Royal College of Physicians, Edinburgh

Robust mortality forecasting in the presence of outliers

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Overview

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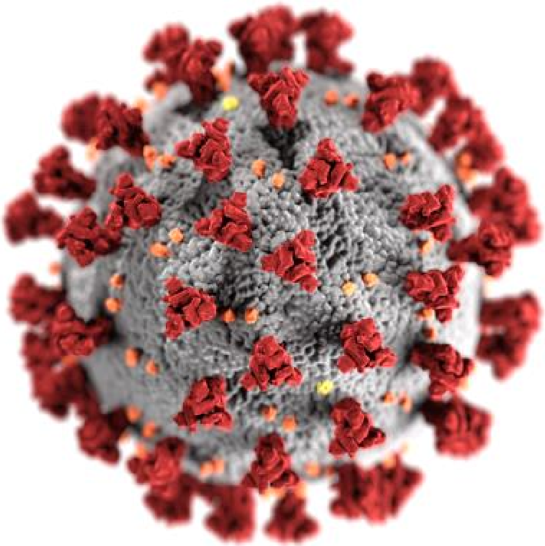
1. Motivation
2. Some alternative methods
3. Univariate forecasts
4. Multivariate forecasts
5. 2D P-spline model
6. Conclusions

1 Motivation



1 Male deaths, England & Wales

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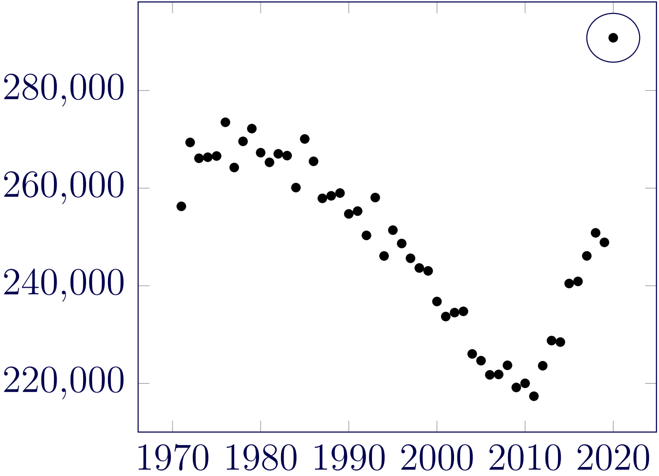
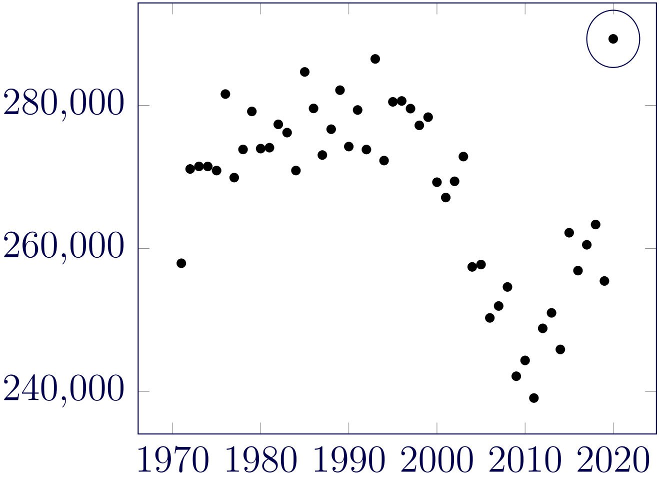


Source: Richards [2024, Figure 1(b)].

1 Female deaths, England & Wales



Source: Richards [2024, Figure 1(d)].

1 Three problems for actuaries

Outliers:

1. Distort forecasts.
2. Biase forecast starting points.
3. Inflate variance, and thus VaR capital.

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Covid-19 distorts central projections...

Using data to 2019

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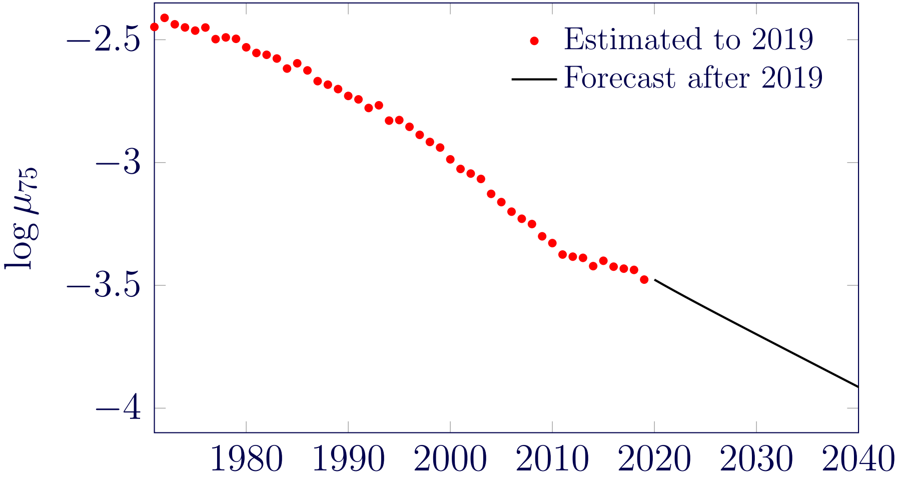
ARIMA forecast of time index in Lee-Carter model:



Source: Data for males in England & Wales, ages 50–105, 1971–2019.

