Institute and Faculty of Actuaries

Transition & Default Risk for Corporate Bonds

Extreme Events Working Party

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Sessional Event, May 2023

Agenda

1. The data used for transition and default risk
2. The Two Parameter model
3. The Vašíček model
4. The K-means model
5. Comparison between the models
6. Discussion
7. Summary

Data and modelling requirements

- The data used to model default and transition risk is the transition matrix
- A matrix with a full range of probabilities for any rated asset moving to any other rating within a defined timeframe
- Historical data has a different transition matrix for each year giving and one of the most difficult risks to model for most insurers
- The challenge is to produce a model that captures the variability in historical transition matrices whilst being relatively simple to implement
- Simplest model – bootstrapping – sampling with replacement

From/to	AAA	AA	A	BBB	BB	B	CCC/C	D
AAA	89.82%	9.42%	0.55%	0.05%	0.08%	0.03%	0.05%	0.00%
AA	0.52%	90.63%	8.17%	0.51%	0.05%	0.06%	0.02%	0.02%
A	0.03%	1.77%	92.30%	5.40%	0.30%	0.13%	0.02%	0.06%
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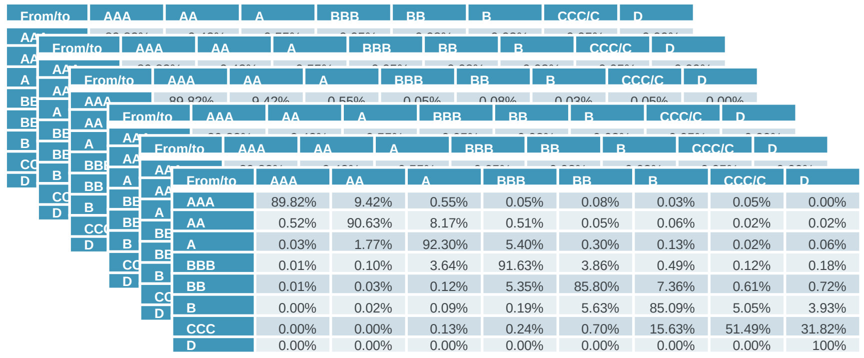
CCC	0.00%	0.00%	0.13%	0.24%	0.70%	15.63%	51.49%	31.82%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100%

1981
1982
1983
…
…
2018
2019



Two factor model - description

A statistical model of a transition matrix. Each transition matrix can be defined by two parameters – Inertia and Optimism which cover the main sources of historical variability.

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Inertia

- The sum of the leading diagonal of the transition matrix
- Gives a measure of how much movement there has been from existing credit ratings

Optimism

- The ratio of upgrades to downgrades/ defaults, weighted by the default amount
- Gives a measure of how upgrades have compared to downgrades

"Stress Testing for Financial Institutions" Rosch et al

Two factor model - calibration

- Calibration is based on historic transition matrices. Each historic matrix is converted to two parameters
- This gives two time series to which a probability distribution or time series model can be fit
- The probability distributions can be combined into a single joint probability distribution using a copula
- This gives a full risk distribution for each of the two parameters
- A base transition matrix is adjusted by simple scaling of the transition matrix elements so that the adjusted matrix has the same Inertia / Optimism as required

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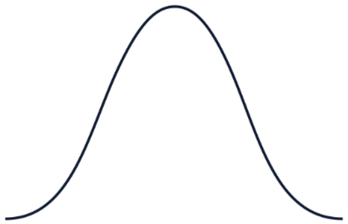
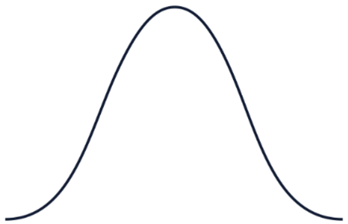
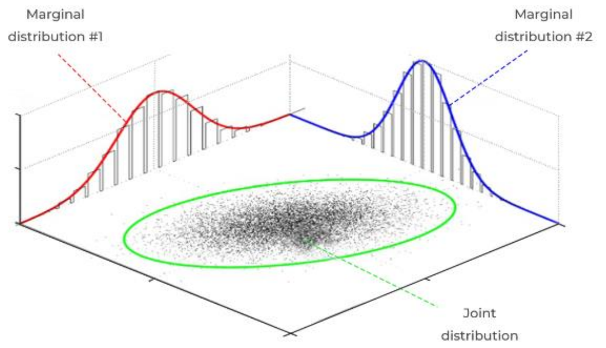
Inertia



