Non-Life Share Price Valuation using Company Financials and Other Methods

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Abstract

This paper focuses on the estimation of the "theoretical" value of a company's share price using methods that might either be used by Sell Side Equity Analysts or those that an Actuary might employ.

A number of different methods have been explored; Dividend Discount Model, Economic Value Added, Free Cash Flow to Equity, Valuation Multiples and Appraisal Values. For each there is a look at how they are implemented, be it Top Down or Ground Up, a discussion of the advantages and disadvantages of each, and how to reconcile the different approaches. This is limited to methods based on company data and related approaches and does not look at what value the market might place on the price of share at any point in time.

Some of the issues associated with Top Down ROE Models are explained using numerical examples. These often highlight the need for further justification in the approaches and assumptions chosen by Equity Analysts in some valuations especially important as these types of model are often sensitive to small changes in key input assumptions.

For the Ground Up approach there is a need to project company financials and this is discussed in detail. The same suite of models being used to project of future capital and solvency ratios as an integral part to any share price and financial evaluation of a company.

Whereas the above has focused on the numerator in the valuation calculation the other key input is the denominator, the risk discount rate. As the Capital Asset Pricing Model ("CAPM") is used often as a basis for discounting risky cashflows it is discussed in some detail. Some of the parameterisation issues, sometimes probably not even realised, in CAPM's use are explored using outputs from a simple model.

Finally there is a detailed Equity Valuation example bringing everything together. The ideas are illustrated throughout with many numerical examples.

Contents

1. Introduction

1.1 Prologue

Newton's Theory of Gravity – a perspective

"The conventional wisdom is that the Apple fell to the Earth under gravity. Now in problem solving it sometimes pays to consider the complete opposite i.e. the Earth moves towards the Apple which is stationary. As one learnt in A Level Mechanics/Applied Mathematics the answer is a bit of both with each moving towards each other. The relative movements dictated by their respective masses so you don't notice the earth moving."

Source: Radio 1, one Tuesday / Thursday night in 1975 with my mother driving me home from a twice-weekly Leicester City schoolboy training session at Belvoir Drive.

This has always remained with me as a useful framework, in particular thinking about what people don't say or write rather than what they do instead.

1.2 Paper Overview

Tabe 1.1 is an overview of the contents of this paper.

No.	Topic	Description
2	A Unifying Framework	A simple diagram that shows how the different components fit together. This covers (i) Initial Capital output at t = 0, (ii) Projection of Company Financials, (iii) Solvency Projections, (iv) Impact of Scenarios and (v) Performance Measurement e.g. EVA, ROE.
3	Different Equity Valuation Models	The high-level approaches of Top Down ROE and Ground Up models have been explored from the perspective of (i) Methodology used, (ii) How implemented and (iii) Advantages and disadvantages. The approaches discussed are Dividend Discount Model, Economic Value Added, Free Cash Flow to Equity, Valuation Multiples and Appraisal values
4	Top Down ROE Models – Some Issues	Some of the issues associated with using Top Down ROE models are explained using numerical examples. Eight specific examples are discussed.
5	Projection of Company Financials	This covers the projections of the (i) Profit and Loss Account, (ii) Balance Sheet, (iii) Cashflow Statement and (iv) Gross / Net Reserves with a discussion of some of the considerations involved.
6	Solvency Ratio Forecasting	The different regulatory solvency capital projections models are grouped into four types. A simple example based on the SII SF SCR is explored. In addition there is an ORSA scenario where the impact on the financial statements, solvency capital and solvency ratios is explained. One may argue that including this is perhaps beyond the intended scope of the paper but its indirectly relevant given that projection methods implicitly assume that an insurance company's solvency ratio risk appetite will always be met.
7	Risk Discount Rate	As the Capital Asset Pricing Model ("CAPM") is used a lot as a basis for discounting risky cashflows it is covered. Some issues in CAPM's use in are discussed using outputs from a simple model.
8	Equity Valuation – Bringing it all together	This is an Equity Valuation using a Ground Up approach. Also shown is a reconciling of the Ground Up and Top Down ROE Models. Different Ground Up methods are presented.. A standard Valuation Summary with values for key metrics is shown.

Table 1.1

This paper goes over and beyond one that just looks at the share price valuation aspect in isolation. The methods used (3 and 4), projection of future company financials (5), projection of future solvency ratios (6) and the risk discount rate (7) are all interrelated.

I have tried to make sections 5, 6 and 7 self-contained as far as possible so each in themselves might be a useful reference point for the reader.

1.3 Background

Some History

I first became interested in the topic in 1997 after coming across Goldman Sachs ("GS") Dynamic ROE model and their paper. This was what I call a "Top Down ROE" model. It was a two period Economic Value Added ("EVA") model where the Return on Equity ("ROE") <> Cost of Equity ("COE") for the first N years and for time period (N+1) and thereafter the ROE = COE in a "Steady State" condition[1].

Why this Paper

This paper was initially meant to be one focused on multi-year Solvency ratio forecasting under various scenarios e.g. those typically used in a UK insurance company's Own Risk Solvency Assessment ("ORSA"). During August 2022, after a few day's work on a model I had the idea of quantifying the impact on a "theoretical" Share Price for each scenario. I thought that this would be key information for any Board of an insurance company.

Then during the Autumn of 2022 as I was implementing Goldman Sach's Dynamic ROE methodology some questions began to surface, (i) What do the P&Ls look like, (ii) Are they reasonable, (iii) How do the annual premiums increase year after year, (iv) What does the Combined Ratio and Premium Growth look like when ROE = COE, (v) Why can't the ROE fall below the COE in the "Steady State" condition.

After reviewing some Insurance Equity Analyst reports in 2022 I came to the conclusion that perhaps something more interesting could be said if I focused on the Share price valuation side of the work instead, hence this paper.

1.4 The concept of a Theoretical Share Price

The "theoretical value" or "fair value" of a share price is a rather ambiguous expression as some of the models, which we will see later, can be very sensitive to modest changes in the input assumptions, or involve model inputs that are very subjective. This leads to situations where it is possible to produce for a company a wide range of share prices for different sets of parameters that may each appear reasonable and justifiable to the reader.

Nevertheless I think it important to understand what a theoretical "fair value" would be at any point in time, in particular when benchmarking company performances and evaluating buying or selling company share opportunities. This becomes important when assessing insurance company start up valuations in recent years and cutting through some of the sales talk.

The words "Theoretical share price" and "Target share price", so often used in Equity Analyst reports, is one and the same for the purposes of this paper.

[1] I do not know whether this approach is still being used by GS.

1.5 Intended Audience

This paper is aimed at the following audience:

Who	Reason Why
Non-Life / Investment Actuaries	Valuation of Non-life insurers and performance metrics
Equity Analysts (Sell side)	Equity Analyst reports for investment clients. Framework to relatively rank Non-Life insurers
Investment Managers (Buy side)	Buy / Sell decisions, identification of potential mismatches between current and theoretical share prices. Framework to relatively rank Non-Life insurers
Insurance CRO / Risk Management	Projection of future financials, capital, solvency ratios under both best case and scenario assumptions
Board / Senior Management	Performance metrics, medium-longer term decision making.
University / MBA Students	Valuation of companies in general and wanting to understand the nuances of Non-Life insurance.

Table 1.2

1.6 Approach and History

I have approached this topic from first principles, using my years of experience in the development of multi-year projection models and time spent briefly as a Sell side Equity Analyst at an investment bank.

The first models that I developed in this area stretch as far back as1984 (nineteen eighty four). I was fortunate enough to work under Alan Spence FIA at Royal London Mutual where the small actuarial team developed a suite of menu driven models for the Life, Pensions and General Insurance sides of the business, not in spreadsheets, but in Basic / Basic A within MS DOS. I based my original 10-year financials planning forecast model, with input from Alan, on the paper by W.M. Abbott, T.G. Clarke and W.R. Treen (1981). Some Financial Aspects of a General Insurance Company. Journal of the Institute of Actuaries.

Indeed the many models that I have developed since have their origins from that time.

1.7 Existing Literature

There does seem to have been an historical shortage of GIRO papers / workshops on Equity valuation. This surprises me given the Board / Senior Management attention to company share prices. The following is a list of some of the literature that I have consulted in the course of writing this paper. It is not meant to be exhaustive:

- W.M. Abbott, T.G. Clarke and W.R. Treen (1981). Some Financial Aspects of a General Insurance Company. Journal of the Institute of Actuaries
- J.P. Ryan, and W.P. Larner (1990). The Valuation of General Insurance Companies. Journal of the Institute of Actuaries
- G. Warren (1997). GIRO 1997 Workshop. An Investment Analyst Values a European Insurance Company.
- R. Rodriguez, R. Bland, G. Fulcher, R. Kelsey, S. Laird and R. Shaw (2000). GIRO paper and Workshop. Shareholder Value Measures in General Insurance Working Party
- R. Goldfarb (2005). CAS Exam 8 Study Note: P&C Insurance Company Valuation
- S. Dias S, F. Giovanni, R. Burden and K. Gill. (1999). Dynamic ROE Model Update. Goldman Sachs Investment Research.
- R. Shaw (1993). Optimum Portfolio Selection Methods. Bournemouth and Norwich Actuarial Societies, March, 1993. Institute of Actuaries.

2. A Unifying Framework

Figure 2.1 below shows some of the inter relationships that exist within an insurance / reinsurance company. Future year financial projections of the P&L, Balance Sheet and Cashflow statements will be needed for the assessment of future solvency ratios, performance metrics and theoretical shares prices.

Business Plan / Capital relationships

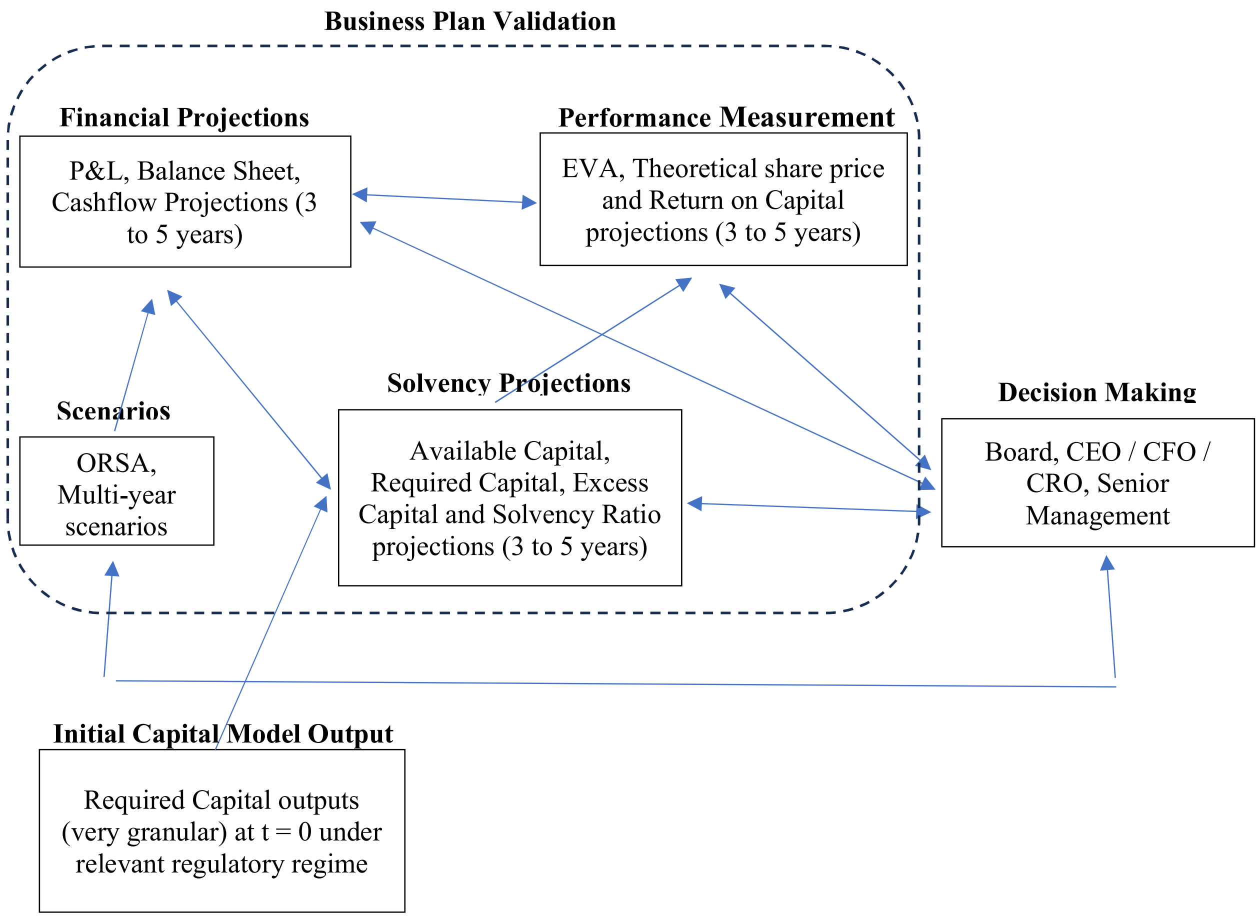

Figure 2.1

3. Different Equity Valuation Models

3.1 Introduction

Section 3 provides an overview of the main equity valuation methodologies.

Two key themes are explored:

- The Calculation Basis – e.g. Economic Value Added ("EVA"), Dividend Discount Model ("DDM"), Free Cash Flow to Equity ("FCFE")…..etc..
- How have these been implemented – I have decided to distinguish between what I have termed "Top Down ROE" vs "Ground Up" approaches.

Top Down ROE:

- These are models where the inputs are variables such as the Return on Equity ("ROE") and Net Asset Value ("NAV") growth rates.
- The same values are in perpetuity or values that vary over different projection periods e.g. two periods or more. There is an example below:
- The model origins are non-industry specific and so may not capture the nuances of Non-Life insurance companies.

An example of such input assumptions can be seen in Table 3.1.

ROE Basis	Period	ROE	Dividend %	Dividend	NAV Growth	ROE - k
1	1 - 5	18.0%	50.0%	9.0%	9.0%	7.0%
2	6 - 15	14.0%	50.0%	7.0%	7.0%	3.0%
3	TV (16+)	12.5%	50.0%	6.3%	6.3%	1.5%

1st Period	5
k initial	11.0%
k increment	0.00%
NAV_0	1,000

Table 3.1

In this example k is the Cost of Capital. It's value through the valuation period is a level 11% p.a. The "k increment" is an option when thinking about a varying Cost of Capital by year. This is discussed further in section 7.6.

Ground Up:

This is where the projected Net Income (after tax, but before dividend) for each year is directly calculated from the constituent parts including:

- Gross written premium, ceded written premium, gross and net premium earnings patterns
- Gross and ceded loss ratios perhaps broken down into loss types of attritional, large and catastrophe
- Gross and ceded acquisition cost %, administration expenses etc
- Investment return assumptions
- Dividend payables etc.

This may or may not involve the projection of the associated balance sheets and cashflow statements.

3.2 The different models covered

The different models considered in this paper are:
- Dividend Discount Model
- Economic Value Added
- Free Cash Flow to Equity
- Valuation Multiples
- Appraisal Values

Within each of these methods the following topics are discussed:
- Methodology
- How implemented
- Advantages
- Disadvantages

Table 3.2 below shows how each of the five methods are usually modelled. The approaches are viewed from the perspective of (i) Calculation Bases (Top Down ROE vs Ground Up) and (ii) How implemented (Cashflow or Earnings based).