Using data to 2020

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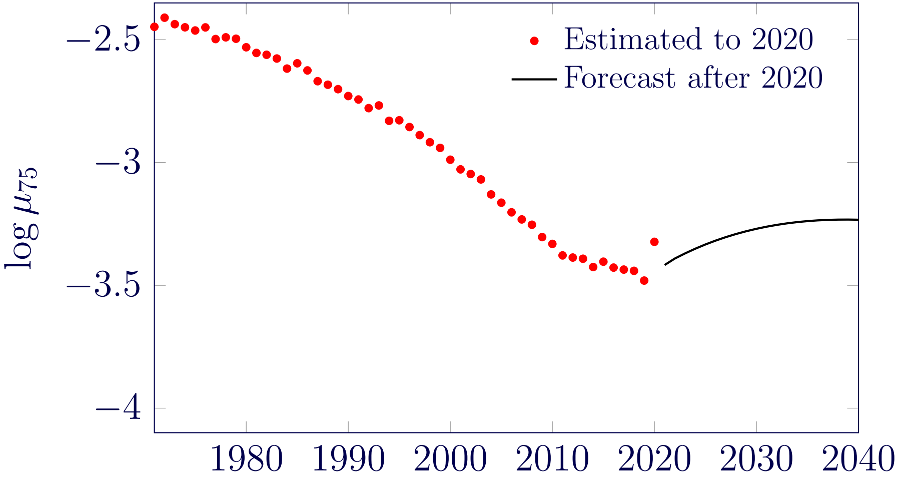
ARIMA forecast of time index in Lee-Carter model:



Source: Data for males in England & Wales, ages 50–105, 1971–2020.

1 Biased starting points

Covid-19 leads to biased starting points. . .

Using data to 2019

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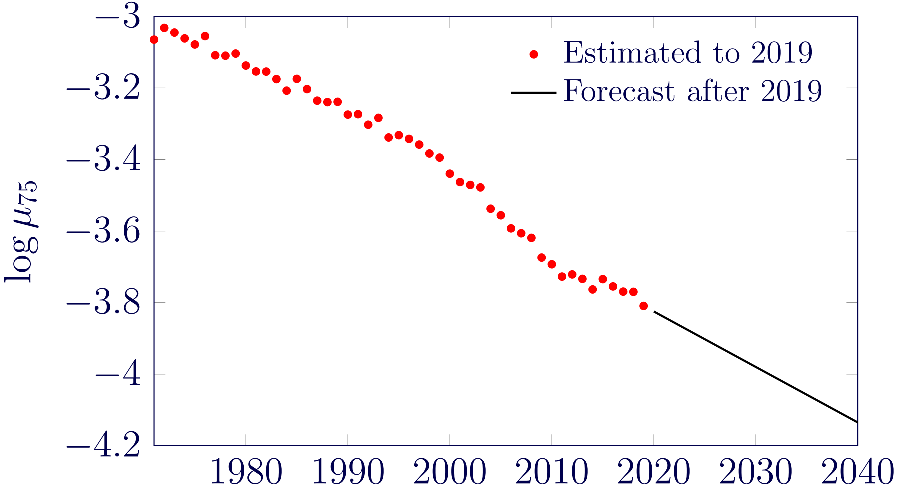
Bivariate random-walk forecast under M5 model:



Source: Data for females in England & Wales, ages 60–105, 1971–2019.

Using data to 2020

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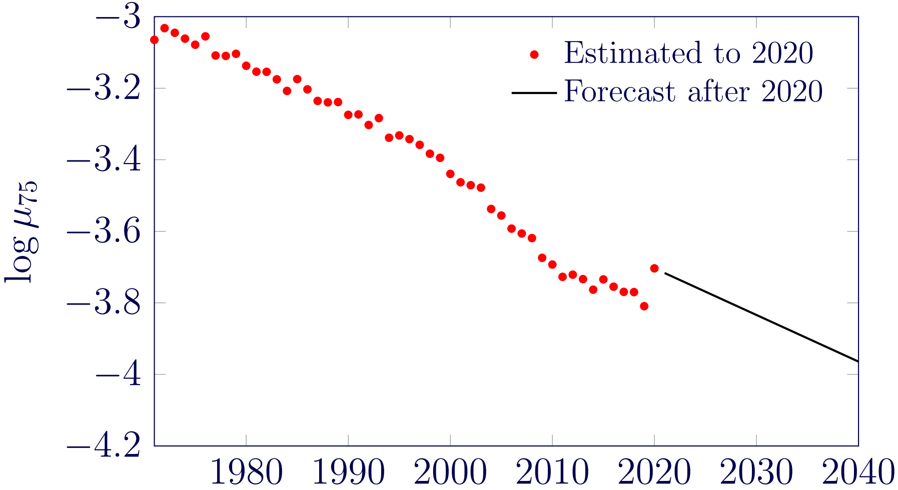
Bivariate random-walk forecast under M5 model:



Source: Data for females in England & Wales, ages 60–105, 1971–2020.

1 Inflated variance

Outliers increase VaR capital requirements. . .

