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Bootstrapping the Cape Cod method

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25 April 2023

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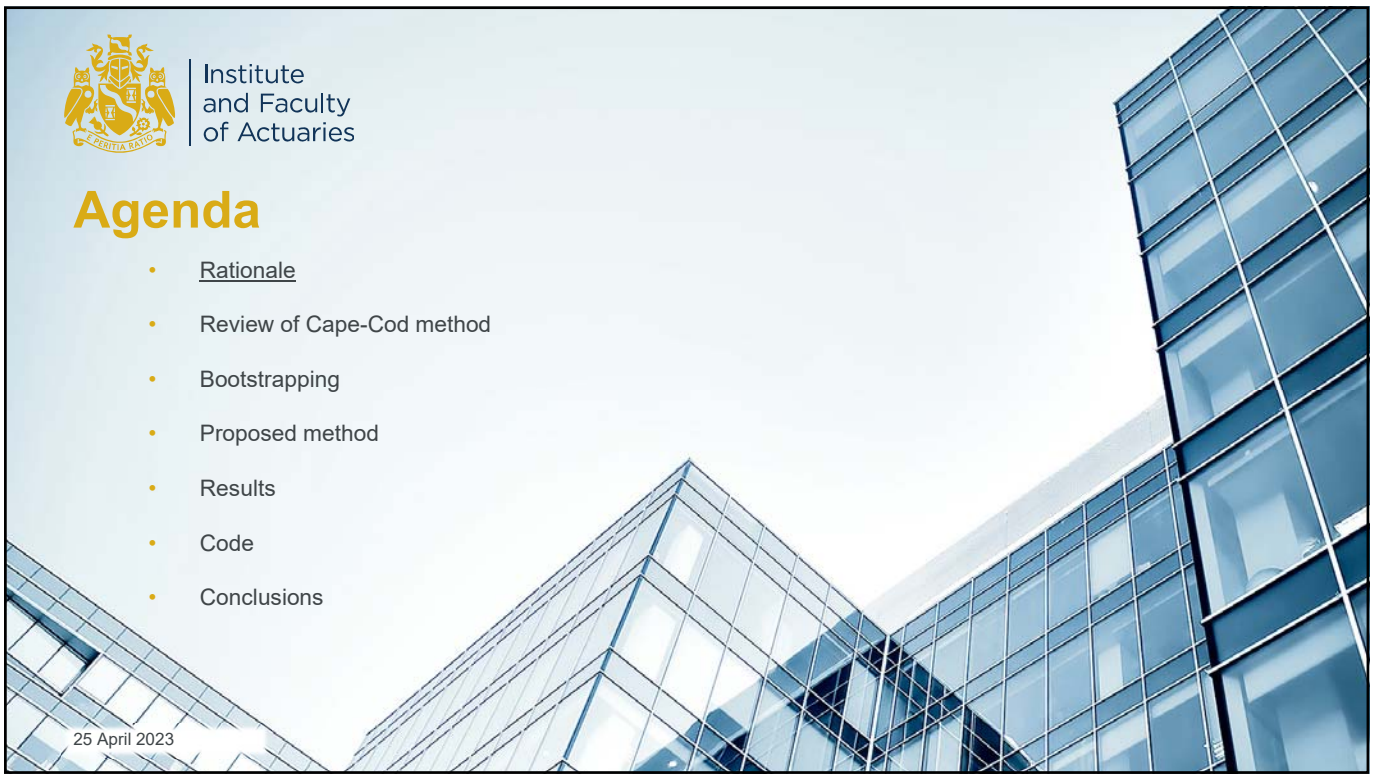
Agenda

- Rationale
- Review of Cape-Cod method
- Bootstrapping
- Proposed method
- Results
- Code
- Conclusions



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Introduction

- Reserving governed by modern solvency/accounting regimes (e.g. SAM/SII and IFRS 17)
- Non-life insurers need to calculate two main items:
  - Best estimate IBNR reserves
  - Reserve risk capital/margins
- Best estimate: range of reserving methods e.g. chain ladder, Bornhuetter Ferguson, Cape Cod
- Capital/margins: uncertainty quantifications methods e.g. Mack's method and bootstrapping
- Bootstrapping usually formulated with respect to the chain ladder method…
- … whereas a different method likely used for the best estimate reserves
- Basic problem: not quantifying uncertainty of the reserves we actually hold on balance sheet (see nice paper by Dal Moro and Lo (2014))

