%**

- A hard income target:
  - Increases the chance to hit the target,
  - Can offset the cost of the minimum income.

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Derivation of the investment strategy

- Black-Scholes market
  - Risky stock price dynamics $\frac{dS(t)}{S(t)} = \mu\, dt + \sigma\, dW(t)$,
  - Risk-free bond price dynamics $\frac{dB(t)}{B(t)} = r\, dt$,
  - Process *W* a standard Brownian motion.

Derivation of the investment strategy

- Initial wealth $x_0 > 0$.
- Find an optimal strategy π^* that maximises

$$E[\frac{1}{\gamma} X^{\pi}(T)^{\gamma}]$$

subject to $X^{\pi}(T) \in [Minimum, Target]$, a. s.

- An optimal strategy is the hedging strategy that gives wealth

$$X^{\pi^*}(T) = z_0 Z(T) - [z_0 Z(T) - Target]_+ + [Minimum - z_0 Z(T)]_+.$$

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Remove minimum

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Remove minimum

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$$X^{\pi^*}(T) = z_0 Z(T) - [z_0 Z(T) - Target]_+.$$

- For $t \leq T$, $X^{\pi^*}(t) = z_0 Z(t) - call\big(t, z_0 Z(t)\big)$.

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$$X^{\pi^*}(T) = z_0 Z(T) - [z_0 Z(T) - Target]_+.$$

- At $t = 0,\ x_0 = z_0 - call(0, z_0)$.

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- Find an optimal strategy π^* that maximises

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subject to $X^{\pi}(T) \leq \text{Target}, \text{a. s.}$

- An optimal strategy is the hedging strategy that gives wealth

$$X^{\pi^*}(T) = z_0 Z(T) - [z_0 Z(T) - Target]_+.$$

- At $t = 0,\ x_0 = z_0 - call(0, z_0) \ \Rightarrow\ z_0 \geq\ x_0$

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Interpretation of z_0

- Quantile uplift z_0/x_0.
- *p*-quantile

$$Q_p = \inf\{y \in \mathbb{R}: \mathbb{P}[X^\pi(T) \leq y] \geq p\}.$$

- Without Target constraint: $Q_p = x_0\beta_p$
- With Target constraint *K*: $Q_p = \min\{K, z_0\beta_p\}$

Initial wealth \$300, r=0, σ =0.1544, T=30 years

p	Target=587	Target=1039	Target=2360	No Target
5%	179	154	147	146
25%	408	350	334	332
50%	587	616	590	587
75%	587	1039	1044	1039
95%	587	1039	2360	2360
Prob. hit Target	60%	27%	5%	N/A
Quantile uplift z_0/x_0	123%	106%	101%	N/A

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Add back in the minimum

Initial wealth \$100, $\gamma = -0.44$, T=30 years

p	Theoretical	Annual rebalancing
1%	83.3	80.2
5%	83.3	82.9
25%	111.5	113.5
50%	138.3	138.3
75%	138.3	139.2
95%	138.3	144.0
99%	138.3	150.8

- Target=138.3,
- Minimum=83.3,
- [0%,100%] in stocks in annual rebalancing.
- $r = 0, \mu = 0.0343, \sigma$=0.1544.

- Prob. hit Target=66%.
- Prob. hit Minimum=15%.

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Initial wealth \$100, $\gamma = -0.44$, T=30 years

Probability of hitting	σ =0.0722	σ =0.1544	σ =0.3088
Target	96%	66%	31%
Minimum	2%	15%	24%

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Initial wealth \$100, $\gamma = -0.44$, *T*=30 years

p	S&P500 annual rebalancing
1%	75.5
5%	81.9
25%	108.1
50%	137.5
75%	138.5
95%	139.5
99%	140.6

- Annual rebalancing.
- [0%,100%] in stocks.
- Bootstrap S&P500 annual returns from 1982-2012 (but then lose serial correlation).
- Subtracted 0.0599 from each return so average return=0.0343.
- Risk-free rate 0%.