Value-at-risk capital at 99.5%

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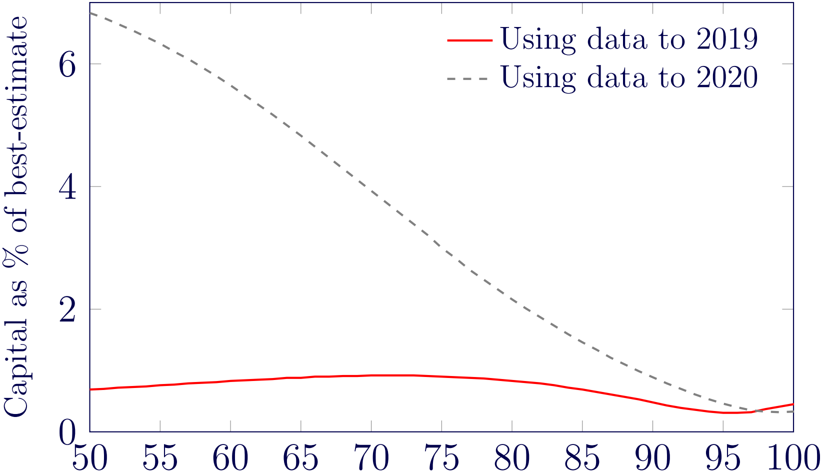

Source: 10,000 recalibrations of Lee-Carter model using data for males in England & Wales. Annuity cashflows discounted at 0% per annum.

1 Problem

- Covid-19 breaks forecasting models in three important ways.
- How can we robustify forecasts for actuarial tasks?

1 Robustification requirements

1. Remove distortion in parameter estimates.
2. Calculate “clean” starting points for forecasts.
3. Estimate variance robustly.
4. Need objective methodology for (1)-(3) to allow repeated recalibration under VaR-style simulations.

1 Solution

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- Identify outliers with statistical tests.
- Co-estimate outlier effects with other parameters.

2 Some alternative methods

2 Ignoring affected data

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- Workable temporary solution just to use data to 2019.
- ✗ No longer workable in 2023.
- ✗ Affected data no longer at end of data series.

2 Weighting the log-likelihood

- ✗ Subjective in identifying outlier location.
- ✗ Weights set arbitrarily.
- ✗ Does not estimate outlier effects.
- ✗ Tells you nothing about the nature of the outlier.
- ✗ Doesn't fully protect log-likelihood against distortion and bias.

2 Negative log-likelihood

$$-\ell(z) \propto \frac{1}{2}z^2 \qquad (1)$$

- Called loss function.
- Contribution to normal log-likelihood is quadratic.
- Outliers distort quadratically.

2 Robust log-likelihoods

Early attempts at robustification replaced $-\ell$ with a function, ρ, that behaved similarly for non-outliers, but which reduced or even eliminated the contribution from outliers. See Martin et al. [1983].

2 Negative log-likelihood

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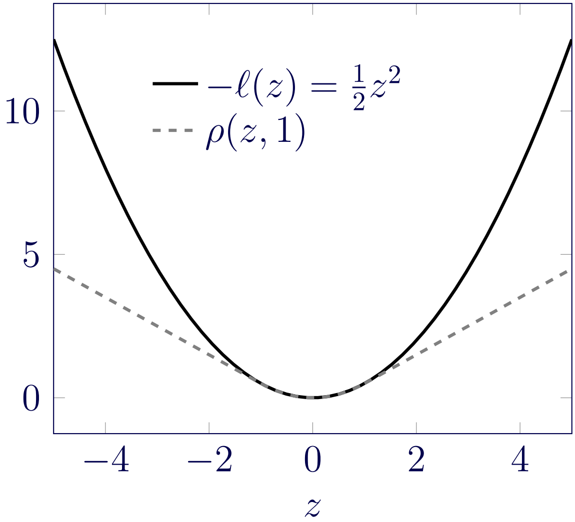

Source: Richards [2024, Figure 2].

3 Univariate forecasts

3 Univariate forecasting

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- A univariate model has multiple parameter vectors, but only one represents a time index to forecast.
- Example from Lee and Carter [1992]:

$$\log \mu_{x,y} = \alpha_x + \beta_x \kappa_y \qquad (3)$$

- $\hat{\alpha}_x$ and $\hat{\beta}_x$ are held constant in the forecast.
- An ARIMA model is fitted to the $\hat{\kappa}_y$ time index to forecast the trend.

3 Outlier definition

Outlier

An observation that is further from the one-year-ahead forecast than is consistent with the noise variance.