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Introduction(2)

- Can we resolve this issue by using a single method for both best-estimate and margins?
- Among exposure-based methods (BF/CC), difficult to see how to bootstrap BF…
- … as it relies on subjective choice of the loss ratios used in the method (however, see the stochastic BF approach of Alai, Merz, & Wüthrich (2011))
- Here we focus on the Generalized Cape Cod (GCC) method
- The GCC method appears to be a better choice…
- … derives loss ratios using an algorithm, based on single parameter defining how much weight to give to older accident years
- However, methods for bootstrapping the GCC have not yet been explored

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Contributions

- Bootstrapping the Cape Cod method: A working paper
- Joint work with OMI ACF team

Bootstrapping the Cape Cod method: A working paper

By Nikita Harilal, Karen Muyengwa, Kovlin Perumal, Ronald Richman, Itumeleng Sefako, Rowald van der Walt

Presented at the Actuarial Society of South Africa's 2022 Convention
Cape Town 25–28 October 2022

ABSTRACT

Methods for estimating the reserve risk of incurred but not reported provisions are usually based on the assumption that these provisions were calculated using the chain ladder method. Thus, in practice, these methods relate to various ways of quantifying the prediction error of the chain ladder estimates, often the bootstrap procedures of England and Verrall (1999) or England and Verrall (2006) for the chain ladder method. On the other hand, IBNR estimates are often quantified using exposure-based methods such as the Bornhuetter-Ferguson or Cape Cod methods. In this working paper, we provide an initial view of how bootstrapping can be applied to the Cape Cod method. We define bootstrap procedures for the Cape Cod method, show how these can be applied and how the results compare to more established methods used in practice. We consider which of the methods are more appropriate for use in reserve risk estimation under Solvency II and accounting estimates in the context of IFRS 17, with a focus on stability and realism of the results. Finally, we provide R code to reproduce these methods on representative data.

- Contributions of our paper:
  1. Define an approach to bootstrapping the Cape Cod method, allowing for the same method to be used for best estimates and margins
  2. Show how to choose the parameters of the Cape Cod method in an optimal manner
  3. Provide a procedure for deriving the one-year and ultimate risk margins
  4. Use the outputs of the approach for estimating premium (prospective) risk

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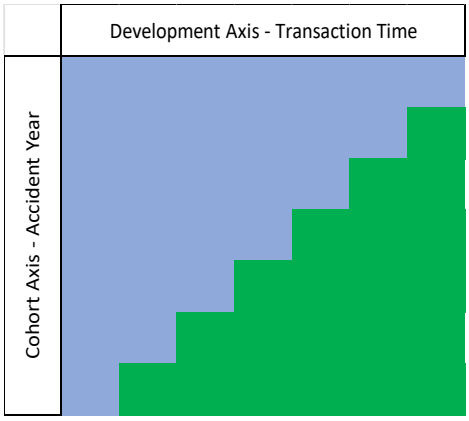
Refresher on IBNR reserving

- Quantification of reserves to back IBNR claims usually performed by actuaries
- Key principles:
  - Aggregate multiple claims events to derive "patterns" with which claims occur
  - Differentiate between cohorts of claims
  - Differentiate between claims development at different stages
- Aggregated claims development collected in claims triangle
- Useful for performing analysis where development happens over time to different cohorts
  - Insurance claims
  - Number of infections
  - Number of current loans



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Example Triangle – Liability