Method	**Top Down ROE**	**Ground Up**	**Basis**
Dividend Discount Model	Yes	Yes	Cashflow
Economic Value Added	Yes	Yes	Earnings
Free Cashflow to Equity	No	Yes	Cashflow
Valuation Multiples	Yes	No	Earnings
Appraisal Values	No	Yes	Earnings

Table 3.2

In this paper I have tended to focus more on the Dividend Discount Model and Economic Value Added Model, one being a popular cashflow based model and the other an earnings based model. A full discussion of all five approaches along the lines of the four topics can be found in the appropriate sections of this paper.

3.3 Why Ground Up over Top Down ROE models

I prefer the Ground Up over Top Down ROE approaches:

Advantages:
- Greater transparency
- More intuitive as the modelling reflects insurance risk drivers e.g.. GWP growth, premium earnings patterns, loss ratios etc.
- The future P&Ls are modelled – The Top Down ROE approach doesn't tell you what the future P&Ls look like beyond t=1
- Balance sheets are projected – Needed for assessing future capital needs
- More meaningful sensitivity tests as changes in insurance risk drivers are directly modelled e.g. net/gross premium ratios, reserve deterioration etc.
- Minimises the risk of unreasonable models – See section 4. "Top Down ROE Models – Some Issues".

Disadvantages:
- More data inputs as values are needed for future periods
- Translating the detailed modelling outputs into simple and understandable metrics e.g. ROE, NAV growth over different timeframes, Price / NAV and P/E ratios.
- More time consuming
- Doesn't lend itself easily to a quarterly roll-forward where say the ROEs assumptions may be preserved.

3.4 Valuation Methods

This section reference "R. Goldfarb (2005). CAS Exam 8 Study Note: P&C Insurance Company Valuation".

There are two types of valuation that reflect the stakeholder interests of the equity shareholders and debt holders:
1. Equity Value – Value of Equity = Share price x Number of shares
2. Enterprise Value = Equity Value + Value of Debt

If the valuation is based on determining the Enterprise Value then the Equity Value is calculated by subtracting the market value of debt.

One tends to find that for each of the main valuation types there are variants for either Equity or Enterprise Value models:

Cash Flow based methods
1. Equity Value
    - Dividend Discount Model or
    - Free Cash Flow to Equity

2. Enterprise Value
    - Free Cash Flow to company

<u>Dividend Discount Model ("DDM")</u>
The valuation based on the present value of future dividends is relatively straightforward however the difficulty arises because one also has to determine the percentage increases in dividends, independent of any other modelled dynamics of a company. Furthermore, dividend payments are discretionary. This limitation can be overcome by thinking first of all projecting the underlying drivers of the dividend e.g. net income after tax and then letting the dividend payment be a function of this.

<u>Free Cash Flow</u>
There are two approaches, one that considers the Free Cash Flow to the company (shareholders and debt holders) and one that considers Free Cash Flow to the equity holders.

Free Cash Flow to Equity holders – The valuation is the present value of cashflows that could in theory be paid out as dividends after adjustments to reflect amounts needed to support the capital expenditure and future growth. The discounting is done using a discount rate that reflects the risk to equity holders only.

Free Cash Flow to Company – This first of all values the total company by discounting the Free Cash Flows to equity and debt holders and then the market value of debt is subtracted from the resultant figure to derive the equity portion. The discounting is done using a different rate, one that reflects the overall risk to both equity and debt holders, the Weighted Average Cost of Capital ("WACC").

Earnings based methods

3. Equity Value
    - Economic Value Added

4. Enterprise Value
    - Economic Value Added

The Economic Value Added approach uses an accounting based earnings / net profit measure of income to determine the value of a company as opposed to cash flow based methods in 1. and 2. The models focus on the capital and differences between returns and cost of capital.

The only differences between the two models 3. and 4. are the definitions of capital and the basis for the return on capital, cost of capital and the discount rate.

Equity Value basis

The Equity Value equals the opening book value (or NAV) plus the present value of future excess returns (i.e. the difference between the return on equity capital and the cost of equity capital). The discounting is done using a discount rate that reflects the risk to equity holders only.

Enterprise Value basis

The Enterprise Value equals the opening total capital stock plus the present value of future excess returns (i.e. the difference between the return on total capital and the cost of total capital). The discounting is done using a different rate, one that reflects the overall risk to both equity and debt holders, the WACC.

As with the Free Cash Flow approach, if the Enterprise Value is first calculated then the market value of debt is subtracted from the resultant figure to derive the equity portion.

Multiples

5. Equity Value
    - Price / Earnings ratio
    - Price / NAV ratio
    - Dividend yield

6. Enterprise Value
    - Enterprise Value to EBITDA
    - Enterprise Value to Capital
    - Free Cash-flow yield.

These same approaches are often used across different industries.

3.5 Dividend Discount Model ("DDM")

Methodology
The value of a share is the present value of all expected future dividends.

$$Value = \sum_{t=1}^{\propto} \frac{Dividend_t}{(1+k)^t}$$

- $Dividend_t$ = Expected Dividend during the period (t-1,t)
- k = Cost of Equity i.e. discount rate

If one assumes, as is commonly the case, that dividends increase at an annual constant growth rate of g per annum in perpetuity then the formula becomes:

$$Value = \frac{Dividend_1}{k-g}$$

<u>N stage growth model</u>
It is possible to specify a two or more stage growth model where different starting dividends and/or dividend growth rates are specified in each of these periods. The valuation of these cashflows being represented by a series of simple annuity formulas.

<u>Forecast Period and Terminal Value</u>
An alternative representation is to consider two separate modelling periods, (i) Forecast Period and (ii) Terminal Value where different assumptions are assumed to hold for t = 1 to N and for t = N+1 onwards a Terminal Value ("TV"). The DDM formula then becomes:

$$Value = \sum_{t=1}^{N} \frac{Dividend_t}{(1+k)^t} + \frac{TV}{(1+k)^N}$$

where

$$TV = \frac{Dividend_{N+1}}{k-g}$$

where g = NAV growth rate in perpetuity

How implemented
The model can be implemented either as a Top Down ROE or Ground Up model.

<u>Top Down ROE</u>
The initial dividend, or dividends (if an N stage model, N>1) are prescribed and annual growth rates thereafter. The formulae in the Methodology is an example of the Top Down ROE approach.

<u>Ground Up</u>
The ground up approach involves the projection of Net Income after Tax for each future year with the dividend being defined as

$$Dividend_t = Dividend\%_t \times Net\ Income_t$$

$$Dividend\%_t = 1 - \frac{g_t}{ROE_t}$$

where $Dividend\%_t$ = Dividend % payout ratio for period (t-1,t).

In this case it is assumed that “g” is a model input which is quite common in Equity Analyst reports. Otherwise the Dividend % could be an input with the value of g then calculated as:

$$g_t = ROE_t \times (1 - Dividend\%_t)$$

Dividend Discount Model “Bad Press” Paradox
The problem with the DDM is the prediction of future dividends based off an initial dividend and assumed future growth rate, or say differential growth rates in a two or more stage Model. Then there are complications from Share Buy Backs etc.

Given this Equity Analysts seem to turn to more so-called sophisticated approaches e.g. Economic Value Added ("EVA"), Appraisal Values, Price / NAV ratios etc.

But herein lies the Paradox. Each of these other methods rely on either explicit or implicit assumptions for future dividends so if the prediction of future dividends is flawed for the Dividend Discount Model it can’t magically right itself when used in each of these other methods.

The issue is not the DDM itself but how it is used. If a Ground Up approach is followed with projections of future Net Income after tax for each future year and assumed dividend payout ratios then the same level of sophistication would exist as with other models.

Advantages
- Models widely used and understood
- Easy to communicate
- Dividends represent the only cash-flows that are meaningful and tangible to investors as opposed to more uncertain cash flows in the Free Cash Flow to Equity method
- One can derive valuations using pencil, paper and a calculator if the assumptions are the same for a number of years
- Benchmarking e.g. initial dividend yield, dividend growth rates.

Disadvantages (if a Top Down ROE Model * is used)
- Doesn’t tell you what the implied individual P&L components look like in future years, and whether these are reasonable or not
- In the Top Down ROE approach the assumption of an initial dividend increasing at an annual growth rate is perhaps too simplistic
- Models are very sensitive to changes in the key assumptions, e.g. dividend growth rates and length of periods if more than one period is used
- Does not lend itself to meaningful sensitivity tests specified in terms of changes to the key insurance drivers e.g. gross written premium growth, loss ratios etc.

One could argue that there is a lot of uncertainty in trying to estimate future dividends using projected Net Income after Tax and an assumed dividend payout ratio in the Ground Up approach.

However, this uncertainty is present in each of the other methods too, as expected dividends are necessary in order to calculate the retained profit for the year end NAV, which will then influence the following year's premium writing.

* If a Ground Up Model is used then modelling is at the Net Income after Tax level already.

3.6 Economic Valued Added ("EVA") Model

Methodology

The method relies more directly on accounting measures of Net Income after Tax rather than cash flow methods as with the DDM and FCFE methods. However this distinction is perhaps a bit misleading if one considers that in a DDM Ground Up approach there is also a requirement to forecast the Net Income after Tax.

In the EVA model we are interested in the sum of the present value of the Economic Profit ("EP") i.e. net income after tax in excess of the cost of capital employed over each future time period (t-1,t).

$$Value = NAV_0 + \sum_{t=1}^{\propto} \frac{EP_t}{(1+k)^t}$$

The EP_t is defined as:

$$EP_t = Net\ Income_t - (k \times NAV_{t-1})$$

- EP_t = Economic Profit during the period (t-1,t)
- NAV_{t-1} = Adjusted NAV beginning of the period (t-1,t)
- k = Cost of Equity ("COE") i.e. discount rate

Net Income here being used to mean Net Income after Tax.

If a company can earn a return on capital equal to a "normal" return demanded by its shareholders then the market value of the company should equal its book value. This is similar to the idea that the market value and face value of bond are equal if the coupon rate and yield are the same.

The formula suggests that positive or negative deviations from the NAV must be due to a company's ability to earn more or less than the "normal" return demanded by shareholders.

Adjusted NAV

The NAV used is an Adjusted NAV which is derived from the GAAP NAV after various adjustments such as:

- Goodwill and other intangibles
- Unrealised Gains
- Other

How implemented
This can be implemented in one of two ways, Top Down ROE or Ground Up approach.

Top Down ROE
This uses an ROE_t as an input

$$EP_t = (ROE_t \times NAV_{t-1}) - (k \times NAV_{t-1}) = (ROE_t - k) \times NAV_{t-1} \quad \text{.... (1)}$$

$$NAV_t = (1 + g_t) \times NAV_{t-1}$$

where:
$g_t = ROE_t \times (1 - Dividend\%_t)$

The formula then becomes:

$$EVA\ Value = NAV_0 + \sum_{t=1}^{\infty} \frac{(ROE_t - k) \times NAV_{t-1}}{(1+k)^t}$$

The formulation of economic profits as (ROE – k) x NAV allows one to think of economic profits / losses in terms of an ROE > k / ROE < k.

Ground Up
The EP_t is defined as:

$$EP_t = Net\ Income_t - (k \times NAV_{t-1}) \quad \text{.... (2)}$$

$$Dividend_t = Dividend\ \%_t \times Net\ Income_t$$

$$NAV_t = NAV_{t-1} + Net\ Income_t - Dividend_t$$

N stage growth model
It is possible to specify a two or more stage growth model where different ROE and/or NAV growth rates are specified in each of these periods. The valuation being represented by a series of simple annuity formulas.

Forecast Period and Terminal Value
An alternative representation is to consider two separate modelling periods, (i) Forecast Period and (ii) Terminal Value where different assumptions are assumed to hold for t = 1 to N and for t = N+1 onwards a Terminal Value ("TV"). The EVA formula then becomes:

$$Value = NAV_0 + \sum_{t=1}^{N} \frac{EP_t}{(1+k)^t} + \frac{TV}{(1+k)^N}$$

where

$$TV = \frac{EP_{N+1}}{k - g}$$

Advantages
- Models widely used and understood
- Easy to communicate
- Identifies whether value above book value is being created and in which years according to whether ROE > k or ROE < k

Disadvantages (if a Top Down ROE Model * is used)
- Doesn't tell you what the implied individual P&L looks like in future years, and whether these seem reasonable or not
- The NAV future growth assumption, either direct or via a dividend payout % determines the future NEP growth.
- EVA models that assume that the ROE equals the COE (Price / Book ratio = 1) in the steady state condition at some point in the future provide no information on the implied combined ratio and/or NEP growth rate at that point. The same comment applies to any other ROE "steady state" assumptions
- Models are very sensitive to changes in the key assumptions, e.g. ROE, NAV growth rates and length of periods if more than one period is used
- Does not lend itself to meaningful sensitivity tests specified in terms of changes to the key insurance drivers e.g. gross written premium growth.

* If a Ground Up Model is used then the modelling is at the Net Income level already

3.7 Free Cash Flow to Equity Model

Methodology

The methodology description is taken from "R. Goldfarb (2005). CAS Exam 8 Study Note: P&C Insurance Company Valuation".

The FCFE focuses on the free cash flows rather than dividends at each future point of time.

The Free Cash Flow ("FCF") represent the maximum cash that could be paid out as dividends to equity holders at each point in time, after making appropriate adjustments to reflect amounts needed to support current operations and generate growth at the expected rates assumed in forecasts.

Expenses that are deducted under UK GAAP accounting but do not represent actual cash expenditures are added back to the Net Income after Tax to determine the cash flow available to be paid to equity holders.

The typical textbook definition of FCFE is shown in Table 3.3 below:

Net Income after Tax	**Comments**
+ Non-Cash Charges (Expenses)	The most significant is the increase in net claims reserves
- Net Working Capital Investment	Cash flow needed to facilitate company operations, e.g. premium receivables. This is not typically significant.
- Capital Expenditures	There are two main components, (i) change in net claims reserves and (ii) increase in capital held to meet regulatory and/or rating agency capital requirements
+ Net Borrowing	
FCFE	

Table 3.3

Dealing with Net Working Capital Investment and Capital Expenditures

Capital Expenditure – Increase in Net Claims Reserves
In the simple case of a two year insurance policy where a company collects premium net of expenses upfront and pays claims at the end of the second period it would not be sufficient to treat the net premiums as positive cash flow during the first period and the claim payments as negative cash flow during the second because part of the premium collected is not free to be paid to shareholders as monies will need to be set aside for claims reserves etc.

As such increases in net claims reserves are included within the definition of capital expenditure.

The overall impact of the increase in net claims reserves on the FCFE is zero as the positive change under Non-Cash Charges is offset by the negative change under Capital Expenditures

Capital Expenditure – Increase in Capital
As insurers are subject to regulatory and/or rating agency capital requirements there is a need at each future point in time to be able to meet regulatory minimum capital requirements.

Any additional monies required to maintain future levels of capital should be treated as capital expenditures in the FCFE.

There are likely to be multiple constraints on free cash flow as a result of having to hold capital in a company. It is necessary to determine the most binding constraint on capital, which might be regulatory, rating agency or a company's own management's assessment of capital as per its Risk Appetite.