3 Four outlier types

IO	Innovation outlier
AO	Additive outlier
TC	Temporary change
LS	Level shift

3 Illustrating outlier types

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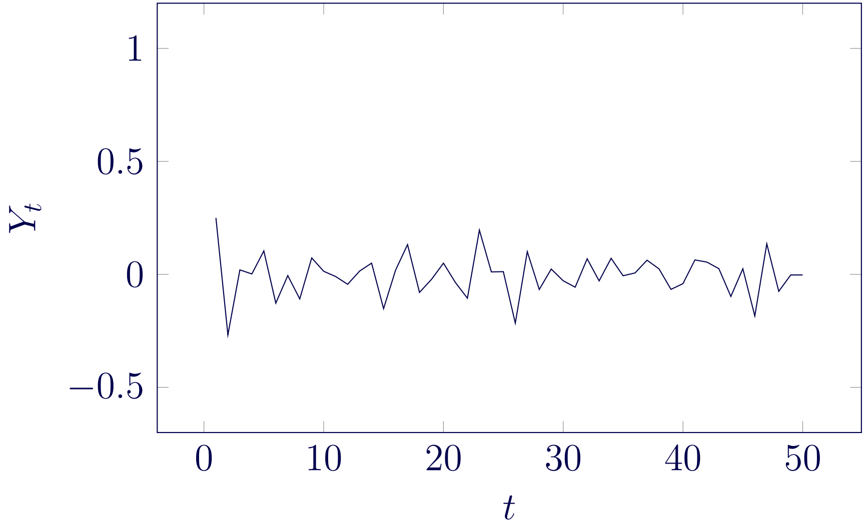
Consider a moving-average (MA) process:

$$Y_t = \epsilon_t - 0.8\epsilon_{t-1} \qquad (5)$$

3 Uncontaminated MA process



Source: Richards [2024, Figure 3].

3 IO — Innovation Outlier

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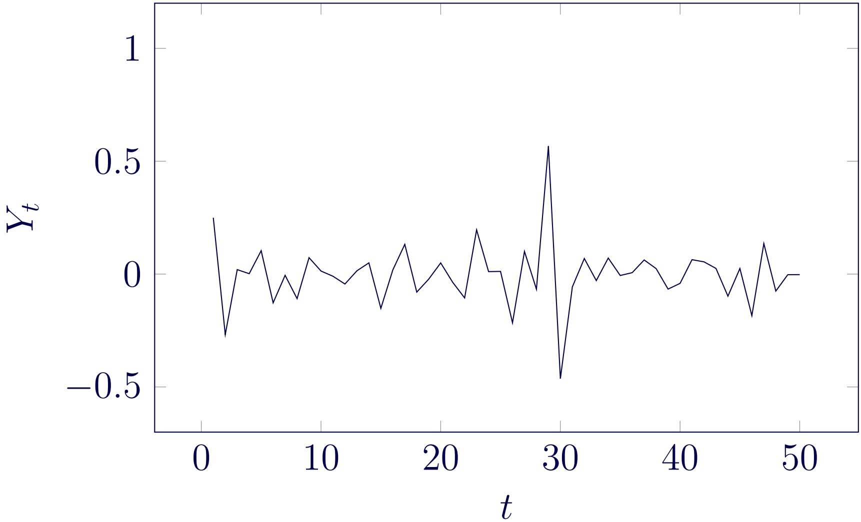


Source: Richards [2024, Figure 3].

3 Innovation outlier

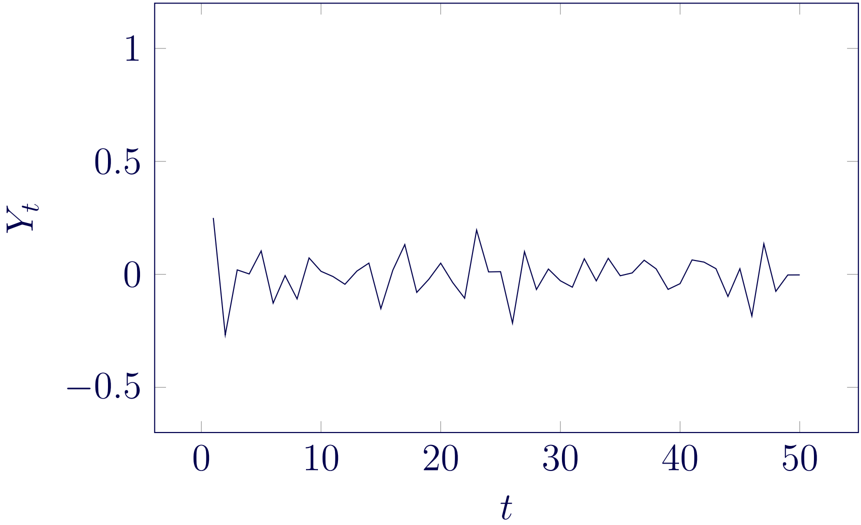
A modest outlier that is nevertheless integrated into the process.

Example

A year with heavy winter mortality due to influenza, possibly with lighter mortality the following year.

Handling: leave alone.

3 Uncontaminated MA process



Source: Richards [2024, Figure 3].

3 AO — Additive Outlier

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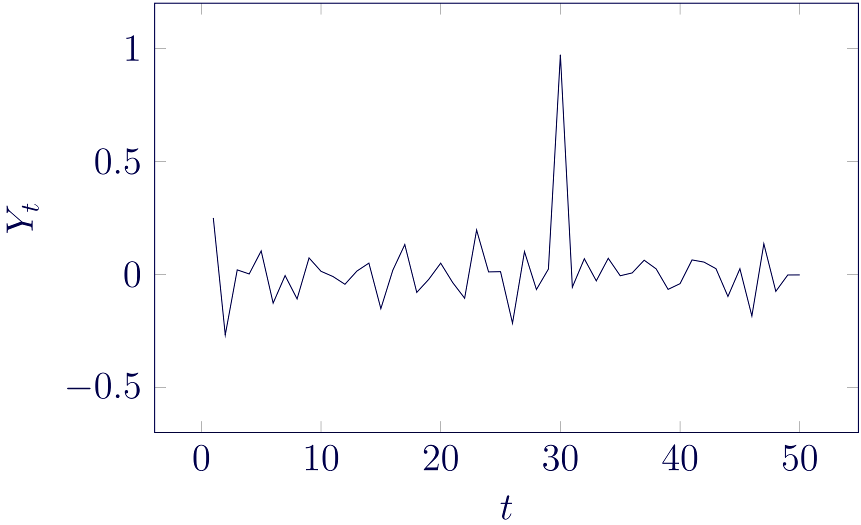


Source: Richards [2024, Figure 3].