	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63
2010Q1	2 014	3 925	6 027	6 477	5 624	5 159	5 214	6 241	6 705	6 716	7 373	7 696	9 011	9 230	8 409	8 448	8 824	9 202	9 388	9 350	9 456
2010Q2	2 531	4 102	3 675	3 868	4 796	6 866	6 072	6 014	5 986	5 788	5 951	5 958	6 048	6 065	5 877	5 969	5 987	5 997	5 767	5 808	5 803
2010Q3	2 373	4 426	7 274	6 514	6 699	7 063	7 603	7 727	7 130	7 655	7 949	8 587	9 113	7 730	7 925	8 276	8 180	7 821	7 822	7 821	8 156
2010Q4	3 665	6 028	5 679	5 793	5 158	5 203	4 824	4 749	5 152	5 240	5 261	5 361	5 262	5 606	5 473	5 359	6 051	5 921	6 166	6 196	6 026
2011Q1	3 685	6 009	6 655	5 535	6 292	6 972	6 793	6 362	6 510	6 457	6 734	6 836	7 817	8 117	8 602	8 508	8 709	9 368	9 671	10 319	10 079
2011Q2	5 624	9 330	8 640	7 676	7 066	6 535	7 004	6 965	7 007	7 244	7 124	7 119	7 294	7 251	6 981	7 051	6 988	7 025	7 182	6 672	6 676
2011Q3	6 360	8 485	9 147	6 628	6 883	6 990	7 417	7 768	8 117	7 775	7 908	7 946	7 905	7 504	8 561	8 369	8 323	8 495	8 286	8 255	8 536
2011Q4	3 655	4 338	4 000	4 568	6 659	6 571	6 599	6 949	7 303	7 384	8 192	8 635	8 644	9 207	9 792	9 953	10 002	9 407	9 452	8 273	8 371
2012Q1	1 203	2 897	3 289	2 418	2 197	3 144	3 153	3 073	3 109	2 367	1 992	2 083	2 505	2 678	2 594	2 648	2 661	2 521	2 529	2 370	2 370
2012Q2	1 110	2 111	3 328	5 474	5 593	5 660	4 710	4 874	4 569	4 547	4 601	4 720	5 137	5 208	5 589	6 254	6 507	6 507	6 593	6 604	6 659
2012Q3	776	3 290	6 315	7 099	7 206	6 951	6 911	6 848	6 906	6 826	7 025	7 046	7 394	10 204	9 915	9 643	9 929	10 269	10 221	10 808	10 913
2012Q4	1 858	3 064	3 965	3 906	4 058	4 270	4 019	3 867	5 107	4 984	4 972	5 328	5 752	5 802	5 554	6 083	6 530	6 523	6 565	6 570	6 520
2013Q1	1 608	4 000	4 989	4 800	4 795	5 482	4 778	4 847	4 767	5 147	5 977	5 955	6 004	6 077	6 228	6 502	6 590	6 603	7 607	7 754	7 754
2013Q2	3 120	3 906	7 002	9 182	9 680	7 916	10 125	10 614	11 271	11 901	11 493	11 493	11 778	11 796	11 238	11 358	11 099	11 107	11 450	11 413	11 607
2013Q3	1 187	1 949	3 062	4 515	4 628	4 755	4 882	4 970	4 877	5 969	6 023	6 168	6 384	7 815	7 804	7 262	8 047	8 042	8 288	8 340	8 330
2013Q4	508	1 844	4 445	6 456	7 419	7 357	7 118	7 671	7 984	8 776	9 575	9 615	9 884	9 908	7 871	8 005	7 982	7 898	7 900	7 894	7 732
2014Q1	632	1 951	2 235	4 025	5 071	5 654	6 389	7 468	7 458	7 303	7 230	7 366	7 498	8 081	8 313	8 779	8 854	9 396	9 567	10 067	10 908
2014Q2	576	3 531	8 148	10 026	10 225	10 713	11 674	11 833	11 639	12 485	12 279	12 327	11 754	11 809	11 776	11 854	11 687	11 709	11 365	11 469	10 940
2014Q3	1 549	4 807	6 178	8 651	10 214	11 283	11 203	11 552	12 697	12 468	12 828	14 163	13 962	14 114	14 932	16 581	16 279	15 668	15 625	17 347	17 165
2014Q4	1 877	4 342	4 787	6 310	7 943	8 482	8 670	8 747	9 150	9 374	9 823	10 712	11 522	11 242	12 279	13 806	13 721	13 858	14 244	16 558	16 406

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Example Triangle – Property (SME)