Optimism



Copula



Years	Inertia	Optimism
2019	6.3	0.4
2018	6.1	0.6
2017	5.8	0.7
2016	5.9	0.4
2015	6.2	0.5
2014	6.2	1.1
2013	6.1	1.0
2012	5.9	0.7
2011	5.5	1.3
2010	5.7	1.8
2009	5.5	0.1
2008	5.7	0.3
2007	6.1	2.0
2006	6.2	1.9
2005	5.9	1.6
2002	5.5	0.3
2001	5.7	0.2
2000	6.0	0.3
1999	6.1	0.3
1998	5.8	0.7
1997	6.1	1.4
1996	6.2	1.9
1995	6.1	1.3
1994	6.2	1.1
1993	5.7	3.2
1992	5.8	1.0
1991	5.7	0.4
1990	5.7	0.4
1989	5.7	1.1
1988	5.7	0.8
1987	6.1	0.9
1986	5.9	0.3
1985	5.8	0.6
1984	6.0	1.5
1983	6.1	0.7
1982	5.9	0.4
1981	6.0	1.3
1932	4.0	0.02

Vašíček's model – description

- Merton's model of a company's asset return (1974):

$$\ln A(\mathrm{T}) = \ln A + \mu_i \mathrm{T} - \frac{1}{2}\sigma^2 \mathrm{T} + \sigma\sqrt{T}X$$

  Where X represents a firm's asset return, and X follows a Standard Normal distribution.

- Oldřich Vašíček (1987) extended the Merton's model to a portfolio of assets (i = 1, …, n):

$$\ln A_i(\mathrm{T}) = \ln A_i + \mu_i \mathrm{T} - \frac{1}{2}{\sigma_i}^2 \mathrm{T} + \sigma_i\sqrt{T}X_i$$

- Vašíček also observed the following property of equi-correlated Standard Normal variables:

$$X_i = \mathrm{Z}\sqrt{\rho} + Y_i\sqrt{1-\rho}\text{, i = 1,…, n}$$

  Where $Z, Y_1, Y_2, \ldots, Y_n$ are mutually independent Standard Normal variables, n the number of firms in a portfolio

- Vašíček replaced X_i in Merton's model with $X_i = \mathrm{Z}\sqrt{\rho} + Y_i\sqrt{1-\rho}$

  Where:
  - » Variable Z is as common across the entire portfolio
  - » Variables Y_i are i^th^ firm's specific variables
  - » Parameter ρ is the asset portfolio correlation (and it's as an important driver of credit risk as it gives a measure of joint probability of default).
  - » A portfolio asset correlation is like Modern Portfolio Theory.

- A key result in Vašíček's framework follows – a firm's probability of default conditional on Z in a large portfolio of assets:

$$P(firm\ i\ defaults \mid Z) = \Phi\left(\frac{x_i - Z\sqrt{\rho}}{\sqrt{1-\rho}}\right)$$

- For the avoidance of doubt, Vašíček's framework introduced in our slides is different to Vašíček's model for interest rates.

Vašíček's model – application & calibration

- A firm's probability of default conditional on Z can be applied to transition rates in a transition matrix.

- Fitted transition rates for the 'G to g' credit rating can be written as:

$$\Phi\left(\frac{x^G_{g+1} - Z\sqrt{\rho}}{\sqrt{1-\rho}}\right) - \Phi\left(\frac{x^G_{g} - Z\sqrt{\rho}}{\sqrt{1-\rho}}\right)$$

  Where:
  - » Φ(•) represents the standard normal cumulative distribution function, with Φ(∞) = 1 and Φ(-∞) = 0
  - » G is the credit rating at the beginning of the year (G = AAA, AA, …, CCC/C)
  - » g is the credit rating at the end of the year (g = AA, A, …, CCC/C, and g+1 = AAA, AA, …, B).

- Belkin (1998) introduced a statistical method to estimate the correlation parameter ρ and common factors Z in Vasicek's model based on historical transition matrices subject to Z restricted to unit variance on the Standard Normal distribution.