3 Additive outlier

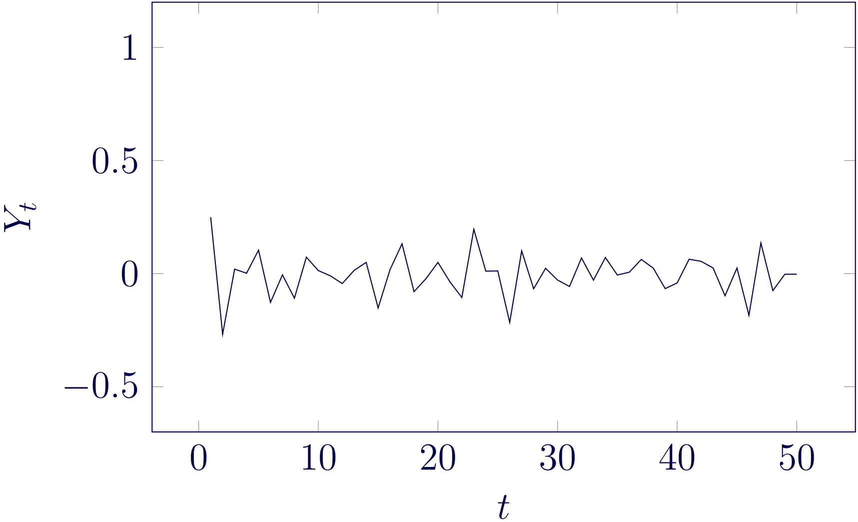
A more extreme outlier that is not integrated into the process.

Example

War or pandemic in a single year.

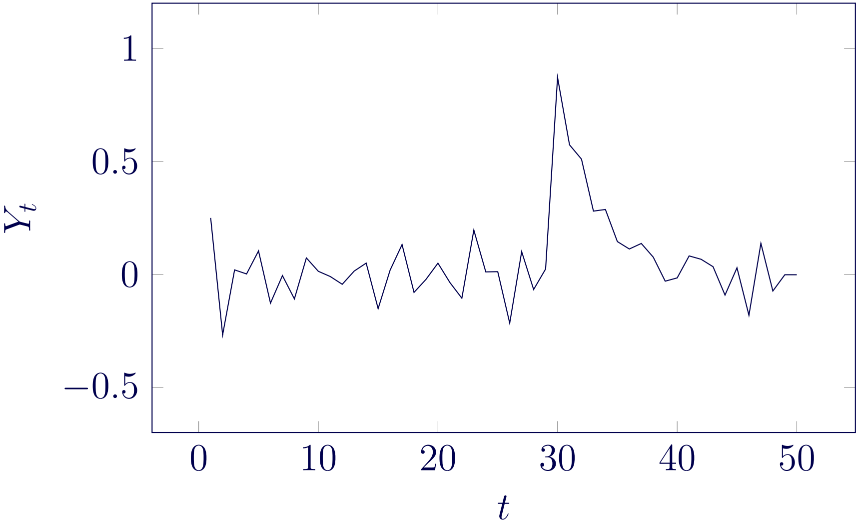
Handling: co-estimate the outlier effect to remove bias in other parameters.

3 Uncontaminated MA process



Source: Richards [2024, Figure 3].

3 TC — Temporary Change



Source: Richards [2024, Figure 3].

3 Temporary change

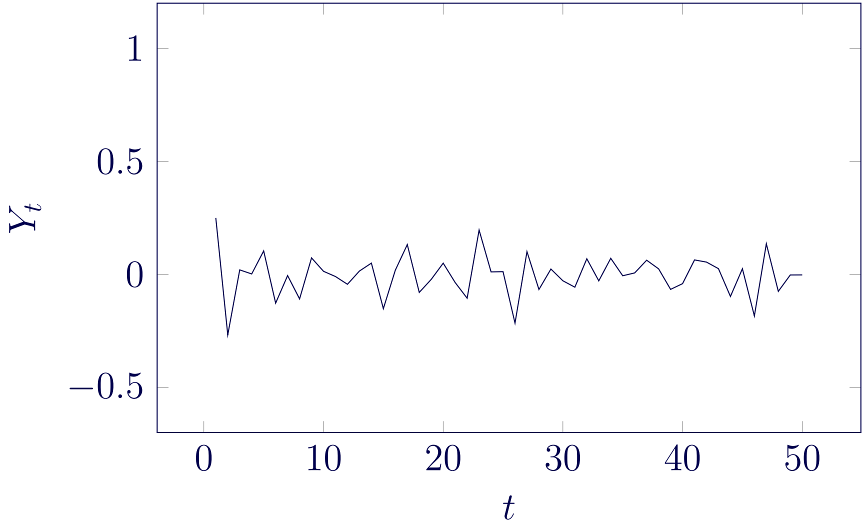
Two or more consecutive outliers that are not integrated into the process.