	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63
2010Q1	15 886	25 178	26 572	25 520	25 047	25 025	24 962	24 448	24 412	24 305	24 305	24 285	24 201	24 159	24 171	24 108	24 097	24 097	24 097	24 097	24 097
2010Q2	18 400	24 210	22 953	22 781	22 710	22 939	22 277	22 395	22 332	22 615	22 615	22 615	22 615	22 615	22 594	22 590	22 592	22 592	22 592	22 592	22 590
2010Q3	15 765	18 860	17 972	18 029	18 100	17 915	17 919	17 802	17 854	17 854	17 868	17 844	17 829	17 829	17 762	17 762	17 754	17 754	17 754	17 754	17 754
2010Q4	18 049	24 586	24 558	24 342	23 850	23 923	23 912	23 959	23 949	23 823	23 921	23 836	23 863	23 863	23 853	23 830	23 830	23 830	23 830	23 829	23 774
2011Q1	22 716	28 799	28 139	26 890	27 203	27 014	26 813	26 542	26 364	26 523	26 259	26 257	26 051	26 001	26 001	26 001	26 001	26 001	26 001	26 001	26 001
2011Q2	17 630	24 164	23 773	23 061	22 775	22 659	22 349	22 242	22 255	22 229	22 229	22 198	22 114	22 062	22 062	22 062	22 062	22 062	22 062	22 062	22 062
2011Q3	20 852	28 044	27 322	27 051	26 750	26 293	26 137	26 133	26 310	26 224	25 943	25 952	25 942	25 940	25 940	25 940	25 940	25 940	25 928	25 921	25 932
2011Q4	21 489	32 919	32 872	32 830	32 575	32 553	32 536	32 444	32 453	32 324	32 324	32 496	32 474	32 464	32 464	32 464	32 464	32 464	32 464	32 464	32 464
2012Q1	18 254	29 871	30 149	29 904	30 122	30 282	30 004	30 047	29 793	29 748	29 722	29 755	29 766	29 765	29 764	29 764	29 764	29 764	29 760	29 760	29 760
2012Q2	21 812	29 826	28 047	27 960	28 009	27 813	27 605	27 431	27 503	27 207	27 184	27 184	27 184	27 184	27 044	27 044	27 044	27 044	27 044	27 044	27 044
2012Q3	25 062	35 859	36 143	35 174	34 845	34 032	33 252	33 128	32 079	32 282	32 480	32 317	32 442	32 413	32 454	32 454	32 561	32 565	32 565	32 561	32 561
2012Q4	27 479	41 905	42 867	42 488	42 368	41 576	41 451	40 987	41 404	41 486	41 476	41 551	41 551	41 551	41 551	41 583	41 583	41 583	41 256	41 214	41 214
2013Q1	26 771	40 484	39 968	40 058	40 011	38 823	38 481	38 506	38 508	38 516	38 527	38 511	38 511	38 511	38 777	38 729	38 727	38 727	38 727	38 727	38 737
2013Q2	20 362	26 805	26 388	26 454	26 380	26 321	26 310	26 323	26 318	26 309	26 621	26 621	26 640	26 640	26 624	26 624	26 624	26 624	26 624	26 624	26 624
2013Q3	19 766	29 065	28 267	27 856	27 475	27 402	27 365	27 263	27 244	27 246	27 257	27 246	27 246	27 259	27 259	27 259	27 259	27 259	27 259	27 259	27 259
2013Q4	27 035	43 553	45 623	45 858	45 471	45 806	44 433	44 634	44 638	44 694	44 694	44 684	44 481	44 529	44 567	44 676	44 707	44 790	44 837	44 883	44 814
2014Q1	19 600	35 432	37 811	38 379	37 947	37 692	37 686	37 629	37 634	37 634	37 612	37 612	37 600	37 600	37 600	37 600	37 600	37 600	37 600	37 600	37 600
2014Q2	22 220	35 361	38 686	38 612	37 948	38 008	38 109	38 099	38 210	38 179	38 179	38 178	38 178	38 178	38 178	38 178	38 178	38 178	38 175	38 175	38 186
2014Q3	22 583	34 819	35 282	34 872	35 781	35 467	35 473	35 477	35 450	35 663	35 628	35 558	35 558	35 564	35 564	35 576	35 576	35 571	35 571	35 569	35 556
2014Q4	25 473	35 455	36 730	37 066	36 677	36 631	36 392	36 361	36 376	36 387	36 420	36 420	36 431	36 445	36 450	36 450	36 437	36 437	36 437	36 437	36 437

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Refresher on Chain Ladder

- Derive chain-ladder factors and make predictions of future development for each cohort
- Easiest way: work out percentage developed and use ratio to derive 100% development for each cohort
- In this example, require R2.6m of IBNR reserves
- Ultimate = estimates of final claims for each cohort