The revised FCFE definition then becomes as in Table 3.4:

Net Income after Tax
+ Non-Cash Charges (Expenses) – excluding changes in Net Claims Reserves
- Net Working Capital Investment
- Increased in Required Capital
+ Net Borrowing
FCFE

Table 3.4

Using the example throughout the paper Figure 3.5 shows the FCFE

Two methods have been used to calculate the FCFE:

Method 1
$$FCFE_t = Net\ Income\ after\ Tax_t - Increase\ in\ Required\ Capital_t$$

Method 2
$$FCFE_t = Equity\ c/f - before\ dividends_t - Minimum\ Capital\ (Based\ on\ Target\ Capital)_t$$

Methods 1 and Methods 2 produce the same values of FCFE given that:

$$Increase\ in\ Required\ Capital_t = Minimum\ Capital\ (Based\ on\ Target\ Capital)_t - Equity\ b/f$$

and

$$Equity\ c/f - before\ dividends_t = Equity\ b/f + Net\ Income\ after\ Tax_t$$

A numerical value of the calculation is shown in Figure 3.5.

YE	2021	2022	2023	2024	2025
AY	2022	2023	2024	2025	2026
FCFE					
Equity b/f	1,016	1,136	1,169	1,211	1,261
Net Income after Tax	123	127	133	139	146
Equity c/f - before dividends	1,139	1,263	1,302	1,350	1,407
Minimum Capital - Based on Target Capital	1,136	1,169	1,211	1,261	1,321
Equity b/f	1,016	1,136	1,169	1,211	1,261
Increase in Required Capital	121	33	42	50	60
FCFE - Method 1					
Net Income after Tax	123	127	133	139	146
Increase in Required Capital	121	33	42	50	60
FCFE	**2**	**94**	**91**	**89**	**86**
FCFE - Method 2					
Equity c/f - before dividends	1,139	1,263	1,302	1,350	1,407
Minimum Capital - Based on Target Capital	1,136	1,169	1,211	1,261	1,321
FCFE	**2**	**94**	**91**	**89**	**86**

Figure 3.5

This example is independent of the example shown in Section 6 which is based on a Solvency II measure of Solvency. The Equity balance sheet numbers are consistent however the Minimum Capital numbers are not.

You will notice that the Equity b/f (x + 1) = Minimum Capital – Based on Target Capital (x)

How implemented

The modelling is a Ground Up approach given the nature of the adjustment necessary to determine the free cash flow from the Net Income after Tax

Advantages

- Modelling is Ground Up and granular

Disadvantages

- The FCFE is more abstract and complex than the DDM cashflow alternative
- The Free Cashflow for each future year depends on the estimation of future capital needs which is more uncertain than other aspects of the modelling
- Investors can argue that they cannot lay claim to these cash flows.

3.8 Valuation Multiples

Methodology

There are two main valuation multiples that I would like to discuss:

- Price / Earnings Ratio
- Price / NAV Ratio

For both of these ratios it is easy to derive their respective formulae using assumptions used in either of the DDM or EVA methods. For each of these ratios the DDM assumptions will be used as the starting point.

Price / Earnings Ratio

Formula:

$$\frac{Price}{Earnings} = \frac{Dividend\ \%}{(k-g)}$$

Proof:

Starting with the DDM:

$$\text{Value} = \frac{Earnings_1 \times Dividend\ \%}{(k-g)}$$

$$\frac{Value}{Earnings_1} = \frac{Dividend\ \%}{(k-g)}$$

Dividing the Value and Earnings_1 by the number of shares we end up with:

$$\frac{Price}{Earnings} = \frac{Dividend\ \%}{(k-g)}$$

where

$$\text{Earnings} = \text{Earnings per Share} = \frac{Earnings_1}{No.of\ shares}$$

Example

Dividend payout ratio = Dividend % = 60%

ROE = 15%

Cost of Capital = 10%

g = 15% x (1 – 60%) = 6.0%

$$\frac{Price}{Earnings} = \frac{60\ \%}{(0.1-0.06)} = 15$$

In this example the Earnings are prospective. The P/E Ratio could also be calculated using the prior period's annual earnings up until time t=0.

Price / NAV Ratio

Formula:

$$\frac{Price}{NAV} = \frac{(ROE \ - g)}{(k - g)}$$

Proof:
Starting with the DDM:

$$\text{Value } = \frac{Dividend_1}{(k - g)} \ = \frac{\text{NAV}_0 \times ROE \times Dividend\,\%}{(k - g)}$$

$$\text{Value } = \frac{\text{NAV}_0 \times \big(ROE \ - ROE \ \times \ (1 \ - \ Dividend\,\%)\big)}{(k - g)}$$

$$\text{Value} = \frac{\text{NAV}_0 \times (ROE - g)}{(k - g)}$$

as

$$\text{g} = \ \text{NAV growth} \ = ROE \ \times \ (1 \ - \ Dividend\,\%)$$

$$\frac{Value}{\text{NAV}_0} = \frac{(ROE - g)}{(k - g)}$$

Dividing the Value and NAV_0 by the number of shares we end up with:

$$\frac{Price}{NAV} = \frac{(ROE \ - g)}{(k - g)}$$

where

$$\text{NAV} = \text{NAV per Share} = \ \frac{NAV_0}{No.of\ shares}$$

Example
Dividend payout ratio = Dividend % = 60%
ROE = 15%
Cost of Capital = 10%

g = 15% x (1 – 60%) = 6.0%

$$\frac{Price}{NAV} = \frac{(15\% \ - 6.0\%)}{(0.1 - 0.06)} = 2.25$$

Advantages
- Easy to understand and communicate
- A method a third party can use if they don’t have access to data to parameterise a model
- Avoids the need to make explicit financial forecasts
- Can compare calculated P/E ratios against the P/E ratios of comparable companies using market data

- If two companies have comparable growth rates, dividend payout ratios and discount rates then P/E ratios should be comparable
- A P/E ratio can be calculated from the Value derived using either of the DDM or EVA methods.

Disadvantages

- Can be thought of as a bit too simplistic
- It assumes the ROE and NAV growth rate is the same from t=1 in perpetuity and does not allow for any changes over time
- Does not lend itself to meaningful sensitivity tests specified in terms of changes to the key insurance drivers e.g. gross written premium and growth
- Models are very sensitive to changes in the key assumptions, e.g. ROE, NAV growth rates used.

3.9 Appraisal Values

In reading Equity Analyst reports I sometimes came across a reference to the use of Appraisal values with no detail. In the early to mid-1990s Embedded Values and Appraisal Values was common in Life assurance and to an emerging extent in Non-Life insurance as described in Ryan (1990) however in recent years one doesn't tend to hear too much about them.

In the paper by Goldfarb (2005) there was a section on Economic Value Added in the guise of Abnormal Earnings but not on Appraisal Values so this got me thinking that perhaps Appraisal Values had evolved into EVA.

Methodology

The methodology is described in the paper "J.P. Ryan, and W.P. Larner (1990). The Valuation of General Insurance Companies. Journal of the Institute of Actuaries"

In simple terms this could be thought of as the sum of the first three items less the Cost of Capital. These items are briefly described in Table 3.6:

Item	Description
Adjusted Net Asset Value Sections 2.4.2, 7.2	Generally taken from the balance sheet with adjustments considered for any assets not stated at market value. Assets such as goodwill or past research and development costs are normally given a nil value. Discounting is applied using selected risk discount rates.
Value from Past Business Sections 2.4.2, 7.3	Future earnings are derived from claims reserves, premium reserves and insurance funds covering past and future exposure periods for business written in the past. Assessments are needed of the redundancy or deficiency in held reserves, future attributable investment income and capital gains and other outgoings. Discounting is applied using selected risk discount rates.
Value from Future Business Section 2.4.2, 7.4	Present value of net earnings streams from future written business, both new and renewal business, with discounting at appropriate risk discount rates. The Profits after Tax will generally be projected for N years (say 3 to 5 years) and any future values beyond this, e.g. Year (N+1) onwards will be capitalised. See the example below.
Cost of Capital Section 2.4.2	Reductions from the value should be made for the cost of any restrictions to investment policy and the return needed to cover capital allocated to the insurance operation.

Table: 3.6

<u>Example</u>

1. Using the long-term assumptions a net profit margin is established per unit of gross written premium, or such other measure of business activity if judged more appropriate.
2. The multiplier represents a capitalisation factor based upon expected future growth rates and risk discount rates in future years.

The following example is taken from Section 7.7 “Simplified Approach” of the paper:

- Gross written premium in year (x+1) = 100
- Net Profit margin = 5%
- Net present value of earnings in year (x+1) at the start of year (x+1) = 5

The Value of Future Written Business under three scenarios is shown in Table 3.7.

Item	Scenario 1	Scenario 2	Scenario 3
Growth rate p.a.	10%	10%	10%
Risk discount rate	15%	20%	25%
Multiplier	21.4	11.0	7.5
Value of Future Written Business	107	55	37

Table: 3.7

The multiplier, risk discount rate and growth rate are inter-related. The value of the multiplier is more dependent on the difference between risk discount rate and growth assumptions than on the individual assumptions themselves.

A company with high growth expectations, which are more uncertain, will likely warrant a higher discount rate and so what one finds is that the margin between the numbers is more stable than the variables modelled.

How implemented
The modelling is more of a Ground Up approach.

Similarities with Economic Value Added
Methodology wise the approach has similarities to the Economic Value Added method.
The differences appear to be in terms of granularity whether it being the financial items being discounted or the discount rates used. A comparison between the two is provided in Table 3.8

Appraisal Value	Economic Value Added
Adjusted Net Asset Value	Adjusted Net Asset Value
Value from Past Business	Embedded within the Net Income after Tax component of the Economic Profit
Value from Future Business	Embedded within the Net Income after Tax component of the Economic Profit
Cost of Capital	The k x NAV deduction component of the Economic Profit.
Varying Discount rates	Discounting of Economic Profit at the Cost of Capital

Table: 3.8

Advantages
- Modelling is Ground Up and granular
- Greater flexibility than EVA e.g. different discount rates
- Lends itself to a wide range of sensitivity tests

Disadvantages
- Doesn't necessarily demonstrate the link with the Net Income after Tax
- More difficult to understand than EVA e.g. different discount rates

3.10 How different are the results from EVA and DDM Models

Introduction
How different are results from the EVA and DDM models if they use consistent assumptions. To answer this question I decided to build a simple three period model in Excel.

To my initial surprise the EVA and DDM models give exactly the same results, even after sensitivity testing. This then led me to investigate whether this could be proved algebraically which I eventually managed to do.

Methodology
The model is relatively straight forward. Economic Profits and Dividends are calculated separately for each of the time periods t = 1 to N. For periods t = N+1 and later the values are represented by the Terminal Value ("TV"), an annuity function of future values.

Ensuring EVA and DDM Model consistency
To ensure that we have consistency between the two models I defined the following:

$$Dividend_t = Dividend\ Payout\%_t \times Net\ Income_t$$

or alternatively:

$$Dividend_t = Dividend\ Payout\%_t \times\ ROE_t \times NAV_{t-1}$$

$$g_t = ROE_t \times (1 -\ Dividend\ Payout\%_t)$$

Results
The Inputs and Outputs are shown in Tables 3.9 to 3.11.

Input

Input

ROE Basis	Period	ROE	Dividend %	Dividend	NAV Growth	ROE - k
1	1 - 5	18.0%	50.0%	9.0%	9.0%	7.0%
2	6 - 15	14.0%	50.0%	7.0%	7.0%	3.0%
3	TV (16+)	12.5%	50.0%	6.3%	6.3%	1.5%

1st Period	5
k initial	11.0%
k increment	0.00%
NAV_0	1,000

Table 3.9

I have assumed that N =15 and that there are three periods each with separate assumptions.

The first 15 years are split into two separate time periods. The length of the 1st period is a model variable, with the length of time of the 2nd period being the remaining time until 15. The final 3rd period involves the calculation of the TV for periods 16 years and later.

Note: I have used a Top Down ROE approach.

The column headings used in Table 3.9 are as follows:
- ROE = Annual ROE for each of the periods shown
- Dividend% = Dividend payout ratio for each of the periods shown

The final three columns are calculated metrics:
- Dividend = Part of the ROE which is represented by the dividend.
- NAV Growth = ROE – Dividend = percentage growth in the NAV
- ROE – k = Excess of the ROE over the Cost of Equity

Other assumptions are:
- 1st Period = Length of Period 1. This is a variable. In this example it is set to 5.
- k initial = Cost of Equity
- k increment = Annual change in the assumed 12-month Cost of Equity. In this example it is 0% and so the Cost of Equity is the same in each 12-month time period.

For the 1st period:
- Dividend = 18.0% x 50.0% = 9.0%
- NAV Growth = 18.0% - 9.0% = 9.0%
- ROE – k = 18.0% - 11.0% = 7.0%

I have assumed that the 1st period will have a higher ROE than the later periods as it is assumed over time that any initial excess returns reduce through competition and that future premium growth will be at the expense of higher combined ratios and lower profit margin, all other things being equal.

To assume that the same high ROE of 18.0% exists in perpetuity would be rather optimistic. In such a scenario this would lead to a Price / NAV ratio of 4.50x.

Output

Output

Period	t = 0	1 - 5	6 - 15	TV	Total	Price / NAV	PE
EVA	1,000	304	210	200	1,714	1.71	9.5
DDM	0	391	491	832	1,714	1.71	9.5
EVA %	58%	18%	12%	12%	100%		
DDM %	0%	23%	29%	49%	100%		

Table 3.10

Table 3.10 shows the key outputs for each of the EVA and DDM models.

The column headings in Table 3.10 are as follows:

- EVA or DDM = present values of future net profits / dividends for each of the four periods shown, t = 0, t = 1 - 5, 6 - 15 and the TV. The Total equals the sum of the values in the four periods.
- EVA % or DDM % = EVA or DDM values / Total in each interval of time
- Price / NAV = Total / NAV
- P/E = Total / 1st Year Net Income

As can be seen from the table the EVA has four components and the DDM three components, the latter always one less as there is no opening NAV.

	NAV Undisc	ROE	Dividend %	Net Income Undisc	Dividend Undisc	NAV Change Undisc	k COE	EP Undisc	EVA Disc	DDM Disc	Discount Rate	Discount Factor
Period t									**1,714**	**1,714**		
0	1,000								1,000	0		
1	1,090	18.0%	50%	180	90	90	11.0%	70	63	81	111.0%	111.0%
2	1,188	18.0%	50%	196	98	98	11.0%	76	62	80	111.0%	123.2%
3	1,295	18.0%	50%	214	107	107	11.0%	83	61	78	111.0%	136.8%
4	1,412	18.0%	50%	233	117	117	11.0%	91	60	77	111.0%	151.8%
5	1,539	18.0%	50%	254	127	127	11.0%	99	59	75	111.0%	168.5%
6	1,646	14.0%	50%	215	108	108	11.0%	46	25	58	111.0%	187.0%
7	1,762	14.0%	50%	230	115	115	11.0%	49	24	56	111.0%	207.6%
8	1,885	14.0%	50%	247	123	123	11.0%	53	23	54	111.0%	230.5%
9	2,017	14.0%	50%	264	132	132	11.0%	57	22	52	111.0%	255.8%
10	2,158	14.0%	50%	282	141	141	11.0%	61	21	50	111.0%	283.9%
11	2,309	14.0%	50%	302	151	151	11.0%	65	21	48	111.0%	315.2%
12	2,471	14.0%	50%	323	162	162	11.0%	69	20	46	111.0%	349.8%
13	2,644	14.0%	50%	346	173	173	11.0%	74	19	45	111.0%	388.3%
14	2,829	14.0%	50%	370	185	185	11.0%	79	18	43	111.0%	431.0%
15	3,027	14.0%	50%	396	198	198	11.0%	85	18	41	111.0%	478.5%
TV = 16+		12.5%	50%		3,983		11.0%	956	200	832		

Discounted Value to t = 15

Table 3.11

Differences in EVA and DDM Value profiles

Even though the profile of the amounts by year are markedly different the overall values for each method are identical.