Example

War or pandemic spread over more than one year.

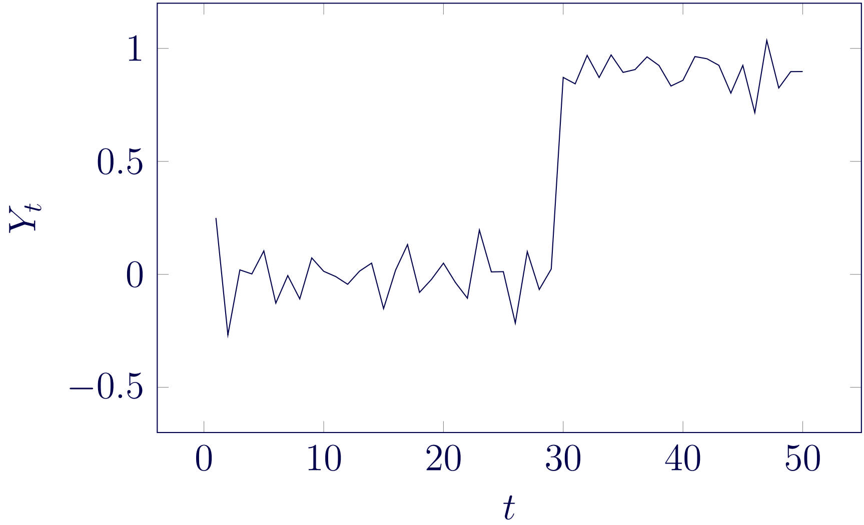
Handling: co-estimate the outlier effects to remove bias in other parameters.

3 Uncontaminated MA process



Source: Richards [2024, Figure 3].

3 LS — Level Shift



Source: Richards [2024, Figure 3].

3 Level shift

Permanent change in level of process.

Example

After German reunification in 1990, old-age mortality in East converged rapidly on levels in the West [Grigoriev et al., 2021].

Handling: review model or data period.

3 Univariate outliers

To robustify an ARIMA model, Chen and Liu [1993]:

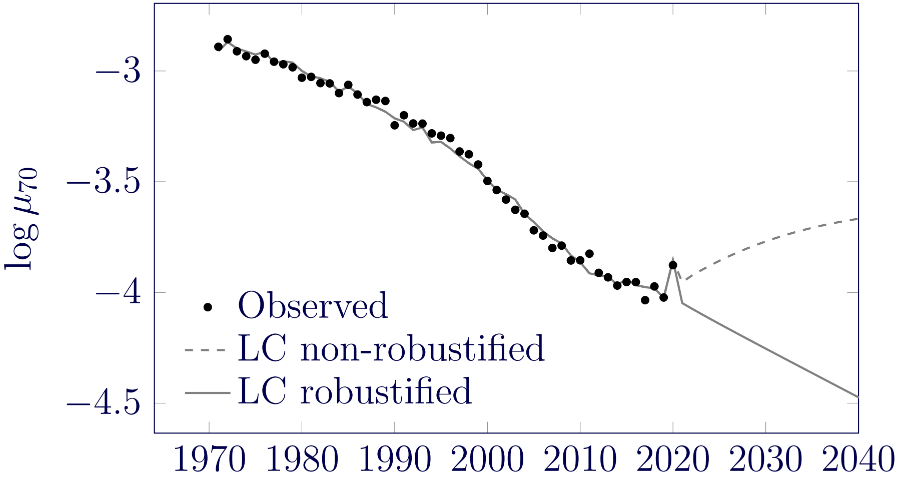
1. Proposed test statistics to identify outliers.
2. Proposed further test statistics to *classify* outliers.

3 Chen and Liu [1993]

Note

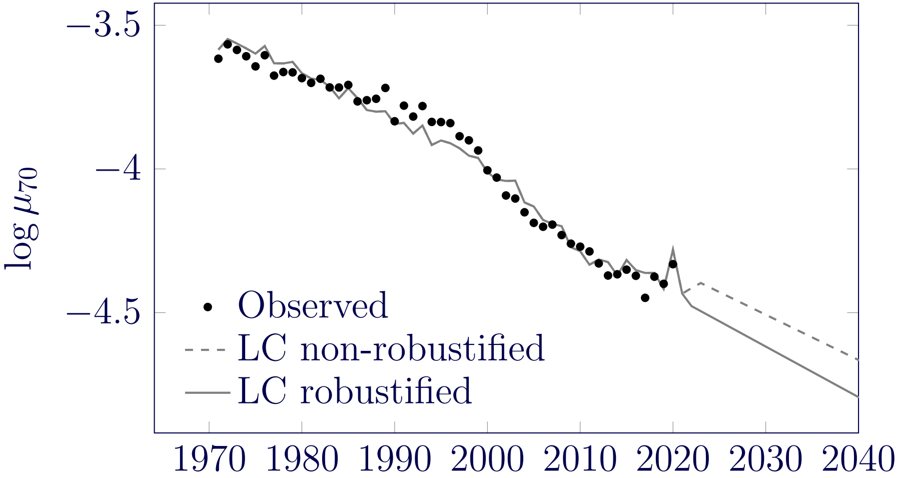
An outlier can be detected anywhere, but "it is impossible to empirically distinguish the type of an outlier at the very end of a series" [Chen and Liu, 1993, page 286].