Row Labels	Min of premium	Latest	% to Ult	IBNR - CL	ULT - CL	ULR - CL
201003	51 347	24 097	100%	-	24 097	47%
201006	53 877	22 592	100%	-	22 592	42%
201009	54 577	17 754	100%	-	17 754	33%
201012	55 527	23 830	100%	-	23 830	43%
201103	56 768	26 001	100%	- 5	25 995	46%
201106	57 244	22 062	100%	- 22	22 041	39%
201109	58 405	25 940	100%	- 42	25 898	44%
201112	59 912	32 474	100%	- 81	32 393	54%
201203	60 339	29 755	100%	- 134	29 621	49%
201206	61 308	27 184	100%	- 116	27 067	44%
201209	60 262	32 282	101%	- 199	32 083	53%
201212	60 513	41 404	101%	- 280	41 124	68%
201303	62 668	38 506	101%	- 360	38 146	61%
201306	63 109	26 310	101%	- 362	25 949	41%
201309	62 934	27 402	102%	- 592	26 811	43%
201312	63 694	45 471	103%	- 1 361	44 110	69%
201403	64 467	38 379	104%	- 1 349	37 029	57%
201406	66 843	38 686	105%	- 1 704	36 982	55%
201409	68 594	34 819	104%	- 1 343	33 477	49%
201412	69 621	25 473	71%	10 479	35 952	52%
Total	1 212 010	600 423		2 529	602 952	50%

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Cape-Cod

- Principled way of deriving loss ratios from triangles
- Due to Bühlmann and Straub (1983); Story of origins

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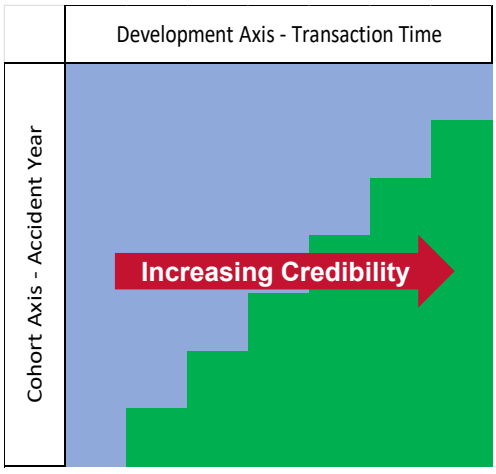
Cape-Cod (2)

- Credibility ~= extent to which we can rely on data versus prior knowledge
  - Credibility of estimates from a triangle increases as development period increases
  - i.e. more developed accident years likely to provide better estimates of ultimates
- Cape-Cod – method for calculating loss ratios from triangles applying credibility weighting =>
- Weight **estimates of ultimate** by **extent of development**
- $IELR = \frac{\sum_{i=1}^{J} C_{\{i,J\}}.\%Dev}{\sum_{i=1}^{J} EP_j.\%Dev}$
- $IELR = \frac{\sum_{i=1}^{J} C_{\{i,j\}}}{\sum_{i=1}^{J} EP_j.\%Dev}$



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Cape-Cod (3)

- IELR derived using CC method is 49.3%
- Leads to slightly higher IBNR than the chain-ladder method
- How reasonable is the assumption of a constant loss ratio?
- Not too realistic!
- Depends on how many accident years included
- Can we automate the process of how much weight to give to older years?