The most obvious difference is the proportion of the overall valuation represented by future time periods t > 0, i.e. t=1, 2, 3 and so on……

For the EVA model the future value as a proportion of the overall value < 100% whereas in the case of the DDM model it is equal to 100%. Table 3.12 below shows how the proportion for periods t > 0 depends on the values of Price / NAV.

$$EVA\,\% \;= 1 - \frac{1}{\left(\frac{Price}{NAV}\right)}$$

Percentage t > 0

Price / NAV	EVA	DDM
1.00	0%	100%
1.25	20%	100%
1.50	33%	100%
1.75	43%	100%
2.00	50%	100%
2.25	56%	100%
2.50	60%	100%
2.75	64%	100%
3.00	67%	100%

Table 3.12

In essence the DDM valuation is 100% based on future assumptions whereas the EVA valuation is made up of an opening value (fixed) plus a proportion that increase with an increase in the Price / NAV ratio.

For example, the value from future assumptions is 60% (if Price / NAV ratio = 2.5).

3.11 Which Model to use

My preference is the Economic Value Added model using a Ground Up Basis:

1. The EVA and DDM values are the same (see Section 3.10)
2. Identifies the sources of Economic Profit / Loss over time.
3. The Future Value (for t > 0) < 100% of the Total whereas with the DDM the Future Value equals the Total Value.

A Free Cash Flow to Equity ("FCFE") model is a plausible alternative cash-flow based model to the DDM however it more complex and uncertain when one thinks about the underlying assumptions needed. Furthermore, investors can argue that they cannot lay claim to these cash flows.

Besides, a DDM implemented using a ground-up basis involving the forecasting of Net Income and appropriate dividend philosophies appears to me to be a more relevant cash-flow valuation method. Dividends represent the only cash-flows that are tangible to investors.

3.12 EVA vs DDM Theoretical Proof of Equivalence

The general proof is shown below. The interesting thing about this proof is that the equivalence is independent of any specific formula for the Net Income or Dividend whether the former is derived using an ROE or the latter derived using a dividend payout % assumption.

$$EVA\,Value = NAV_0 + \sum_{t=1}^{\propto} \frac{EP_t}{(1+k)^t}$$

$$\mathrm{EP}_t = \mathrm{NI_t} - k \times NAV_{t-1}$$

$$\mathrm{NI}_t = \mathrm{D_t} + NAV_t - NAV_{t-1}$$

i.e. Net Income = Dividend + Change in NAV

$$\mathrm{EP}_t = \mathrm{D_t} + NAV_t - (1 + k) \times NAV_{t-1}$$

$$\sum_{t=1}^{\infty} \frac{EP_t}{(1+k)^t} = \sum_{t=1}^{\infty} \frac{D_t}{(1+k)^t} + \sum_{t=1}^{\infty} \frac{NAV_t}{(1+k)^t} - (1+k) \times \sum_{t=1}^{\infty} \frac{NAV_{t-1}}{(1+k)^t}$$

$$\sum_{t=1}^{\infty} \frac{EP_t}{(1+k)^t} = \sum_{t=1}^{\infty} \frac{D_t}{(1+k)^t} + \sum_{t=1}^{\infty} \frac{NAV_t}{(1+k)^t} - \sum_{t=1}^{\infty} \frac{NAV_{t-1}}{(1+k)^{t-1}}$$

$$\sum_{t=1}^{\infty} \frac{EP_t}{(1+k)^t} = \sum_{t=1}^{\infty} \frac{D_t}{(1+k)^t} - \mathrm{NAV}_0$$

$$\mathrm{NAV}_0 + \sum_{t=1}^{\infty} \frac{EP_t}{(1+k)^t} = \sum_{t=1}^{\infty} \frac{D_t}{(1+k)^t}$$

i.e. EVA = DDM

The proof itself assumes that the cost of capital k for each time period (t-1, t) is the same.

On review of this proof it appears that the equivalence will still hold even if the 12-month cost of capital varies for each time period (t-1,t). See Section 7.8 for the more generalised proof where the Cost of Capital varies for each 12-month time period.

4. Top Down ROE Models – Some Issues

4.1 What are They

This section describes some of the issues that I have thought about when reviewing Equity Analyst reports. It goes some way to explaining why I prefer Ground Up modelling working with the drivers of the underlying profit.

The following is list of these issues as I see them. These are discussed in the subsequent sections 4.2 to 4.8:

1. A level ROE does not mean a level Combined Ratio "through the cycle"
2. No information on the Combined Ratio and NEP growth after the initial period N
3. Difficult to tell whether future P&Ls are reasonable for t > 1
4. Dividend payout ratio determines the following year's Net Income and NEP
5. Not possible to project future capital needs and hence future Solvency Ratios
6. Models are very sensitive to a limited number of key assumptions e.g. ROE, COE
7. Valuation Multiples – Dividend % inconsistencies
8. Contradictory Dividend Sensitivity Results for Top Down ROE and Ground Up Models.

4.2 A level ROE does not mean a level Combined Ratio "through the cycle"

A "through the cycle ROE" is typically derived from a "through the cycle Combined Ratio". A calculation of the ROE from the Combined Ratio can be seen in Figure 4.1. In this example a level combined ratio of 85% results in a level ROE of 15.8%.

Simple Model - Top Down

Time t	**1**
NAV (Beginning year)	1,000
NEP	1,000
Combined Ratio	85%
UW Profit	150
Average Investments	2,400
Investment return	48
Other	0
Finance Costs	0
Profit Before Tax	198
Tax	-40
Net Income After Tax	158
ROE	15.8%
Dividend	95
Retained Profit	63
Dividend Growth p.a.	
NAV Growth	6.3%
NEP Growth	6.3%
Average investment return	2.0%
Tax Rate	20%
Dividend	60%
Cost of Equity	9.5%

Figure 4.1

Problems:
1. It tells you nothing about the Net Earned Premium growth from year to year and whether this is consistent with an assumed level Combined Ratio. A higher NEP growth is often at the expense of future profitability e.g. a higher combined ratio, other things being equal.
2. Does a level Combined Ratio mean a level ROE ?

Let’s look at point 2.

In a simplified P&L model for a company in a “Steady State” condition where the ratios of (i) Net Reserves to NEP and (ii) Investments to Net Reserves over time are consistent then the implied NEP Growth rate is given by:

$$NEP\ Growth\ rate = Initial\ ROE\ \times (1\ - Dividend\ \%)\ = NAV\ Growth\ rate$$

But companies won’t necessarily be in a Steady State condition or close to one, especially for a new and/or rapidly growing company.

As one can see from the formula above:
1. A lower dividend payout ratio will lead to higher NEP growth and higher NAVs consistent with larger capital needs, and
2. A higher dividend payout ratio will lead to lower NEP growth and NAVs consistent with lower capital needs.

Figure 4.2 shows the evolution of the P&L over time when the NEP Growth rate is assumed equal to the growth in the NAV.

Using the formula above the growth in the NAV = 15.84% x (1 – 0.6) = 6.3% p.a.

If we assume that the NEP Growth = 6.3% p.a., then the calculated retained profits over time leads to a calculated increase in the NAV of 6.3% p.a. too, consistent with the original assumption.

I have assumed an average investment return on the opening net investments each year of 2.0% p.a. and a tax rate of 20%.

Simple Model - Top Down								
		NEP = NAV Growth (t=0)						
Time t	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**TV**
NAV (Beginning year)	1,000	1,063	1,131	1,202	1,279	1,360	1,446	1,537
NEP	1,000	1,063	1,131	1,202	1,279	1,360	1,446	
Combined Ratio	85%	85%	85%	85%	85%	85%	85%	
UW Profit	150	160	170	180	192	204	217	
Average Investments	2,400	2,552	2,714	2,886	3,069	3,263	3,470	
Investment return	48	51	54	58	61	65	69	
Other	0	0	0	0	0	0	0	
Finance Costs	0	0	0	0	0	0	0	
Profit Before Tax	198	211	224	238	253	269	286	
Tax	-40	-42	-45	-48	-51	-54	-57	
Net Income After Tax	158	168	179	190	203	215	229	
ROE	15.8%	15.8%	15.8%	15.8%	15.8%	15.8%	15.8%	11.0%
Dividend	95	101	107	114	122	129	137	
Retained Profit	63	67	72	76	81	86	92	
Dividend Growth p.a.		6.3%	6.3%	6.3%	6.3%	6.3%	6.3%	
NAV Growth	6.3%	= 0.1584 x (1 - 0.6)						
NEP Growth	6.3%							
Average investment return	2.0%							
Tax Rate	20%							
Dividend	60%							
Cost of Equity	9.5%							

EVA Valuation								
Time t	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**TV**
EP	63.4	67.4	71.7	76.2	81.1	86.2	91.7	23.1
Discounted EP	57.9	56.2	54.6	53.0	51.5	50.0	48.6	386.1
EVA Value	**1,758**							
Price / Book Ratio	**1.76**							
Price / Earnings Ratio	**11.10**							

Figure 4.2

In this example N = 7.

The Economic Profit (“EP”) = 23.1 for t = 8 using a TV calculation. The value being noticeably lower as the ROE is assumed to be 11.0% and not 15.8%. The NAV value of 1,537 is the NAV at Year End t = 7.

The EVA Value of 1,758 is derived using the EVA formula:

$$EVA\,Value = NAV_0 + \sum_{t=1}^{N} \frac{(ROE_t - k) \times NAV_{t-1}}{(1+k)^t} + \frac{(ROE_{N+1} - k) \times NAV_N}{(k-g)} \times \frac{1}{(1+k)^N}$$

Example 2: NEP Growth < Initial implied NAV Growth
What happens if the NEP Growth rate is less than the Initial implied NAV Growth rate. In this example the NEP Growth rate of 4.5% p.a. has been assumed which is less than the previous 6.3% p.a.

The example in Figure 4.3 shows that when the NEP Growth rate is less than the Initial implied NAV Growth rate a level combined ratio of 85% will result in a <u>lower ROE</u> over time. The ROE here is calculated and not assumed.

Simple Model - Top Down							
	NEP < NAV Growth (t=0)						
Time t	**1**	**2**	**3**	**4**	**5**	**6**	**7**
NAV	1,000	1,063	1,130	1,200	1,273	1,350	1,430
NEP	1,000	1,045	1,092	1,141	1,193	1,246	1,302
Combined Ratio	85%	85%	85%	85%	85%	85%	85%
UW Profit	150	157	164	171	179	187	195
Average Investments	2,400	2,552	2,712	2,879	3,055	3,239	3,432
Investment return	48	51	54	58	61	65	69
Other	0	0	0	0	0	0	0
Finance Costs	0	0	0	0	0	0	0
Profit Before Tax	198	208	218	229	240	252	264
Tax	-40	-42	-44	-46	-48	-50	-53
Net Income After Tax	158	166	174	183	192	201	211
ROE	15.8%	15.6%	15.4%	15.3%	15.1%	14.9%	14.8%
Dividend	95	100	105	110	115	121	127
Retained Profit	63	66	70	73	77	81	84
Dividend Growth p.a.		4.9%	4.9%	4.9%	4.9%	4.9%	4.9%
NAV Growth	6.3%	= 0.1584 x (1 - 0.6)					
NEP Growth	4.5%						
Average investment return	2.0%						
Tax Rate	20%						
Dividend	60%						

Figure 4.3

Conclusion:
The only way for a level calculated ROE to be 15.8% p.a. is if the Combined Ratio decreases over time. Therefore, assuming a level ROE in the equity valuation will overstate a company's value.

Example 3: NEP Growth > Initial implied NAV Growth
Conversely when the NEP Growth rate is greater than the Initial implied NAV Growth rate using a level Combined Ratio of 85% would lead to a higher ROE over time.

Assuming a level ROE would implicitly assume successively increasing combined ratios higher than 85% for t = 1 to 5.

Conclusion:
The only way for a level calculated ROE to be 15.8% p.a. is if the Combined Ratio increases over time. Therefore, assuming a level ROE in the equity valuation will understate a company's value.

4.3 No information on the Combined Ratio and NEP growth after N

A particular problem arises in models where the assumed ROE transitions from one value to another at some point in the future, N years. For example, the ROE might equal 15.8% p.a. for each of the first 3 years but reduce to 12% p.a. say for the remainder of the term.

I have often seen in reports that is assumed that in a steady state condition the ROE will not be lower than the cost of capital, but this may not necessarily be true if one considers ground up models.

Problems:

1. If the combined ratio remains unchanged there is a large negative and unrealistic reduction in the NEP at the point of transition from ROE to another. This is because the cause of a lower ROE in the first place would be a transition to a higher combined ratio.
2. There are many possible combinations of the NEP growth and combined ratio at the point of transition. Some combinations might be more feasible than others.
3. A calculated ROE at some point in the future can fall below a long term assumed ROE.

Example 4: Combined ratio of 85% unchanged

An example of this can be seen in Figure 4.4. If the combined ratio of 85% is unchanged after year 4 then the only way for the ROE to be 12% for year 5 is if the NEP growth between years 4 and 5 is -27.7%.

Simple Model - Top Down

Time t	**1**	**2**	**3**	**4**	**5**	**6**	**7**
NAV (Beginning year)	1,000	1,063	1,131	1,202	1,279	1,340	1,404
NEP	1,000	1,063	1,131	1,202	869	911	955
Combined Ratio	85%	85%	85%	85%	85%	85%	85%
UW Profit	150	160	170	180	130	137	143
Average Investments	2,400	2,552	2,714	2,886	3,069	3,216	3,370
Investment return	48	51	54	58	61	64	67
Other	0	0	0	0	0	0	0
Finance Costs	0	0	0	0	0	0	0
Profit Before Tax	198	211	224	238	192	201	211
Tax	-40	-42	-45	-48	-38	-40	-42
Net Income After Tax	158	168	179	190	153	161	169
Assumed ROE	15.8%	15.8%	15.8%	15.8%	12.0%	12.0%	12.0%
Dividend	95	101	107	114	92	96	101
Retained Profit	63	67	72	76	61	64	67
Dividend Growth p.a.		6.3%	6.3%	6.3%	-19.4%	4.8%	4.8%
Calculated ROE		15.8%	15.8%	15.8%	12.0%	12.0%	12.0%
NEP Growth		6.3%	6.3%	6.3%	-27.7%	4.8%	4.8%
Average investment return	2.0%						
Tax Rate	20%						
Dividend	60%						

Figure 4.4

Note for years 6 and 7 onwards the NEP growth is 4.8% which is simply the formula we had before, namely:

NEP growth = 4.8% = 12.0% x (1 – 0.6)

Example 5: NEP growth for year 5 onwards the same

An ROE of 12.0% is consistent with a higher combined ratio of 90%. This can be seen in Figure 4.5. In this scenario the NEP growth is now the same for year 5 and onwards.