3 Lee-Carter forecast — males



Source: Richards [2024, Figure 4(b)].

3 Lee-Carter forecast — females



Source: Richards [2024, Figure 4(a)].

3 Chen and Liu [1993]

Objective procedure allows for:

1. Identification of outlier location,
2. Classification of outlier type,
3. Estimation of outlier effect,
4. Calculation of "clean" starting point, and
5. Unbiased and undistorted forecast parameters.

4 Multivariate forecasts

4 Multivariate forecasting

- A multivariate model has two or more time indices.
- Example from Cairns et al. [2006]:

$$\log \mu_{x,y} = \kappa_{0,y} + (x - \bar{x})\kappa_{1,y}$$

- $\hat{\kappa}_{0,y}$ and $\hat{\kappa}_{1,y}$ are forecast jointly as a bivariate random walk with drift.

4 M9 example, $\hat{\kappa}_{0,y}$

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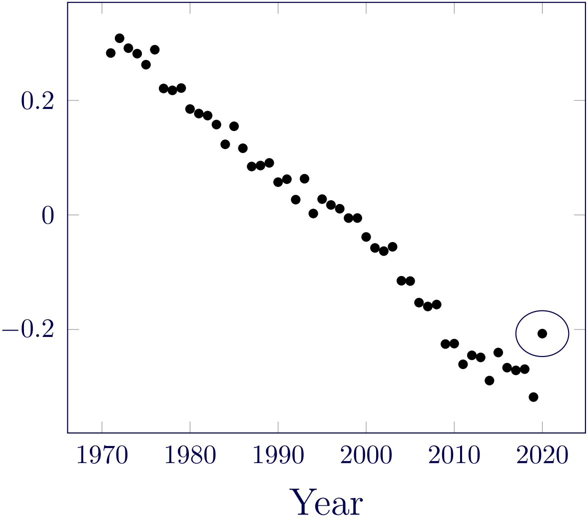

Source: Richards [2024, Figure 6, left panel].

4 Multivariate robustification

Various ways to identify outliers in multivariate data:

- Mahalanobis distance.
- Hadi [1992], Hadi [1994].
- Galeano et al. [2006].

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- CBD models project $\boldsymbol{\kappa}$ using a p-dimensional random walk with drift.
- Calculate first difference $\boldsymbol{z}_j = \Delta\boldsymbol{\kappa}_j$.
- Then $\boldsymbol{z}_j$ has a multivariate normal distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$.

4 Mahalanobis distance

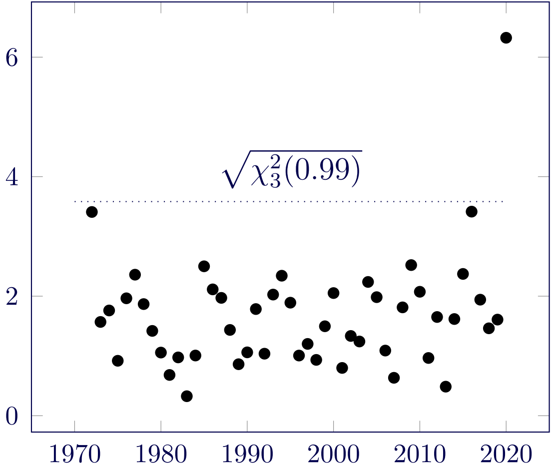
$$D_j = \sqrt{(\boldsymbol{z}_j - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{z}_j - \boldsymbol{\mu})} \qquad (14)$$

$$D_j^2 \sim \chi_p^2$$

.

4 Mahalanobis distance

Mahalanobis distance for $\Delta\hat{\kappa}$ for M9.



Source: Richards [2024, Figure 8].

4 Mahalanobis distance

"Mahalanobis distance is not robust, as it is affected by masking and swamping" Hadi et al. [2009]

Masking

An outlier is hidden because it inflates the estimate of variance used to detect outliers.

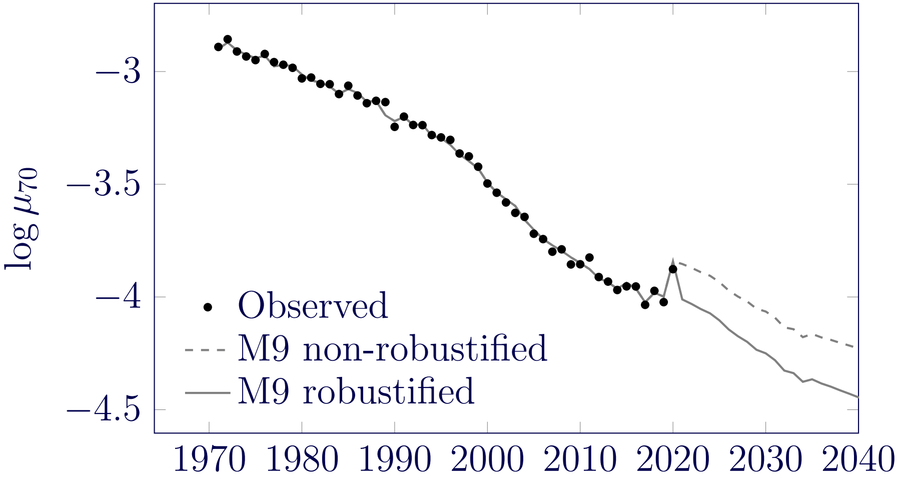
Swamping

Non-outliers have a large Mahalanobis distance because an outlier has distorted the mean of the process.