AY	Row Labels	Min of premium	Latest	% to Ult	IBNR - CL	ULT - CL	ULR - CL	IELR - CC	IBNR - CC	Ult - CC	ULR - CC
1	201003	51 347	24 097	100%	-	24 097	47%	49.8%	-	24 097	47%
2	201006	53 877	22 592	100%	-	22 592	42%	49.8%	-	22 592	42%
3	201009	54 577	17 754	100%	-	17 754	33%	49.8%	-	17 754	33%
4	201012	55 527	23 830	100%	-	23 830	43%	49.8%	-	23 830	43%
5	201103	56 768	26 001	100%	- 5	25 995	46%	49.8%	- 5.54	25 995	46%
6	201106	57 244	22 062	100%	- 22	22 041	39%	49.8%	- 27.86	22 035	38%
7	201109	58 405	25 940	100%	- 42	25 898	44%	49.8%	- 46.70	25 893	44%
8	201112	59 912	32 474	100%	- 81	32 393	54%	49.8%	- 74.65	32 400	54%
9	201203	60 339	29 755	100%	- 134	29 621	49%	49.8%	- 135.89	29 619	49%
10	201206	61 308	27 184	100%	- 116	27 067	44%	49.8%	- 131.23	27 053	44%
11	201209	60 262	32 282	101%	- 199	32 083	53%	49.8%	- 186.48	32 096	53%
12	201212	60 513	41 404	101%	- 280	41 124	68%	49.8%	- 205.22	41 199	68%
13	201303	62 668	38 506	101%	- 360	38 146	61%	49.8%	- 294.27	38 212	61%
14	201306	63 109	26 310	101%	- 362	25 949	41%	49.8%	- 437.76	25 873	41%
15	201309	62 934	27 402	102%	- 592	26 811	43%	49.8%	- 691.35	26 711	42%
16	201312	63 694	45 471	103%	- 1 361	44 110	69%	49.8%	- 978.33	44 493	70%
17	201403	64 467	38 379	104%	- 1 349	37 029	57%	49.8%	- 1 169.30	37 210	58%
18	201406	66 843	38 686	105%	- 1 704	36 982	55%	49.8%	- 1 533.20	37 153	56%
19	201409	68 594	34 819	104%	- 1 343	33 477	49%	49.8%	- 1 369.21	33 450	49%
20	201412	69 621	25 473	71%	10 479	35 952	52%	49.8%	10 100.05	35 573	51%
	Total	1 212 010	600 423		2 529	602 952	50%		2 813	603 236	50%

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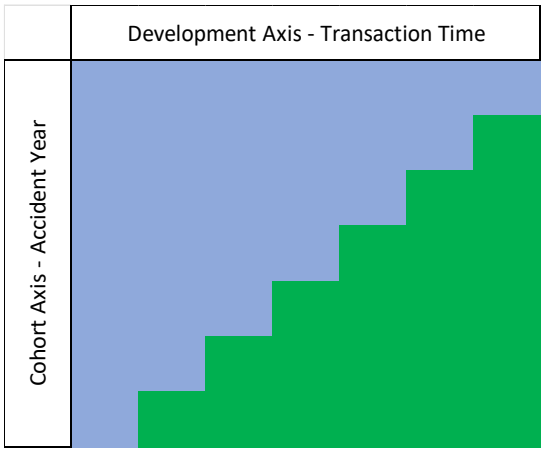
Generalized Cape-Cod

- Due to Gluck (1997)
- Weight estimates of ultimate by extent of development as well as recency of information

$$IELR_j = \frac{\sum_{i=1}^{J} C_{\{i,J\}} \cdot \%Dev \cdot \gamma^{|i-j|}}{\sum_{i=1}^{J} EP_j \cdot \%Dev \cdot \gamma^{|i-j|}}$$

$$IELR_j = \frac{\sum_{i=1}^{J} C_{\{i,j\}} \cdot \gamma^{|i-j|}}{\sum_{i=1}^{J} EP_j \cdot \%Dev \cdot \gamma^{|i-j|}}$$

- $\gamma = 0$ => Chain-ladder as all weight given to current period
- $\gamma = 1$ => Cape-Cod as all weight given to current period



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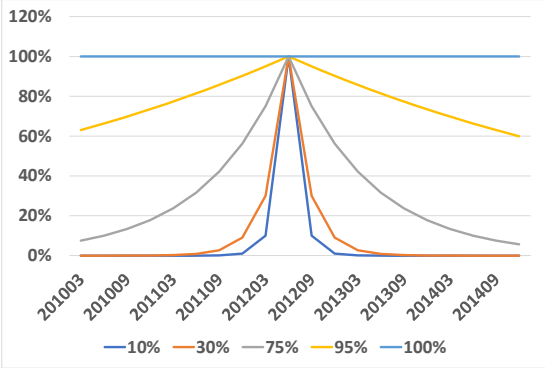
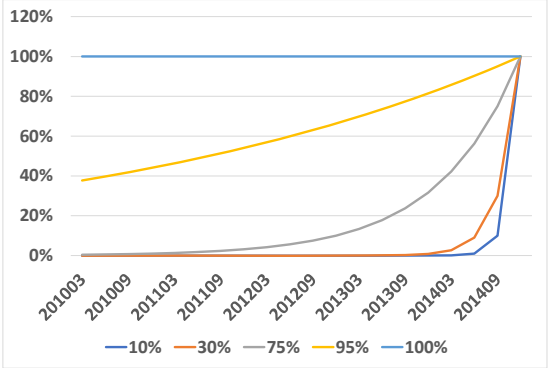
Reweighting accident years