Simple Model - Top Down

Time t	1	2	3	4	5	6	7
NAV (Beginning year)	1,000	1,063	1,131	1,202	1,279	1,340	1,404
NEP	1,000	1,063	1,131	1,202	1,304	1,367	1,432
Combined Ratio	85%	85%	85%	85%	90%	90%	90%
UW Profit	150	160	170	180	130	137	143
Average Investments	2,400	2,552	2,714	2,886	3,069	3,216	3,370
Investment return	48	51	54	58	61	64	67
Other	0	0	0	0	0	0	0
Finance Costs	0	0	0	0	0	0	0
Profit Before Tax	198	211	224	238	192	201	211
Tax	-40	-42	-45	-48	-38	-40	-42
Net Income After Tax	158	168	179	190	153	161	169
Assumed ROE	15.8%	15.8%	15.8%	15.8%	12.0%	12.0%	12.0%
Dividend	95	101	107	114	92	96	101
Retained Profit	63	67	72	76	61	64	67
Dividend Growth p.a.		6.3%	6.3%	6.3%	-19.4%	4.8%	4.8%
Calculated ROE		15.8%	15.8%	15.8%	12.0%	12.0%	12.0%
NEP Growth		6.3%	6.3%	6.3%	4.8%	4.8%	4.8%
Average investment return	2.0%						
Tax Rate	20%						
Dividend	60%						

Figure 4.5

4.4 P&L reasonableness

Where an ROE is a model input for each future year there is little understanding of whether the implied future premium, claims and expense numbers on a gross, net and ceded basis are reasonable or not.

Furthermore, it is difficult to interpret what a change in the future ROE means in practice e.g. an ROE changing from 11% to 12% p.a. However, if ground up modelling was performed any changes that would imply a higher ROE are easier to validate and challenge if necessary.

4.5 Dividend Payout

With a Ground Up approach we start off with an NEP, derive the Net Income after Tax and use the dividend payout ratio to determine the Dividend and Retained Profit.

In the Top Down ROE approach the future Net Income and NEP is a function of the assumed Dividend payout ratio. This is counter intuitive.

4.6 Future Solvency Ratios

There is a need to project future Capital and Solvency Ratios over the valuation time horizon.

This will be necessary so as to avoid situations where a favourable "Buy", "Add" or similar equity valuation recommendation arises whilst at the same time the projected future Solvency ratios cause potential concerns.

Many Top Down ROE valuation assumptions involve higher ROEs in the earlier years and lower ROEs in later years, and not vice versa, so this is a potential risk. A lower ROE in later years will put future Solvency Ratios under pressure, the impact being greater for those writing larger proportions of Long Tail class business all other things being equal.

4.7 Sensitivity Tests

The Top Down ROE models are can be very sensitive to changes in the input assumptions.

This leads to situations where it is possible to derive a wide range of values using different sets of plausible model inputs, because as pointed out in section 4.4 it is far more difficult to challenge the validity of one ROE vs another over time.

The results from the following sensitivity tests are shown. The impact on the Price / NAV is shown using the model described in section 3.10.

1. Sensitivity Test for COE / ROE 1
2. Sensitivity Test for Dividend 1% / ROE 1
3. Sensitivity Test for Dividend 3% / ROE 3
4. Sensitivity Test for 1st Period / ROE 1

Here, ROE m and Dividend n%, means the ROE and Dividend percentage respectively during the n^{th} period.

1. Sensitivity Test for COE / ROE1

Price / NAV					
COE / ROE 1					
	14.0%	15.0%	16.0%	17.0%	18.0%
8.0%	4.02	4.13	4.25	4.36	4.48
9.0%	2.57	2.65	2.73	2.81	2.89
10.0%	1.89	1.96	2.02	2.08	2.15
11.0%	1.50	1.55	1.61	1.66	1.71
12.0%	1.24	1.29	1.33	1.38	1.43

Figure 4.6

The value of 1.71 shown in blue is the base case.

2. Sensitivity Test for Dividend 1% / ROE 1

Price / NAV					
Dividend 1 % / ROE 1					
	14.0%	15.0%	16.0%	17.0%	18.0%
30.0%	1.56	1.62	1.68	1.74	1.81
40.0%	1.53	1.58	1.64	1.70	1.76
50.0%	1.50	1.55	1.61	1.66	1.71
60.0%	1.47	1.52	1.57	1.62	1.67
70.0%	1.45	1.49	1.54	1.58	1.63

Figure 4.7

3. Sensitivity Test for Dividend 3% / ROE 3

Price / NAV					
Dividend 3 % / ROE 3					
	9.0%	10.0%	11.0%	12.0%	12.5%
30.0%	1.25	1.36	1.51	1.76	1.94
40.0%	1.29	1.39	1.51	1.68	1.79
50.0%	1.32	1.41	1.51	1.64	1.71
60.0%	1.34	1.42	1.51	1.62	1.67
70.0%	1.36	1.44	1.51	1.60	1.65

Figure 4.8

4. Sensitivity Test for 1st Period / ROE 1

Price / NAV					
1st Period / ROE 1					
	14.0%	15.0%	16.0%	17.0%	18.0%
4	1.50	1.54	1.59	1.63	1.67
5	1.50	1.55	1.61	1.66	1.71
6	1.50	1.56	1.62	1.69	1.75
7	1.50	1.57	1.64	1.72	1.79
8	1.50	1.58	1.66	1.75	1.83

Figure 4.9

4.8 Valuation Multiples – Parameterisation Inconsistencies

Price / NAV Formula:
Sometimes one comes across valuations in Equity Analyst reports which involve the application of an assumed Price / NAV multiple, e.g. 1.5x say to a company's NAV (or Adjusted NAV).

As we saw in section 3.8 the formula for the Price / NAV is a function of the ROE, g and k, namely:

$$\frac{Price}{NAV} = \frac{(ROE\ - g)}{(k - g)}$$

Problems:

1. If a High ROE is assumed say something of the order of 14% p.a., the Price / NAV ratio may become unrealistically high if a reasonable dividend payout % ratio is used

The problem is not the High ROE in itself, which in multi-period models is often only assumed for an initial short time period but that it is assumed in perpetuity, and doesn't reduce over time.

To compensate for this an unrealistically low value of "g", e.g. 2%, may have been selected. The problem is that this may imply an unrealistically high dividend payout % which will not always be obvious.

A price / NAV ratio in the range, 1.0 < Price / NAV < 2.0 is not unusual for insurance companies.

This is illustrated in Figures 4.10 and 4.11. Figure 4.10 shows the range of Price / NAV ratio for different growth "g" and ROE assumptions. The Cost of Equity is assumed to be 11.0% p.a. This table might look reasonable in its own right as the Price / NAV ratios do not look unreasonable.

However, when one looks at the implied dividend payout ratios in Figure 4.11 a different picture starts to emerge.

Price / NAV					
g / ROE					
	14.0%	15.0%	16.0%	17.0%	18.0%
1.0%	1.30	1.40	1.50	1.60	1.70
2.0%	1.33	1.44	1.56	1.67	1.78
3.0%	1.38	1.50	1.63	1.75	1.88
4.0%	1.43	1.57	1.71	1.86	2.00
5.0%	1.50	1.67	1.83	2.00	2.17
k (= COE)	11.0%				
Price / NAV <= 2					

Figure 4.10

Dividend Payout %					
g / ROE					
	14.0%	15.0%	16.0%	17.0%	18.0%
1.0%	93%	93%	94%	94%	94%
2.0%	86%	87%	88%	88%	89%
3.0%	79%	80%	81%	82%	83%
4.0%	71%	73%	75%	76%	78%
5.0%	64%	67%	69%	71%	72%
Dividend % > 0.8					
Dividend % > 0.7					

Figure 4.11

As one can see once the ROE is 14% or more the necessary dividend payout ratios to derive overall reasonable Price / NAV ratios become rather high. The situation becomes worse the lower the COE. This highlights the issues of assuming an ROE in perpetuity.

Investor Checks
So what checks should a third party e.g. an investor, do when confronted with such valuation assumptions:

1. Calculate $Dividend\ \% = 1 - g/ROE$ then perform the following checks:
2. Compare Dividend % vs (i) Company's historical record and (ii) Peer companies
3. Project Solvency Ratios (or think about it); a Low Retained profits % contribution to NAV might not be high enough to offset the increase in required capital leading to a deteriorating Solvency Ratio over time, especially for longer tail risks.

Solution
The solution to this problem would have been to have used a high ROE for earlier years and a lower one for later years. In this way a more realistic dividend payout ratio could be assumed.

4.9 Dividends – Top Down ROE and Ground Up Model contradictions

Question
What happens if one reduces the Dividend payout ratio.

Answer
Rather surprisingly the direction of the Change in value depends on the type of model one is using, be it a Top Down ROE or a Ground Up model. This is rather strange as one would think that the direction of change should be independent of what type of model one chooses.

The explanation is provided in Table 4.12 and reflects differences in assumptions for the two different models. The direction of change is the same whether a DDM or EVA model is used.

Model	Change in Value	Explanation	Observations	Future Solvency Ratios
Top Down ROE	Increase	**DDM** In the earlier years future dividends are lower, however at some point N in the future a lower payout ratio x a higher NAV will give a higher dividend value than before the change. This impact tends to dominate the NPV calculation. This can be seen in the mathematical proof below. **EVA** Economic Profits are higher as a fixed (ROE – COE) is applied to higher projected future NAVs.	The Net Income will automatically be higher as the same ROE is assumed to apply to a higher NAV. It implicitly assumes that excess writing will earn the same ROE. This may not necessarily be true.	Future Solvency ratios may be broadly unchanged as a higher NAV is offset by higher required capital needs from an increase in the implied larger premiums, reserve and asset exposures. The increase in the Ground Up solvency ratios will be higher than in the Top Down ROE model.
Ground Up	Decrease	**DDM** In earlier years dividend payments will be lower cf before as Net Income is assumed to be unchanged. A NPV impact. **EVA** Economic Profits = Net Income – COE x NAV are lower as Net Income is unchanged but the COE x NAV is higher with a fixed COE.	One can test in isolation the impact of dividend payout ratio. The opposite happens if the dividend payout ratio is increased i.e. future solvency ratios decrease.	Higher future Solvency ratios than before the change as the NAV increases but as the same level of business is assumed to be written I think the required capital changes will be less give that the underwriting risk component should be broadly unchanged with changes only to asset exposures.

Table 4.12

Top Down ROE Model – Proof that Value increases if the Dividend % is reduced

$$Value\ Before\ = \frac{ROE \times NAV_O \times Dividend\ \%_{\ Before}}{\left(k\ - ROE\ \times \left(1 - Dividend\ \%_{\ Before}\right)\right)}$$

$$Value\ After\ = \frac{ROE \times NAV_O \times Dividend\ \%_{\ After}}{\left(k\ - ROE\ \times \left(1 - Dividend\ \%_{\ After}\right)\right)}$$

$\boldsymbol{Value\ After >\ Value\ Before}$

If: $\frac{ROE \times NAV_O \times Dividend\ \%_{\ After}}{\left(k\ - ROE\ \times \left(1 - Dividend\ \%_{\ After}\right)\right)} > \frac{ROE \times NAV_O \times Dividend\ \%_{\ Before}}{\left(k\ - ROE\ \times \left(1 - Dividend\ \%_{\ Before}\right)\right)}$

If: $\frac{Dividend\ \%_{\ After}}{\left(k\ - ROE\ \times \left(1 - Dividend\ \%_{\ After}\right)\right)} > \frac{Dividend\ \%_{\ Before}}{\left(k\ - ROE\ \times \left(1 - Dividend\ \%_{\ Before}\right)\right)}$

If: $Dividend\ \%_{\ After}\ \times \left(k\ - ROE\ \times \left(1 - Dividend\ \%_{\ Before}\right)\right) >\ Dividend\ \%_{\ Before}\ \times \left(k\ - ROE\ \times \left(1 - Dividend\ \%_{\ After}\right)\right)$

If: $Dividend\ \%_{\ After}\ \times (k\ - ROE\ ) >\ Dividend\ \%_{\ Before}\ \times (k\ - ROE\ )$

If: $\boldsymbol{Dividend}\ \%_{\ \boldsymbol{After}}\ <\ \boldsymbol{Dividend}\ \%_{\ \boldsymbol{Before}}$

As: $(k\ - ROE)\ < 0$

5. Projection of Company Financials

5.1 Introduction

In Ground Up modelling there is an advantage to be had in projecting a company's financials over time as the ROE for future years will be calculated values.

5.2 Foxes Capital

For modelling purposes, I have constructed a hypothetical set of historical 2020 and 2021 P&Ls and YE2020 and YE2021 balance sheet data, together with gross and net reserves by AY as at YE2021. All consistent with each other.

The starting position is the balance sheet as at YE2021 with the first projection year being Calendar Year ("CY") 2022. Projections are made for CYs 2022 to 2026 and YEs 2022 to 2026. I am assuming here that a company's accounting period runs from 1st January to 31st December of each year.

5.3 Model Building Blocks

For the purposes of this paper the starting point is a simple model that projects financial statements over future years. The four key building blocks that I have focused on are:

- Profit and Loss Account ("P&L")
- Balance Sheet
- Cashflow Statement
- Gross and Net Claims Reserves Projections

The model works from the ground up with key drivers such as:

- Gross and Net written premiums
- Gross and Net premium earnings patterns
- Gross and net loss ratios
- Gross and net acquisition cost ratios
- Administration / other expenses bases and ratios
- Investment income, realised/unrealised gains and investment expenses ratios
- Tax rate
- Dividend payout basis and payout ratios / amounts

The gross and net reserve projections are a significant driver of future balance sheets and so assumptions are needed for the opening YE2021 gross and net claims reserves by AY, and gross and net payment patterns.

Iterative Build Up

With granular opening Balance Sheet items at YE2021 it is relatively straightforward to project balance sheets for most items for YE2022 and later using P&L and reserve projections. The only potential complication is getting the formula right to calculate the projection of net new cash flow for each future year, which can be the main driver of the increase in investments in the balance sheet between year ends.

In addition, there are some P&L items for future years that depend on beginning of year Balance Sheet items e.g. Investment Income being a function of the opening investment assets and cash balances.

P&L is already provided
An alternative approach would involve an already pre-determined 5-year or 3-year P&L. If this is provided in advance then the balance sheet and cashflow statements can still be calculated in a similar fashion to the above.

P&L / Balance Sheet Validation
One sometimes finds that the pre-determined P&Ls are inconsistent with the projected balance sheets.

One of the inconsistencies that I have come across is the investment income/gains in the plan P&Ls. After using the provided P&Ls to project the implied future balance sheets the implied future investment return ratios (= investment income/gains / opening balance sheet values) can be calculated. These should be tested for reasonableness and checked against assumptions used to construct the P&L.

Similar calculations can be done for other metrics of interest.

Model Granularity
The level of model complexity and granularity will vary according to model use.

Table 5.1 is a simplistic representation of different levels of modelling granularity. For the purposes of this paper, I have adopted the granularity that is highlighted in blue.

Risk	Very High	High	Medium	Low
Premium	Sub-class	Class	Segment	Combined class
Claims Reserves	Sub-class	Class	Segment	Combined class
Investment Assets	Individual assets / Model points	Asset class	Asset class	No Asset class

Table 5.1

Simplifications:
1. For the sake of simplicity, I have assumed one overall class of business that can be viewed as a weighted average of granular classes. It is further assumed that the mix remains unchanged from year to year.
2. Gross and Net incurred losses are considered in total and are not modelled separately by attritional, large and catastrophe claim types.

5.4 Profit and loss Account ("P&L")

The P&L should represent the comprehensive income for any given CY, i.e. the change in Retained Earnings equals the difference between the beginning and end of year Total Equity shown in the balance sheet. This may involve combining an income statement with the statement of comprehensive income.

Figure 5.2 shows the P&L used for the purposes of this paper. If one takes a typical UK non-life insurance company's annual reports and accounts recourse may be needed to the notes to provide the granularity required e.g. investment income items.