4 Multivariate robustification

- Galeano et al. [2006].
- Use projection pursuit.
- See Richards [2024, Appendix C] for details of application to multivariate mortality indices using R.

4 Robust M9 forecast



Source: Richards [2024, Figure 9(b)].

5 2D P-spline model

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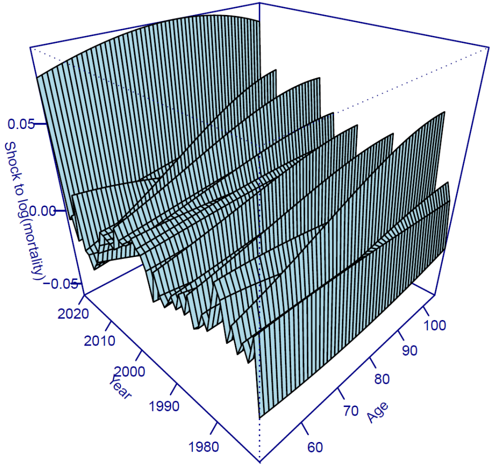
5 2D P-spline model

- Introduced by Currie et al. [2004].
- Extended by Kirkby and Currie [2010] to estimate period shocks...

5 Period shocks (females)



Source: Richards [2024, Figure 10].

5 2D P-spline model

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- Figure 10 identifies every period effect.
- Kirkby and Currie [2010] implemented scaling to dampen minor period effects.
- Figure 12 shows period effects with optimised scaling.

5 Scaled period shocks (females)

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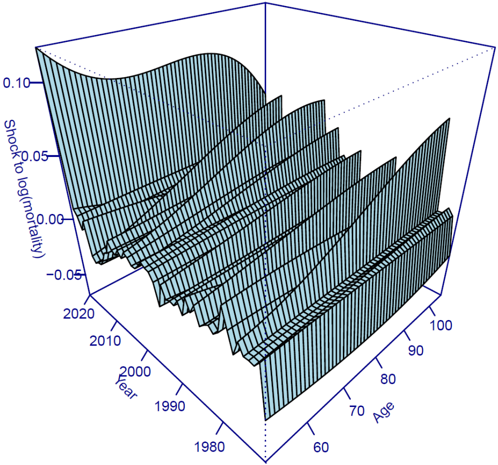

Source: Richards [2024, Figure 12].

5 2D P-spline model

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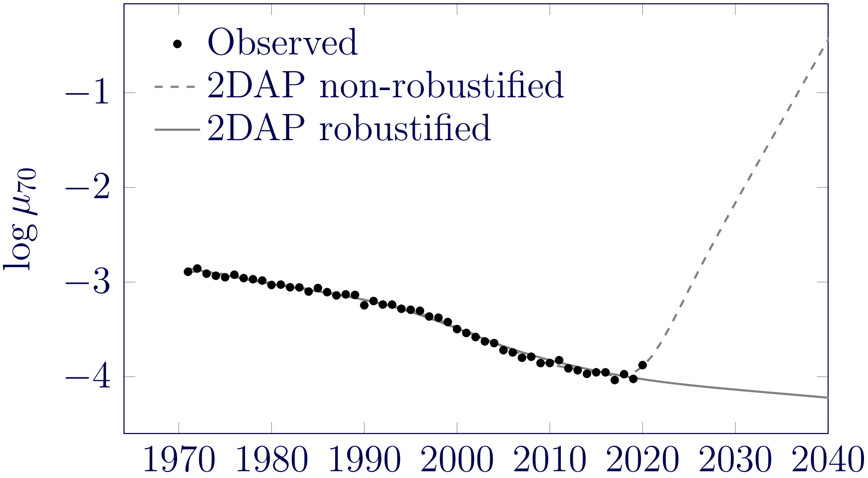
- Forecast using *penalty function*.
- Estimating period shocks robustifies penalty forecast...

5 Robust 2DAP forecast



Source: Richards [2024, Figure 13(b)].

6 Conclusions

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6 Conclusions — I

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Co-estimation of outliers and parameters:

1. Reduces bias in forecasting parameters.
2. Yields better starting points for forecasts.
3. Reduces variance in capital requirements.

6 Conclusions — II

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Univariate forecasting

- Lee-Carter and APC models.
- Use approach of Chen and Liu [1993].

Multivariate forecasting

- Cairns-Blake-Dowd & Tang-Li-Tickle models.
- Use approach of Galeano et al. [2006].

2D *P*-spline

- Currie-Durban-Eilers model.
- Use approach of Kirkby and Currie [2010].

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Coronavirus graphic from CDC

More on robust forecasting at www.longevitas.co.uk/robust-forecasting

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