Weights with respect to:

201206



201412



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Generalized Cape-Cod (2)

- Liability example
- Weighting parameter of 80%
- Leads to higher IBNR than chain-ladder and cape-cod
- Can see how Cape-Cod IELRs vary over time picking up the trend
- Smooth IELRs reflecting classic actuarial prior

AY	Row Labels	Min of premium	Latest	% to Ult	IBNR - CL	ULT - CL	ULR - CL	IELR - CC	IBNR - CC	Ult - CC	ULR - CC
1	201003	25 728	9 350	100%	-	9 350	36%	33%	-	9 350	36%
2	201006	25 975	5 767	100%	- 24	5 743	22%	33%	- 35.18	5 731	22%
3	201009	24 954	7 821	101%	- 55	7 766	31%	33%	- 59.17	7 762	31%
4	201012	24 478	6 051	101%	- 35	6 016	25%	34%	- 48.30	6 002	25%
5	201103	23 899	8 508	97%	249	8 757	37%	36%	242.60	8 751	37%
6	201106	21 730	6 981	96%	259	7 240	33%	37%	288.45	7 270	33%
7	201109	15 232	7 504	98%	149	7 653	50%	39%	115.33	7 619	50%
8	201112	11 990	8 644	100%	12	8 656	72%	40%	6.83	8 651	72%
9	201203	11 731	2 083	95%	109	2 192	19%	40%	235.31	2 318	20%
10	201206	11 503	4 601	92%	385	4 985	43%	41%	366.34	4 967	43%
11	201209	13 317	6 826	89%	802	7 627	57%	42%	590.82	7 417	56%
12	201212	14 270	5 107	90%	559	5 666	40%	42%	597.52	5 705	40%
13	201303	15 193	4 847	87%	693	5 540	36%	43%	815.12	5 662	37%
14	201306	15 077	10 125	86%	1 631	11 756	78%	44%	913.34	11 039	73%
15	201309	18 804	4 755	86%	795	5 550	30%	43%	1 147.01	5 902	31%
16	201312	17 930	7 419	84%	1 458	8 877	50%	42%	1 248.45	8 668	48%
17	201403	18 825	4 025	80%	994	5 019	27%	42%	1 553.94	5 578	30%
18	201406	20 181	8 148	76%	2 603	10 751	53%	42%	2 050.81	10 199	51%
19	201409	20 891	4 807	58%	3 449	8 256	40%	41%	3 608.03	8 415	40%
20	201412	23 654	1 877	32%	3 980	5 857	25%	41%	6 530.29	8 407	36%
	Total	375 364	125 244		18 013	143 258	38%		20 168	145 412	39%

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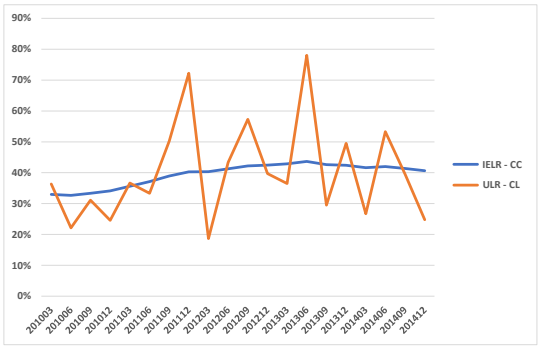
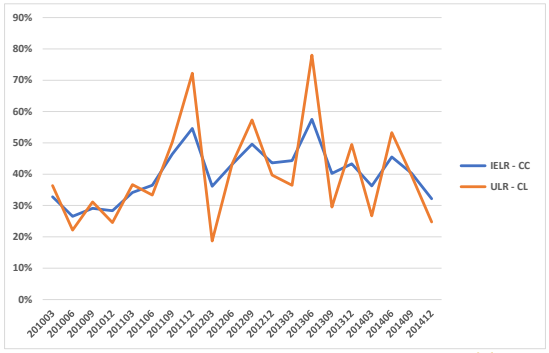
Reweighting accident years (2)

IELR with respect to:

$\gamma = 80\%$



$\gamma = 30\%$



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Agenda

- Rationale
- Review of Cape-Cod method
- Bootstrapping
- Proposed method
- Results
- Code
- Conclusions