The "consolidated statement of change in equity" or "statement of comprehensive income" will enable one to ensure that the P&L represents the comprehensive net income. The item shown "Other Net income" represents the additional amounts so that the P&L is on this basis.

The level of detail will vary from company to company.

This P&L captures the main dynamics. I have made some simplifying assumptions in that some non-core items are level in amount over the projection period e.g. Finance costs. Furthermore, I have assumed the P&L impact from Goodwill & intangibles is zero which would imply that the decrement on beginning of year values is offset by new amounts of equal value.

P&L	
Entity	Foxes Capital
BS Date	31/12/2021
First AY	2022
Currency / Units	£ m

Initial Assumptions

P&L	2020	2021	2022	2023	2024	2025	2026
Gross Written Premium	1,000	1,200	1,260	1,323	1,389	1,459	1,532
Ceded Written Premium	-280	-324	-340	-357	-375	-394	-414
Net Written Premium	**720**	**876**	**920**	**966**	**1,014**	**1,065**	**1,118**
Gross Earned Premium	1,050	1,220	1,233	1,295	1,359	1,427	1,499
Ceded Earned Premium	-335	-345	-333	-350	-367	-385	-405
Net Earned Premium	**715**	**875**	**900**	**945**	**992**	**1,042**	**1,094**
Gross Claims Incurred	-578	-659	-666	-699	-734	-771	-809
Ceded Claims Incurred	177	178	171	179	188	198	208
Net Claims Incurred	**-400**	**-481**	**-495**	**-520**	**-546**	**-573**	**-602**
Gross Acquisition Costs	-189	-220	-222	-233	-245	-257	-270
Ceded Acquisition Costs	60	62	60	63	66	69	73
Net Acquisition Costs	**-129**	**-158**	**-162**	**-170**	**-179**	**-188**	**-197**
Operational expenses	-125	-150	-154	-162	-170	-179	-188
Other expenses (e.g. foreign exchange)	-10	-5	-5	-5	-6	-6	-6
Total Expenses	**-264**	**-313**	**-321**	**-337**	**-354**	**-372**	**-391**
Net UW Result	**51**	**81**	**84**	**88**	**92**	**97**	**102**
Investment Income	38	45	56	57	59	61	64
Realised Gains / Losses	15	11	14	14	15	15	16
Investment expenses	-2	-3	-3	-3	-4	-4	-4
Net Investment Result	**50**	**53**	**67**	**67**	**70**	**73**	**76**
Other Income (Ceding Coom, Broker Fee)	25	28	28	30	31	33	35
Operating Result	**126**	**163**	**179**	**185**	**193**	**203**	**213**
Finance costs	-22	-25	-25	-27	-28	-29	-31
Other	0	0	0	0	0	0	0
Profit/(loss) before tax	**104**	**138**	**154**	**159**	**166**	**174**	**182**
Tax	-10	-14	-31	-32	-33	-35	-36
Net Income after Tax	**94**	**124**	**123**	**127**	**133**	**139**	**146**
Net Income Attributed to non-controlling	0	0	0	0	0	0	0
Other Net Income	50	16	16	16	16	16	16
Net Income before Dividend	**144**	**139**	**138**	**142**	**148**	**155**	**161**
Dividend	-45	-45	-49	-51	-53	-56	-58
Retained Earnings	**99**	**94**	**89**	**92**	**95**	**99**	**103**

Figure 5.2

5.5 Balance Sheet

Figure 5.3 shows the granularity of the Balance Sheet used for the purposes of this paper. As with the P&L recourse may be needed to the notes to the accounts in order to construct the level of granularity required.

The level of detail will vary from company to company.

Gross and Net Claims reserves are separately projected (see Section 5.7) using opening balance sheet gross and net reserves and P&L gross and net incurred claims projections.

Reconciling Total Assets and Liabilities

The Asset and Liabilities are separately calculated and there is a check to ensure that the difference is zero, both on initial amounts and for projected values. In some models there is a balancing item in either the Asset or Liabilities so that the totals are the same.

It should always be possible to calculate the Assets and Liabilities directly and do a test to ensure that the difference is zero. The problem with relying on the balancing item approach is that it may hide unintentional errors in the relationships between P&L, Balance Sheet and Cashflow statements that may cause material issues later on down the line when changing assumptions or modelling scenarios.

Indeed, during April 2024 when developing a far more detailed forecasting model for a client using quarterly projected periods and quarterly accident years my first pass through of the model resulted in a difference between total assets and liabilities. It took me about a day to work out why and resolve. Then differences emerged when layering in more sophisticated model changes so in my opinion this is a very important thing to do.

These differences can easily arise if one represents P&L and Cashflow outgo and Balance Sheet liability items as -ve numbers. For example, rather than let Net Earned Premium ("NEP") = Gross Earned Premium ("GEP") + Ceded Earned Premium ("CEP"), where CEP is negative, let NEP = GEP – CEP where CEP is a positive value.

The real balancing item arises via the Net Cashflow which is discussed in more detail in section 5.5.

Balance Sheet

Entity	**Foxes Capital**
BS Date	**31/12/2021**
First AY	**2022**
Currency / Units	**£ m**

Actuals

Balance Sheet	**31/12/20**	**31/12/21**	**31/12/22**	**31/12/23**	**31/12/24**	**31/12/25**	**31/12/26**	**Change**	**Change p.a.**
Assets									
Investments	2,483	2,795	2,826	2,935	3,065	3,213	3,376	19.5%	3.9%
- Debt / Fixed Income	2,040	2,388	2,386	2,449	2,527	2,617	2,716	13.8%	2.6%
- Equities / Investment Funds	400	372	376	391	408	428	449	19.5%	3.9%
- Other	43	35	64	95	130	169	211	231.7%	43.2%
Cash	302	214	216	225	235	246	259	19.5%	3.9%
Investments and Cash	**2,785**	**3,009**	**3,042**	**3,159**	**3,300**	**3,459**	**3,635**	19.5%	3.9%
Property, plant and equipment	38	25	25	25	25	25	25	0.0%	0.0%
Goodwill & Intangibles	200	211	211	211	211	211	211	0.0%	0.0%
DAC	81	97	102	107	113	118	124	21.6%	5.0%
Ceded UPR	126	146	153	161	169	177	186	21.6%	5.0%
Reinsurance recoveries	608	621	589	589	597	612	632	7.3%	0.4%
Receivables on insurance / reinsurance	200	240	252	265	278	292	306	21.6%	5.0%
Reinsurance recoveries	**808**	**861**	**841**	**853**	**875**	**904**	**939**	11.6%	1.7%
Deferred tax assets	40	63	63	63	63	63	63	0.0%	0.0%
Premium Debtors	250	300	315	331	347	365	383	21.6%	5.0%
Other Assets	12	10	10	10	10	10	10	0.0%	0.0%
Total Assets	**4,338**	**4,721**	**4,761**	**4,919**	**5,110**	**5,331**	**5,574**	**17.1%**	**3.4%**
Liabilities									
Share Capital & Premium	210	210	210	210	210	210	210	0.0%	0.0%
Retained Earnings	600	694	784	875	971	1,069	1,172	49.6%	11.0%
Other	98	106	106	106	106	106	106	0.0%	0.0%
Equity	908	1,010	1,100	1,191	1,287	1,385	1,488	35.4%	8.1%
Non-controlling interest	6	6	6	6	6	6	6	0.0%	0.0%
Total Equity	**914**	**1,016**	**1,105**	**1,197**	**1,292**	**1,391**	**1,494**	35.2%	8.0%
Gross UPR	450	540	567	595	625	656	689	21.6%	5.0%
Gross Claims Reserves	2,250	2,300	2,210	2,233	2,284	2,357	2,447	10.7%	1.3%
Ceded DAC	23	26	28	29	30	32	33	21.6%	5.0%
Creditors insurance / reinsurance	224	259	272	286	300	315	331	21.6%	5.0%
Financial Liabilities	250	255	255	255	255	255	255	0.0%	0.0%
Deferred tax liabilities	3	5	5	5	5	5	5	0.0%	0.0%
Other Liabilities	225	320	320	320	320	320	320	0.0%	0.0%
Liabilities	**3,424**	**3,705**	**3,656**	**3,722**	**3,819**	**3,940**	**4,080**	11.6%	1.9%
Equity and Liabilities	**4,338**	**4,721**	**4,761**	**4,919**	**5,111**	**5,331**	**5,574**	**17.1%**	**3.4%**
Difference: Total Assets - Liabilities	**0**	**0**	**0**	**0**	**0**	**0**	**0**		
Net Cashflow			**33**	**117**	**140**	**160**	**175**		

Figure 5.3

Adjusted Net Value

For Equity valuation modelling purposes there is a need to project the "Adjusted Net Asset Value" which can be derived from the Net Asset Value (or "Equity b/f").

Equity Forecasts	31/12/20	31/12/21	31/12/22	31/12/23	31/12/24	31/12/25	31/12/26
Equity b/f	815	914	1,016	1,105	1,197	1,292	1,391
Net Income after Tax	94	124	123	127	133	139	146
Other Net Income	50	16	16	16	16	16	16
Dividend	-45	-45	-49	-51	-53	-56	-58
Equity c/f	**914**	**1,016**	**1,105**	**1,197**	**1,292**	**1,391**	**1,494**
Unrealised Gains	0	0	0	0	0	0	0
Intangibles	-200	-211	-211	-211	-211	-211	-211
Adjusted NAV	**714**	**806**	**895**	**986**	**1,082**	**1,180**	**1,284**

Figure 5.4

For the purposes of economic modelling the Equity c/f is not used but instead an adjusted net asset value ("Adj NAV"). Items typically deducted to derive the adjusted net value are:

- Goodwill and intangibles
- Unrealised Gains

I have assumed for the purposes of this paper that the Adjusted NAV doesn't include the value of the discount in the net claims reserves.

5.6 Cashflow Statement

Figure 5.5 shows the granularity of the Cashflow statement for the purposes of this paper.

Cashflow

Entity	**Foxes Capital**
BS Date	**31/12/2021**
First AY	**2022**
Currency / Units	**£ m**

Cashflow	2022	2023	2024	2025	2026
Gross Premium Received	1,245	1,307	1,373	1,441	1,513
Gross Paid Acquisition costs	-227	-238	-250	-263	-276
Ceded Premium Paid	-327	-344	-361	-379	-398
Ceded Acquisition costs	61	64	68	71	74
Gross Losses paid	-756	-676	-683	-697	-719
Ceded Losses received	203	180	180	183	187
Receivables on insurance / reinsurance	-12	-13	-13	-14	-15
Operational expenses	-154	-162	-170	-179	-188
Other expenses (e.g. foreign exchange)	-5	-5	-6	-6	-6
Investment Income Received	67	67	70	73	76
Other Income (Ceding Coom, Broker Fee)	28	30	31	33	35
Finance costs	-25	-27	-28	-29	-31
Other	0	0	0	0	0
Tax Paid	-31	-32	-33	-35	-36
Dividend Paid	-49	-51	-53	-56	-58
Increase in Deferred Tax Assets	0	0	0	0	0
Other Cashflows	16	16	16	16	16
Net Cashflow	**33**	**117**	**140**	**160**	**175**

Figure 5.5

The usual representation of the cashflow statement within UK insurance company accounts is to see the Profit/(Loss) before tax at the top with subsequent line entry adjustments either representing changes in the Balance Sheet assets or liabilities or items that pass through the P&L that are either not earned, incurred or accrued. This can be seen in Table 5.6.

Item	**Description**
Cash flow from operating activities	Profit/(loss) before income tax e.g. Depreciation, (increase) in DAC, increase in receivables, tax paid….. ….. Net cash from/(used in) in operating activities
Cash flow from investing activities	e.g. purchase of investments, interest and dividends received, proceeds from sale of investments….. ….. Net cash (used in)/from investing activities
Cash flow from financing activities	e.g. equity raise, finance costs, dividend paid ….. Net cash (used in)/from financing activities
Net increase/(decrease) in cash and cash equivalents	

Table 5.6

The approach taken in this paper is to focus directly on the nature of each cashflow item like gross and ceded paid claims. Table 5.7 shows the relationships between some financial items and their presence in the P&L, Balance Sheet and Cashflow Statement. In this way management and the modelling team gain a better appreciation of the drivers of cashflow.

Profit & Loss Account	**Balance Sheet**	**Cashflow Statement**
Gross Written Premium Ceded Written Premium	Change in Premium Debtors Change in Reinsurance payables	Gross Premium Received Ceded Premium Paid
Gross Incurred Claims Ceded Incurred Claims	Change in Gross Reserves Change in Ceded Reserves	Gross Paid Claims Ceded Paid Claims
Gross Acquisition Costs Ceded Acquisition Costs	Change in Gross DAC Change in Ceded DAC	Gross Paid Acquisition Costs Ceded Paid Acquisition Costs
etc.	etc.	etc.

Table 5.7

For example, Gross Premium Received can be thought of in terms of the relationship between Gross Written Premium (“GWP”) in the P&L and the change in Premium Debtors in the Balance Sheet.

In this example I have assumed that the balance sheet Goodwill & Intangibles is level over the five-year projection period. If this was not the case then a corresponding entry would need to appear in the Cashflow Statement. Indeed, for every item in the Balance Sheet that I have assumed to be level there would need to be a new entry in the Cashflow Statement should their values change between Balance Sheet dates.

It is advisable to start off with the more obvious items, for example as in this table, and then gradually work through a list to pair off matching P&L / Balance Sheet items.

5.7 Gross and Net Claims Reserves

Figure 5.8 show the Gross reserve projections.

Reserve Projections

Entity	**Foxes Capital**
BS Date	**31/12/2021**
First AY	**2022**
Currency / Units	**£ m**

			Projection					
Gross	AY	**Incurred**	**31/12/21**	**31/12/22**	**31/12/23**	**31/12/24**	**31/12/25**	**31/12/26**
	2007		0	0	0	0	0	0
	2008		0	0	0	0	0	0
	2009		0	0	0	0	0	0
	2010		23	0	0	0	0	0
	2011		23	0	0	0	0	0
	2012		46	0	0	0	0	0
	2013		46	18	0	0	0	0
	2014		69	38	15	0	0	0
	2015		92	55	30	12	0	0
	2016		138	98	59	32	13	0
	2017		230	156	111	66	36	14
	2018		253	187	126	90	54	30
	2019		345	268	198	134	95	57
	2020		460	360	280	206	140	99
	2021		575	472	369	287	212	143
	2022	666		559	459	359	279	206
	2023	699			587	482	377	293
	2024	734				616	506	396
	2025	771					647	531
	2026	809						679
	Total		**2,300**	**2,210**	**2,233**	**2,284**	**2,357**	**2,447**

Figure 5.8

The key inputs to any projection model will be the projected gross and ceded claims reserves, the latter either being separately projected or calculated as the difference between the gross and net claims reserves projections. Gross and net reserve projections are based off the YE2021 opening gross and net claims reserves by AY and gross and net claims payment patterns at whatever level of granularity is desired.

Claim payments are projected out until the run-off of the reserves, so beyond the last date shown. The projections are only shown as far as YE2026.

The reserve projections use Unpaid %ages as per the following formula for (AY, YE x).

$$Claims\ Reserves\ (AY, x) = Claims\ Reserves\ (AY, x-1) \times \frac{Unpaid\%(x+1-AY)}{Unpaid\%(x-AY)}$$

where:

$$Unpaid\%(t) = \left(1\ - Cumulative\ Paid\%(t)\right)$$

In this way any claims reserve increase/decrease scenario at any future CY point will be reflected in the subsequent year claims reserve projections.