Average transition matrix (S&P average 1981-2019, average 1932-1935 and 1932

From/to	AAA	AA	A	BBB	BB	B	CCC/C	D
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D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100%

Derive 'bins'

From/to	AAA	AA	A	BBB	BB	B	CCC/C	D
AAA	(∞, -1.27)	[-1.27, -2.41)	[-2.41, -2.81)	[-2.81, -2.88)	[-2.88, -3.15)	[-3.15, -3.28)	[-3.28, -5.61)	[-1.27, -∞)
AA	(∞, 2.57)	[2.57, -1.36)	[-1.36, -2.49)	[-2.49, -2.96)	[-2.96, -3.08)	[-3.08, -3.34)	[-3.34, -3.53)	[2.57, -∞)
A	(∞, 3.42)	[3.42, 2.12)	[2.12, -1.58)	[-1.58, -2.6)	[-2.6, -2.9)	[-2.9, -3.18)	[-3.18, -3.28)	[3.42, -∞)
BBB	(∞, 5.61)	[5.61, 3.1)	[3.1, 1.81)	[1.81, -1.69)	[-1.69, -2.44)	[-2.44, -2.77)	[-2.77, -2.93)	[5.61, -∞)
BB	(∞, 4.25)	[4.25, 3.4)	[3.4, 2.96)	[2.96, 1.63)	[1.63, -1.35)	[-1.35, -2.23)	[-2.23, -2.46)	[4.25, -∞)
B	(∞, 5.61)	[5.61, 3.51)	[3.51, 3.08)	[3.08, 2.78)	[2.78, 1.6)	[1.6, -1.31)	[-1.31, -1.77)	[5.61, -∞)
CCC	(∞, 5.61)	[5.61, 5.61)	[5.61, 3.04)	[3.04, 2.73)	[2.73, 2.34)	[2.34, 1.01)	[1.01, -0.43)	[5.61, -∞)

Calibrated ρ

Calibrated Z values

Fitted transition matrices

1981
1982
1983
…
…
2018
2019

From/to	AAA	AA	A	BBB	BB	B	CCC/C	D
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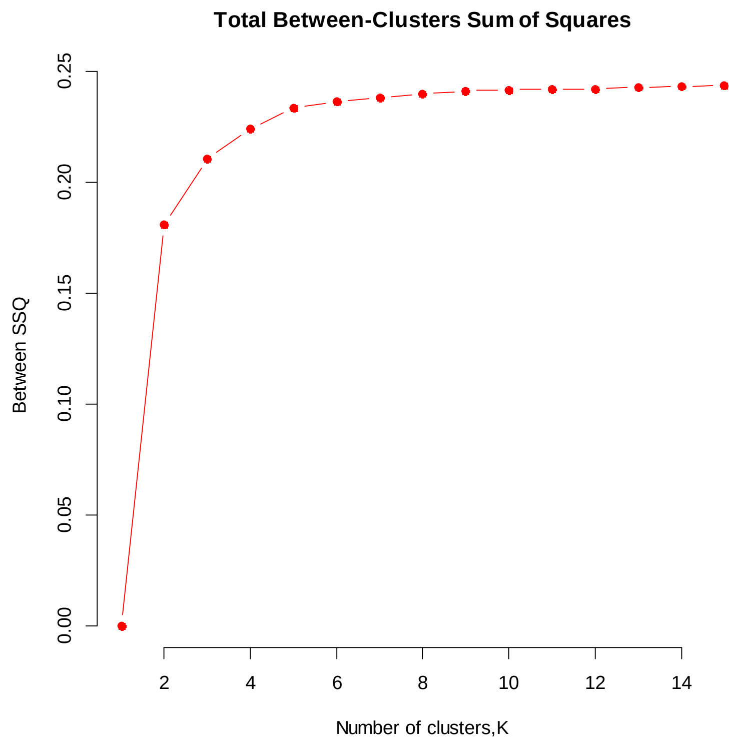
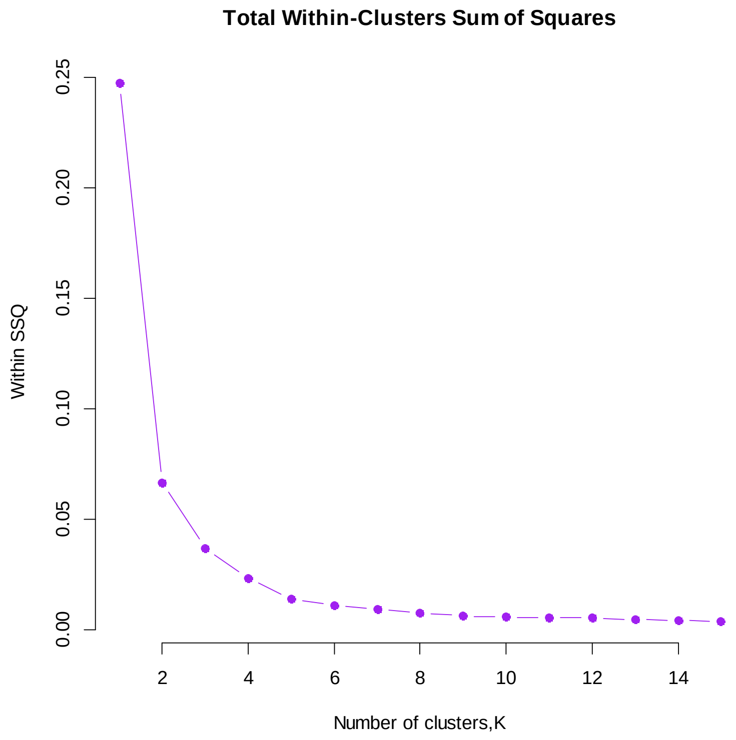
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100%