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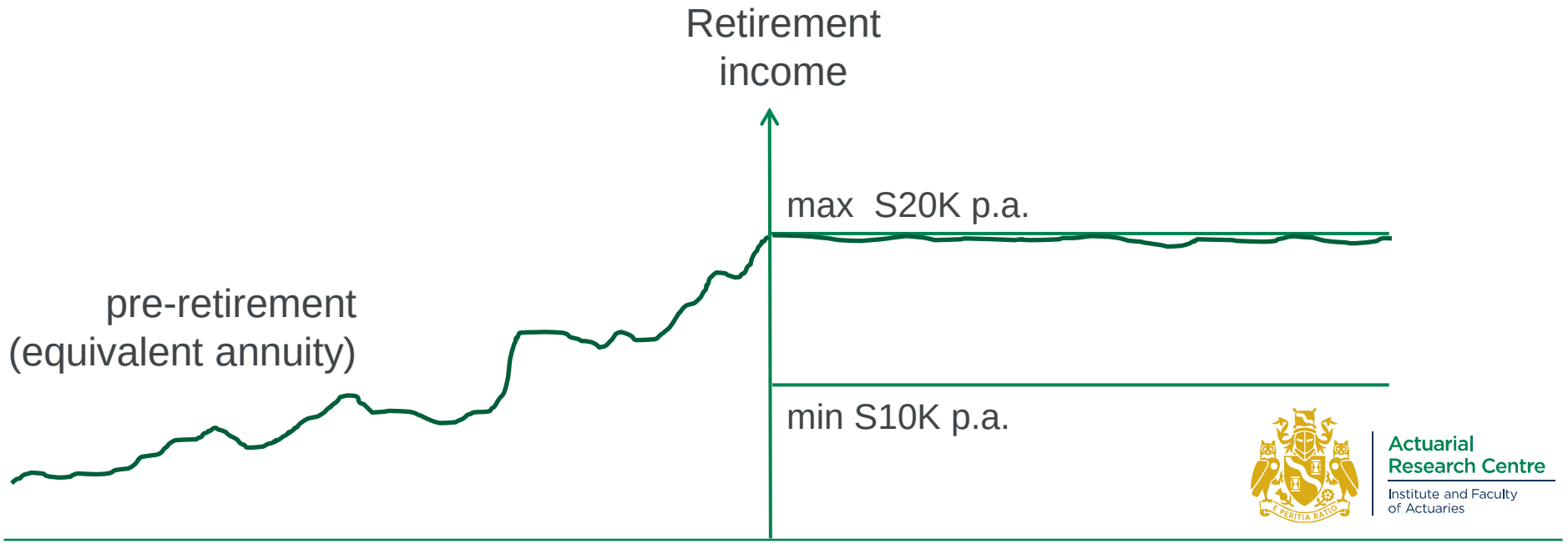
Summary

- Target wealth restriction:
  - Increases certainty of level of retirement wealth,
  - Offsets the cost of a minimum wealth,
  - May aid in communication of risk.

- Follow replicating portfolio strategy, don’t buy options.

Future directions

- Decumulation
  - Remain invested post-retirement,
  - Pool mortality risk for life-long income.



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Future directions

- Investment risk-sharing and/or return smoothing
- Aim for transparency as far as possible.
- Avoid guarantees until they are needed.

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The Actuarial Research Centre (ARC)

A gateway to global actuarial research

The Actuarial Research Centre (ARC) is the Institute and Faculty of Actuaries' (IFoA) network of actuarial researchers around the world.

The ARC seeks to deliver cutting-edge research programmes that address some of the significant, global challenges in actuarial science, through a partnership of the actuarial profession, the academic community and practitioners.

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- Donnelly, C, Guillén, M, Gerrard, R. and Nielsen, J.P. Less is more: Increasing retirement gains by using an upside terminal wealth constraint. *Insurance: Mathematics and Economics* (2015), 64, pp259-267.

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Questions

Comments

The views expressed in this presentation are those of the presenter.

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