<u>Example</u>

Considering AY = 2020 and YE 2023.

$$Claims\ Reserves\ (2020, 2022) = Ultimate\ Claims\ (2020) \times Unpaid\%(3)$$

$$Claims\ Reserves\ (2020, 2023) = Ultimate\ Claims\ (2020) \times Unpaid\%(4)$$

and so:

$$Claims\ Reserves\ (2020, 2023) = Claims\ Reserves\ (2020, 2022) \times \frac{Unpaid\%(4)}{Unpaid\%(3)}$$

For AY = y > 2021:

$$Claims\ Reserves\ (y, y) = Incurred\ Claims\ (y) \times Unpaid\%(1)$$

Claims Reserves (y, a) then follows the formula above when a > y.

5.8 Model Outputs – Summary

The P&L, Balance Sheet and Cashflow Statement outputs shown in Sections 5.4 to 5.6 are very detailed and would not necessarily be presented in this form to key stakeholders.

Figures 5.9 to 5.11 give examples of possible summary exhibits.

This level of granularity would facilitate a comparison of:

- Projected versus Historical data, allowing for known changes in the business
- Values of metrics against those of other companies.

Profit and loss Account

P&L

Entity	**Foxes Capital**
BS Date	**31/12/2021**
First AY	**2022**
Currency / Units	**£ m**

P&L	**2020**	**2021**	**2022**	**2023**	**2024**	**2025**	**2026**
Gross Written Premium	1,000	1,200	1,260	1,323	1,389	1,459	1,532
Net Earned Premium	715	875	900	945	992	1,042	1,094
Net Claims Incurred	-400	-481	-495	-520	-546	-573	-602
Net Acquisition Costs	-129	-158	-162	-170	-179	-188	-197
Operational expenses	-135	-155	-159	-167	-176	-184	-194
Net UW Result	**51**	**81**	**84**	**88**	**92**	**97**	**102**
Net Investment Result	50	53	67	67	70	73	76
Operating Result	**126**	**163**	**179**	**185**	**193**	**203**	**213**
Profit/(loss) before tax	104	138	154	159	166	174	182
Net Income after Tax	**94**	**124**	**123**	**127**	**133**	**139**	**146**
Other Net Income	50	16	16	16	16	16	16
Net Income before Dividend	144	139	138	142	148	155	161
Dividend	-45	-45	-49	-51	-53	-56	-58
Retained Earnings	**99**	**94**	**89**	**92**	**95**	**99**	**103**
Profit Ratios							
ROE (x) = Net Income before Dividend (x) / Adj NAV (x-1)			17.2%	15.9%	15.0%	14.3%	13.7%
ROE Net (x) = Retained Earnings (x) / Adj NAV (x-1)			11.1%	10.2%	9.6%	9.1%	8.7%
ROC (x) = Net Income before Dividend (x) / Capital (x-1)							
Profit Ratio (x) = Net Income After Tax (x) / NEP (x)			13.7%	13.4%	13.4%	13.3%	13.3%
Combined Ratio							
Combined Ratio (x) = Net Loss Ratio (x) + Net Expense Ratio (x)			90.7%	90.7%	90.7%	90.7%	90.7%
Net Loss Ratio (x) = Net Incurred Losses (x) / NEP (x)			55.0%	55.0%	55.0%	55.0%	55.0%
Net Expense Ratio (x) = Net Expenses (x) / NEP (x)			35.7%	35.7%	35.7%	35.7%	35.7%
Net Acquisition Costs Ratio (x) = Net Incurred Losses (x) / NEP (x)			*18.0%*	*18.0%*	*18.0%*	*18.0%*	*18.0%*
Operational Expenses Ratio (x) = Operational Expenses (x) / NEP (x)			*17.7%*	*17.7%*	*17.7%*	*17.7%*	*17.7%*
Net to Gross Ratios							
NEP (x) / GEP (x)			73%	73%	73%	73%	73%
Net Incurred Losses (x) / Gross Incurred Losses (x)			74%	74%	74%	74%	74%
Investments and Cashflow Ratios							
Inv Yield (x) = Net Investment Result (x) / Investments and Cash (x-1)			2.2%	2.2%	2.2%	2.2%	2.2%

Figure 5.9

Balance Sheet

Balance Sheet and Cashflow

Entity	**Foxes Capital**
BS Date	**31/12/2021**
First AY	**2022**
Currency / Units	**£ m**

Balance Sheet	**31/12/20**	**31/12/21**	**31/12/22**	**31/12/23**	**31/12/24**	**31/12/25**	**31/12/26**
Assets							
Investments and Cash	2,785	3,009	3,042	3,159	3,300	3,459	3,635
Goodwill & Intangibles	200	211	211	211	211	211	211
DAC	81	97	102	107	113	118	124
Ceded UPR	126	146	153	161	169	177	186
Reinsurance recoveries	608	621	589	589	597	612	632
Other Assets	539	637	664	692	722	753	786
Total Assets	**4,338**	**4,721**	**4,761**	**4,919**	**5,110**	**5,331**	**5,574**
Liabilities							
Total Equity	914	1,016	1,105	1,197	1,292	1,391	1,494
Gross UPR	450	540	567	595	625	656	689
Gross Claims Reserves	2,250	2,300	2,210	2,233	2,284	2,357	2,447
Ceded DAC	23	26	28	29	30	32	33
Other Liabilities	702	839	852	865	880	895	910
Total Liabilities	**4,338**	**4,721**	**4,761**	**4,919**	**5,111**	**5,331**	**5,574**
Adjusted Net Asset Value	**714**	**806**	**895**	**986**	**1,082**	**1,180**	**1,284**

Debt Leverage	**31/12/21**	**31/12/22**	**31/12/23**	**31/12/24**	**31/12/25**	**31/12/26**
Financial Liabilities	250	255	255	255	255	255
Final Equity	806	895	986	1,082	1,180	1,284
Total	**1,056**	**1,150**	**1,241**	**1,337**	**1,435**	**1,539**
Debt Leverage Ratio %	**23.7%**	**22.2%**	**20.5%**	**19.1%**	**17.8%**	**16.6%**

Investments Coverage	**31/12/21**	**31/12/22**	**31/12/23**	**31/12/24**	**31/12/25**	**31/12/26**
Investments and Cash (x)	3,009	3,042	3,159	3,300	3,459	3,635
Equity (x)	1,016	1,105	1,197	1,292	1,391	1,494
NPV Net Reserves (x)	1,652	1,582	1,598	1,634	1,687	1,751
Investments and Cash (x) / (Equity (x) + NPV Net Reserves (x))	**113%**	**113%**	**113%**	**113%**	**112%**	**112%**
Debt / Fixed Income (x)	2,388	2,386	2,449	2,527	2,617	2,716
NPV Net Reserves (x)	1,652	1,582	1,598	1,634	1,687	1,751
Debt / Fixed Income (x) / NPV Net Reserves (x)	145%	151%	153%	155%	155%	155%

Premium Debtors / Reinsurance Balances Payable Assumptions	31/12/22	31/12/23	31/12/24	31/12/25	31/12/26
Premium Debtors (x) / GWP (x)	25%	25%	25%	25%	25%
Reinsurance Balances Payable (x) / CWP (x)	80%	80%	80%	80%	80%

Figure 5.10

Cashflow

Cashflow	**2020**	**2021**	**2022**	**2023**	**2024**	**2025**	**2026**
Gross Premium Received			1,245	1,307	1,373	1,441	1,513
Gross Paid Acquisition costs			-227	-238	-250	-263	-276
Ceded Premium Paid			-327	-344	-361	-379	-398
Ceded Acquisition costs			61	64	68	71	74
Gross Losses paid			-756	-676	-683	-697	-719
Ceded Losses received			203	180	180	183	187
Operational expenses			-154	-162	-170	-179	-188
Other Cashflows			-11	-14	-16	-18	-19
Net Cashflow			**33**	**117**	**140**	**160**	**175**

Figure 5.11

5.9 Some Model Design points

One of the issues in these projection models is ensuring that they are not unduly complex. The key granularity will be the number of classes of business modelled. This does not necessarily mean modelling down to 20 or more classes, 2 to 4 segments capturing the risk profile may be more than sufficient.

There is no such thing as one balance sheet projection. There are various trade-offs that one should consider.

Premium Debtors / Reinsurance Balances payable
One of the key trade-offs in these models is that between net cashflow and the change in gross and ceded, premium and commission, debtor and creditor items.

An increase in the gross premium receivable will increase the net cashflow but will decrease the premium debtors and vice versa. The level of premium debtors should be reasonable compared to a company's history, peers and knowledge of the business. A similar consideration arises on the reinsurance premium payable and reinsurance balances.

Net Cashflow
The net cashflow also needs to be reasonable from an investment management perspective.

Investments and Cash (YE x) = Investments and Cash (YE x-1) + Net Cashflow (CY x)

Any Net Investment income/gains present in the P&L is assumed to pass through the Cashflow statement hence its absence from the formula above.

Each year new monies will be allocated to investments and cash. Net Cashflow (x) as a percentage of the beginning of year Investments and Cash will be more significant for companies in a growth phase and/or writing longer-tail lines of business.

In such situations year on year projections of investments and cash are more likely to be dominated by the net cashflow contribution and not investment returns on the beginning of year investments.

6. Solvency Ratio Forecasting

6.1 Introduction

The calculation of a share price or equity evaluation should not be divorced from having an understanding of the projected future solvency ratios of a company.

Financial projections may reveal a point of time in the future where the solvency ratio falls below a company's risk appetite or perhaps the regulatory minimum leading to a capital injection between now and then. Any such implied capital injections would need to be reflected in the equity valuation

Solvency and share price forecasting are inextricably linked as they both require granular multi-year financial projections. Even in a Top Down ROE model there are usually short term forecasts of financials to accompany the estimation of value.

6.2 Regulatory Background

Some History

Non-life insurers must hold an adequate level of capital to absorb unexpected losses. The original minimum regulatory requirements were based on EU directives that went back to 1973 which were simplistic formulas involving factors applied to either premiums or claims incurred. These were widely thought to set levels that were too low and were not risk-sensitive.

The UK's Individual Capital Assessment ("ICA") regime came in with effect from 1st January 2005. This was a risk-based capital assessment where the capital of a company was set at a level so that they could survive an event or combination of events with an overall probability of 99.5% over 12 months. Companies often modelled reserve risk to ultimate on the 99.5% basis and in fact this became the standard.

This was then followed by the EU's Solvency II regime which preserved the 99.5% probability over 12 months. However, there are two ways of calculating the Solvency Capital Requirement ("SCR"), (i) Standard Formula ("SF") or (ii) using outputs from an approved Internal Model.

Multi-year solvency forecasting is nothing new. For example in the early days of the FSA's (now PRA) risk-based capital regime, ICAS, there were initial proposals where companies had a choice of capital setting over different modelling time horizons, (i) 99.5% probability of survival over 12 months, (ii) 98.5% over three years or (iii) 97.5% over five years. This was all before 99.5% over one year became the standard.

However, the models that involved capital setting for a time horizon greater than 12 months were very complex Dynamic Financial Analysis ("DFA") models rather than more simplified approaches discussed in this paper.

Regulatory Capital Regimes

Regulatory regimes vary worldwide. In Europe there is Solvency II with a distinction between the SCR capital being calculated using either the Standard Formula or with outputs from an approved Internal Model.

Many other regulatory regimes e.g. US RBC or the BSCR under the Bermudian Monetary Authority ("BMA") are Formula / Factor based models with variation in the granularity of model inputs and risk aggregation using correlation matrices and similar.

These Formula/ Factor based models have inputs that are often exposures, be it gross/net written premium, gross/net earned premium or gross/net claims reserves by class of business with relevant exposures for market risk, credit risk, operational risk etc.

6.3 Solvency Forecasting approaches

Four types of Regulatory Capital Projection

I have grouped into what appear to be four different types of modelling approaches [2]:

A. Formula based models e.g. Turkey, Brazil, Hong Kong, Singapore and Indonesia.
B. Factor based models e.g. SF SCR, BMA BSCR and US RBC
C. Standalone risk capital outputs from Internal Models
D. IM Simulation Scaling

Definition of Available Capital

Whereas this section is focused on the projection of the regulatory capital it should be noted the other important component in any calculation of future solvency ratios is the numerator, the Available Capital. This varies from regime to regime.

For example, under Solvency II the Available Capital will be the Solvency II Balance Sheet Own Funds, whereas anything involving internal model output as per C. and D., will likely involve a variant of an Adjusted Net Asset Value based on GAAP accounts.

Dealing with each of A to D.

A. Formula based models

1. Project Exposures for t =1,2,...n using granular data from P&L, Balance Sheet, Cashflow, Reserve projections
2. Use projected exposures in prescribed formula
3. Aggregate capital amounts using given correlation matrices or specified formulas.

B. Factor based models

Given standalone capital amounts for t = 0 this involves the following steps:

1. Calculate a Risk Factor based on granular capital amounts and exposure at t = 0
2. Project Exposures for t =1,2,...n using granular data from P&L, Balance Sheet, Cashflow, Reserve projections
3. Derive capital amounts at t = n by applying risk factors to projected exposures
4. Aggregate capital amounts using given correlation matrices

[2] The comments are based on my own experience building these models for clients for each of the Regulatory regimes mentioned

Formula wise these steps are for Risk k:

$$Risk\ Factor\ (k) = \frac{Capital\ (k, 0)}{Exposure\ (k, 0)}$$

$$Capital\ (k, n) = Risk\ Factor\ (k)\ \times Exposure\ (k, n)$$

The Risk Factor (k) can vary over time to be different to its value based on data at $t = 0$.
A combination of A. and B. might be used.

Example: SII SF SCR
For example, under the SII SF SCR, one might see a variation in the granularity by risk type, with more granularity in the (i) Non-Life Premium and Reserve risk and (ii) Non-Life Catastrophe risk modules and lesser granularity in Market risk, Counterparty Default risk and Operational risk. This can be seen in Table 6.1 below:

Risks	**How Modelled**
Non-Life Underwriting Risk - Non-Life Premium and Reserve Risk	Premium Risk component using projected exposures for NEP (last 12 months), NEP (next 12 months), FP (Existing), FP (Future) by SII LOB. Gross and Ceded. Reserve Risk component using projected values for Claims Provisions by SII LOB. The Claims Provisions being modelled by separating out into three components, (i) Claims Cost related, (ii) Future Premium and (iii) Future Commission and separately projecting each. Gross and Ceded. Projected Values: Claims Cost related = Factor x Claims Reserves Future Premium – Not Overdue part of projected premium based on premium debtor/creditor triangles (similar to reserve projection in section 5.7) Future Commission – Not Overdue part of projected premium based on commission debtor/creditor triangles (similar to reserve projection in section 5.7) The overdue parts of the above are separate entries in the SII Balance Sheet. The Geographical Diversification Factors are assumed to be constant
Non-Life Underwriting Risk - Non-Life Catastrophe Risk	Calculation of SCR after Risk Mitigation, SCR before Risk mitigation and Risk Mitigation. Separate modelling of Natural Catastrophe risk and Man-made Catastrophe risk at peril level within each. Peril Capital amounts projected in line with changes in appropriate exposures e.g. gross and ceded earned premium. For material Natural Catastrophe perils such as Windstorm and Flood use might be made of the two events scenarios and the accurate modelling of XOL / Quota Share reinsurance treaties.
Counterparty Default Risk - Type 1 and Type 2	Factor based as per B. with the selection of appropriate exposures.
Market Risk - Interest rate, Equity, Property, Spread, Market risk concentration and Currency	Factor based as per B. for each sub-risk type e.g. Interest rate the selection of appropriate exposures.