Years	Z values
2019	0.29
2018	0.1
2017	-0.88
2016	-0.24
2015	0.12
2014	-0.19
2013	0.3
2012	-0.36
2011	-1.16
2010	-0.41
2009	-1.44
2008	-1.26
2007	0.93
2006	0.99
2005	0.78
2002	-1.01
2001	0.38
2000	-0.29
1999	0.2
1998	0.7
1997	0.7
1996	1.9
1995	0.49
1994	0.33
1993	1.17
1992	0.37
1991	-0.09
1990	3.18
1989	0.41
1988	0.32
1987	0.15
1986	-0.48
1985	-0.07
1984	-0.13
1983	1.88
1982	1.48
1981	-0.03
1932	-2.12

Transition Risk – K- Means Approach

- The k-means clustering method is an unsupervised machine learning technique used to identify clusters of data objects in a dataset.
- Selection of initial number of k centroids is one of key expert judgments under this approach. Centroids are data points representing the center of a cluster.



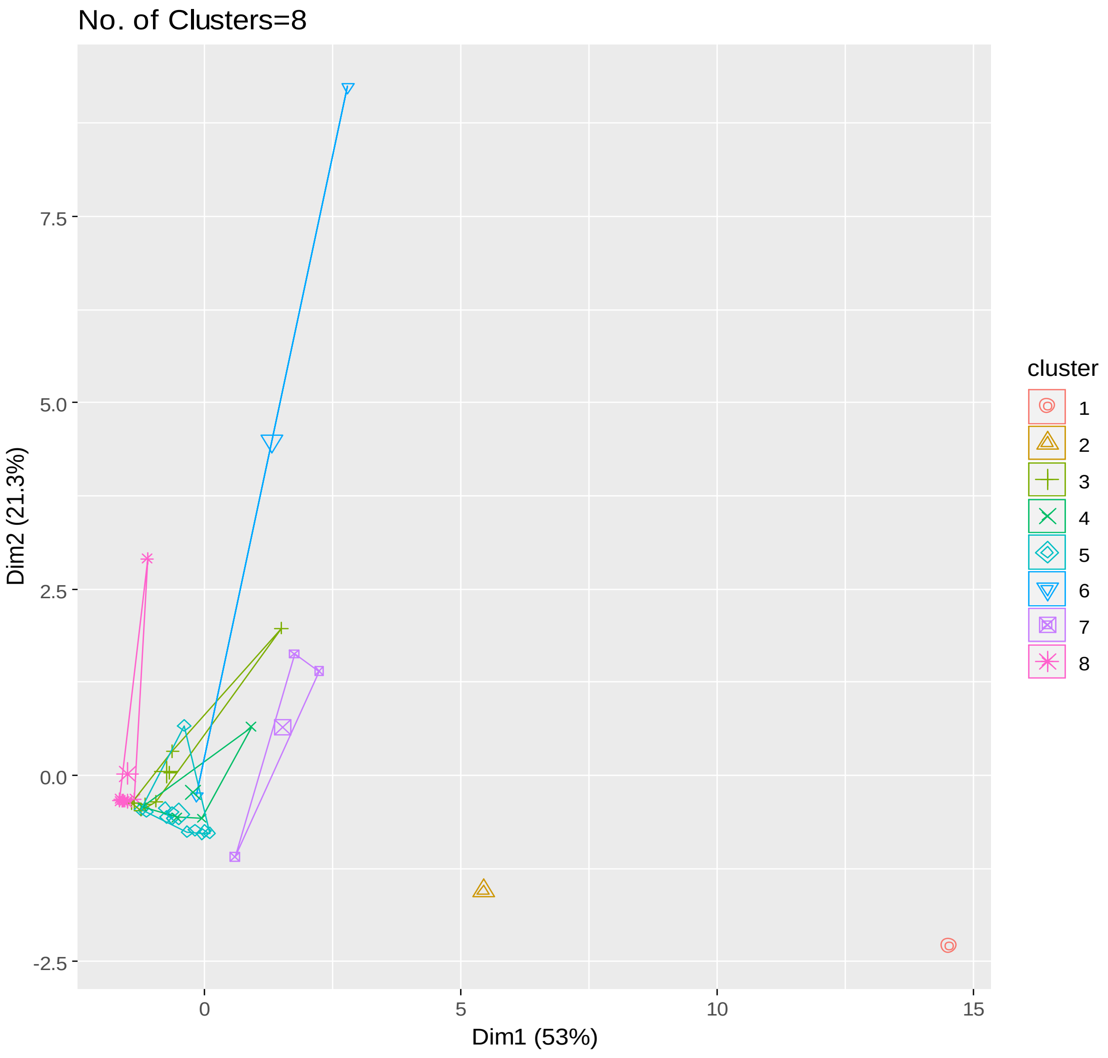


- We have used 1932 matrix, 1931-1935 average matrix and S&P transition matrices from 1981 to 2020.
- Both within and between sum of square charts suggests that around 8 to 9 groups are sufficient to explain the variation in the data.
- Improvements in sum of squares does not improve after 9 clusters.

We given equal weights to all ratings in this analysis. We have considered the following transitions for K-Means analysis:

- AA→A,AA→BBB,AA→BB,AA→B,AA→CCC,AA→D and A→BBB,A→BB,A→B,A→CCC,A→D,
- BBB→BB,BBB→B,BBB→CCC,BBB→D and BB→B, BB→CCC, BB→D.

Transition Risk – K-Means Approach



- The cluster chart is a pictorial representation of these 8 clusters. These are covered in detail in table below.
- It shows 1932 (Grp 1) and 1935 (Grp 2) are separate groups and the farthest from the rest of the groups as they are the extreme matrices that we have observed in the history.
- The next group of extreme transitions is the group which contains 1986, 2002 and 1982 transition matrices (Grp 7).
- Grp 3, Grp 5 and Grp 8 are more within the body.

Group1	Group2	Group3	Group4	Group5	Group6	Group7	Group8
1932	1935	2018	1993	2015	1987	2002	2019
		2016	1988	2012	1985	1986	2017
		2008	1984	2011		1982	2014
		2006	1983	2009			2013
		2005		2003			2010
		1997		2001			2007
		1995		2000			2004
		1992		1999			1996
				1998			1994
				1991			
				1990			
				1989			
				1981			

Model comparison

Comparison	Bootstrapping	K-means model	Vašíček	Two parameter
Replication of historic movements	Near identical replication of underlying data movements	Very close replication of underlying data movements	Poor replication of underlying data movements	Good replication of underlying data movements
1932 Backtest	Would pass a backtest if backtest level is in the historic data; but cannot produce stress worse than anything in the data	Pass by construction	Requires significant additional expert judgement strengthening to pass	Limited expert judgement strengthening to pass
Objectivity	Objective – no expert judgement	Heavy expert judgement in distribution construction	Expert judgement to strengthen to pass backtest	Expert judgement in choice of distributions and copula
Simplicity	Simple	Complex	Complex	Complex
Breadth of uses	Less appropriate for extreme percentiles as cannot produce	Highly flexible and can be set to the required use with appropriate judgements	Is used widely; but does not capture historic movements in the data well	Flexible model for a range of uses; additional parameters to Vasicek allows better replication of historic data movements