Table 6.1

By far the hardest part of the exercise from my experience is the projection of a multi-year SII Balance Sheets in order to derive values of the Own Funds, in other words the Excess of Assets over Liabilities. These amounts cannot be derived from the GAAP Balance Sheet Equity.

C. Standalone risk capital outputs from Internal Models
This is very similar to A except that diversification between risks is taken into account i.e. if the exposure doubles between t = 0 and t = 1 the increase in capital will be less than 100% unless there is 100% correlation between the original and additional exposures.

The only other consideration, dependent on the level of granularity, is to assume the level of diversification between risks. The relationship between the calculated capital at time t = n and time t = 0 depends on the underlying model assumptions.

Formula wise the steps are:

$$Capital\ (k, n) = Capital\ (k, 0) \times \left(\frac{Exposure\ (k,n)}{Exposure\ (k,0)}\right)^{\frac{1}{n}}$$

<u>Scaling Method</u>
Choices here could be Multiplicative, Additive, Power Rule or a weighted average of the first two. The method and weights may vary by class of business. Table 6.2 lists some scaling options.

Method	**Meaning**	**Value of n**
Multiplicative Scaling	Capital / Exposure ratio is preserved	n = 1
Additive Scaling	Capital amount is preserved	1/n = 0
Square Root Scaling	A mixture of Multiplicative and Additive	n = 2

Table 6.2

<u>Risk Aggregation</u>
Risk Aggregation for future years is exactly the same as at t = 0 with use being made of the aggregation rules and implied correlations derived at t = 0.

D Simulation Scaling
The idea here is to calculate future capital numbers by scaling the underlying simulations for exposure changes.

<u>Standalone Capital</u>
- Individual simulations for years t > 0 are calculated by applying Scaling Factors, based on changes in exposure, to the simulations at t = 0.
- The standalone capital for a risk in year t > 0 is derived by taking the percentile, e.g. 99.5%, of the resultant distribution

For Premium risk and Reserve risk this would likely be at the individual class level.

$$Loss\ Sim(k, i, n) = \ Loss\ sim(k, i, 0) \times Scaling\ Factor\ (k, n)$$

Where:
- Loss Sim (k, i, n) = Scaled ith simulation for Risk k in year n
- Exposure (k, n) = Exposure for Risk k in year n

The Loss Sim is the deviation from the mean rather than the absolute amount

The Reserve Risk exposure at t = 0 will be the Balance Sheet reserves whilst the Premium Risk exposure will be the 1st period future premium.

The following formulae show how the Scaling Factor (k, n) varies with the scaling method:

Multiplicative Scaling

$$Scaling\ Factor\ (k, n) = \frac{Exposure\ (k, n)}{Exposure\ (k, 0)}$$

Additive Scaling

$$Scaling\ Factor\ (k, n) = 1$$

Power Rule Scaling:

$$Scaling\ Factor\ (k, n) = \left(\frac{Exposure\ (k, n)}{Exposure\ (k, 0)}\right)^{\frac{1}{n}}$$

e.g. when n = 2, Square Root scaling.

Risk Aggregation
Risk Aggregation for future years is exactly the same as at t = 0 in that the scaled simulations are added to derive capital amounts at the appropriate level of granularity.

If we assumes that there are m sub-risks within Risk Category j then:

$$Loss\ Sim(Risk\ Category\ j, i, n) = \sum_{k=1}^{m} Loss\ sim(k, i, n)$$

where
Loss Sim (Risk Category j, i, n) = Scaled simulation ith simulation for Risk Category j in year n.

For example Risk Category j might be Reserve risk and Risk k the reserve risk capital for class k

Selections
The key selections here are:

Exposures:
The measure plus the level of granularity e.g. for Reserve Risk this might be Net Claims Reserves or for Premium Risk Net Earned Premium

Scaling Method:
Choices here could be Multiplicative, Additive, Power Rule or a weighted average of the former. The weights may vary by class of business.

6.4 SII SF SCR Example – Base Case

I thought it would be useful to give an example of the projection of future solvency ratios under one regulatory capital regime. For this purpose, I have focused on the Solvency II SF SCR calculation.

Each of the standalone risk capital projections can be performed with different levels of granularity. I have focused on a high-level approach. For the purposes of this paper the projection of the SII Own Funds shown is rather crude as I have not attempted to project a SII Balance Sheet.

Table 6.3 shows two sets of projected numbers:

Solvency Ratio

- Ratio of Available Capital ("SII Own Funds") to Regulatory Capital ("SF SCR")
- Comparison of the Available Capital with the Target Capital based on a company's risk appetite. The target capital for instance might be the amount consistent with a particular agency rating.

SF SCR Summary

- The Final SCR is calculated by adding the Basic SCR, Operational Risk SCR and an amount for the Loss absorbing capacity of Technical Provisions / Deferred Tax
- The Basic SCR ("BSCR") is based on the projected standalone capital amounts for Non-Life Underwriting Risk, Market Risk and Counterparty Default Risk. It is assumed that the Life Underwriting SCR and Health Underwriting SCR capital amounts are zero.
- The capital amounts for the risks making up the BSCR are themselves a result of aggregation at a lower level of granularity
- The BSCR correlation matrix is shown in Table 6.4.

Solvency

Entity	**Foxes Capital**					
BS Date	**31/12/2021**					
First AY	**2022**					
Currency / Units	**£ m**					
	YE	**2021**	**2022**	**2023**	**2024**	**2025**
	AY	**2022**	**2023**	**2024**	**2025**	**2026**
Solvency Ratio						
Available Capital (SII Own Funds)		965	1,050	1,137	1,227	1,321
SF SCR		760	757	779	807	841
Excess over SF SCR		205	293	358	420	481
Economic Capital (SCR x 1.2)		913	909	935	969	1,009
Excess over Economic Capital		53	141	202	259	313
Solvency Ratio		**127%**	**139%**	**146%**	**152%**	**157%**
SF SCR Summary						
Undiversified BSCR		923	920	946	980	1,020
Diversification Credit		-199	-200	-206	-213	-222
Basic SCR		724	720	741	767	798
Operational Risk		37	37	39	41	43
Loss absorbing capacity of TPs / Def Tax		0	0	0	0	0
Final SF SCR		**760**	**757**	**779**	**807**	**841**

Table 6.3

Basic BSCR

	Parameters				
$Corr_{ij}$	SCR_{mkt}	SCR_{def}	SCR_{life}	SCR_{health}	SCR_{nl}
SCR_{mkt}	100%	25%	25%	25%	25%
SCR_{def}	25%	100%	25%	25%	50%
SCR_{life}	25%	25%	100%	25%	0%
SCR_{health}	25%	25%	25%	100%	0%
SCR_{nl}	25%	50%	0%	0%	100%

Table 6.4

Table 6.5 shows four sets of projected numbers for the main risks categories Non-Life Underwriting Risk, Market Risk, Counterparty Default Risk and Operational Risk.

The standalone risk capital numbers shown in column (YE 2021, AY 2022) are the initial SII SF SCR amounts as at 31/12/2021 (t = 0).

		YE	2021	2022	2023	2024	2025
		AY	2022	2023	2024	2025	2026
Capital BY Risk Category							
Non-Life Underwritng Risk	Premium and Reserve Risk		529	524	537	555	577
	Catastrophe Risk		0	0	0	0	0
	Lapse Risk		0	0	0	0	0
	Premium Risk		200	210	221	232	243
	Reserve Risk		400	386	392	402	416
	SCR_{nl} Pre-Div		529	524	537	555	577
	SCR_{nl} Div Credit		0	0	0	0	0
	SCR_{nl} Post Div		**529**	**524**	**537**	**555**	**577**
Market Risk	Interest Rate Risk		60	60	62	63	66
	Equity Risk		80	81	84	88	92
	Property Risk		50	51	52	55	57
	Spread Risk		125	125	128	132	137
	Concentration Risk		25	25	25	25	25
	Currency Risk		120	121	126	132	138
	SCR_{mkt} Pre-Div		460	463	477	495	515
	SCR_{mkt} Div Credit		-137	-138	-142	-146	-152
	SCR_{mkt} Post Div		**323**	**325**	**336**	**349**	**364**
Counterparty Default Risk	Type 1 Risk		50	49	50	51	53
	Type 2 Risk		25	26	28	29	30
	SCR_{def} Pre-Div		75	75	78	80	84
	SCR_{def} Div Credit		-4	-4	-5	-5	-5
	SCR_{def} Post Div		**71**	**71**	**73**	**76**	**79**
Operational Risk	GEP year previous		1,220	1,233	1,295	1,359	1,427
	GEP 2 years previous		1,050	1,220	1,233	1,295	1,359
	Operational Risk		**37**	**37**	**39**	**41**	**43**

Table 6.5

A description of each of the calculations is a follows:

<u>Non-Life Underwriting Risk</u>

- Standalone capital amounts are projected for Premium and Reserve Risk, Catastrophe Risk and Lapse Risk
- These standalone capital amounts are aggregated using the Non-Life Underwriting Risk correlation matrix in Table 6.6

- Premium Risk and Reserve Risk are shown separately as the exposure bases used to project future capital amounts vary for each. The actual Non-Underwriting Risk SCR module uses a calculation based on combined risk exposures
- For future YE (> 2021) /AY (> 2022) the Premium Risk and Reserve Risk numbers are first projected and then combined to calculate a Premium and Reserve Risk SCR based on the implied between Premium Risk / Reserve Risk correlation for (YE 2021, AY 2022). A more detailed calculation would involve replicating the Non-life Premium and Reserve Risk model as described in Table 6.1

Non-Life Underwritng Risk

	NL Premium	NL Catastrophe	NL Lapse
NL Premium and Reserve	100%	25%	0%
NL Catastrophe	25%	100%	0%
NL Lapse	0%	0%	100%

Table 6.6

<u>Market Risk</u>

- Standalone capital amounts are projected for Interest Rate Risk, Equity Risk, Property Risk, Spread Risk and Concentration Risk.
- These standalone capital amounts are aggregated using the Market Risk correlation matrix in Table 6.7.

Market Risk

	Parameters					
Corrij	Mktint	Mkteq	Mktprop	Mktsp	Mktconc	Mktfx
Mktint	100%	50%	50%	50%	0%	25%
Mkteq	50%	100%	75%	75%	0%	25%
Mktprop	50%	75%	100%	50%	0%	25%
Mktsp	50%	75%	50%	100%	0%	25%
Mktconc	0%	0%	0%	0%	100%	0%
Mktfx	25%	25%	25%	25%	0%	100%

Table 6.7

<u>Counterparty Default Risk</u>

- Standalone capital amounts are projected for Type 1 Risk and Type 2 Risk
- These standalone capital amounts are aggregated using an SF SCR formula which is the same as assuming a correlation of 75% between Type 1 and Type 2 capital amounts.

<u>Operational Risk</u>

- Operational risk for future years is based on formulae for the prior year and 2 year’s previous Gross Earned Premium (“GEP”) and Gross Claims Provision.

6.5 SII SF SCR Scenario Example – Gross and Net Reserve increase

One may be interested in the impacts on a "theoretical share price" arising from a scenario.

Let's consider a scenario where the gross and net reserves increase by 10% (reserve deterioration) across all AYs during CY 2023, i.e. when t = 2.

Figures 6.8 and 6.9 show the impact on the gross and net reserves respectively for YE 2023 through to YE 2026.

Gross	AY	Incurred	Projection 31/12/21	31/12/22	31/12/23	31/12/24	31/12/25	31/12/26
	2007		0	0	0	0	0	0
	2008		0	0	0	0	0	0
	2009		0	0	0	0	0	0
	2010		23	0	0	0	0	0
	2011		23	0	0	0	0	0
	2012		46	0	0	0	0	0
	2013		46	18	0	0	0	0
	2014		69	38	16	0	0	0
	2015		92	55	33	13	0	0
	2016		138	98	65	36	14	0
	2017		230	156	122	73	40	16
	2018		253	187	139	99	59	33
	2019		345	268	218	147	105	63
	2020		460	360	308	227	153	109
	2021		575	472	406	316	233	158
	2022	666		559	505	359	279	206
	2023	699			645	482	377	293
	2024	734				616	506	396
	2025	771					647	531
	2026	809						679
	Total		**2,300**	**2,210**	**2,456**	**2,366**	**2,412**	**2,482**
Reserve Increase / Decrease %			0%	0%	10%	0%	0%	0%
Reserve Increase / Decrease			0	0	223	0	0	0

Figure 6.8

Net	AY	Incurred	Projection 31/12/21	31/12/22	31/12/23	31/12/24	31/12/25	31/12/26
	2007		0	0	0	0	0	0
	2008		0	0	0	0	0	0
	2009		0	0	0	0	0	0
	2010		17	0	0	0	0	0
	2011		17	0	0	0	0	0
	2012		34	0	0	0	0	0
	2013		34	13	0	0	0	0
	2014		50	28	12	0	0	0
	2015		67	40	24	9	0	0
	2016		101	72	47	26	10	0
	2017		168	114	89	53	29	11
	2018		185	136	101	72	43	24
	2019		252	196	159	107	77	46
	2020		336	263	225	166	112	80
	2021		420	345	297	231	170	115
	2022	495		415	375	267	207	153
	2023	520			480	358	280	218
	2024	546				458	376	294
	2025	573					481	395
	2026	602						505
	Total		**1,679**	**1,621**	**1,809**	**1,747**	**1,785**	**1,840**
Reserve Increase / Decrease %			0%	0%	10%	0%	0%	0%
Reserve Increase / Decrease			0	0	164	0	0	0

Figure 6.9

Figures 6.10 and 6.11 show the movements in the available capital, required capital and solvency ratios:

	Difference				
	2021 2022	2022 2023	2023 2024	2024 2025	2025 2026
Solvency Ratio					
Available Capital (SII Own Funds)	0	0	-75	-74	-74
SF SCR	0	0	40	11	6
Excess over SF SCR	0	0	-115	-85	-80
Economic Capital (SCR x)	0	0	48	14	7
Excess over Economic Capital	0	0	-123	-88	-81
Solvency Ratio	**0%**	**0%**	**-16%**	**-11%**	**-10%**
SF SCR Summary					
Undiversified BSCR	0	0	49	13	6
Diversification Credit	0	0	-8	-1	0
Basic SCR	0	0	40	11	6
Operational Risk	0	0	0	0	0
Loss absorbing capacity of TPs / Def Tax	0	0	0	0	0
Final SF SCR	**0**	**0**	**40**	**11**	**6**

Figure 6.10

		Difference				
		2021 2022	2022 2023	2023 2024	2024 2025	2025 2026
Capital BY Risk Category						
Non-Life Underwritng Risk	Premium and Reserve Risk	0	0	37	13	9
	Catastrophe Risk	0	0	0	0	0
	Lapse Risk	0	0	0	0	0
	Premium Risk	0	0	0	0	0
	Reserve Risk	0	0	39	14	10
	SCR_{nl} Pre-Div	0	0	37	13	9
	SCR_{nl} Div Credit	0	0	0	0	0
	SCR_{nl} Post Div	0	0	37	13	9
Market Risk	Interest Rate Risk	0	0	2	0	-1
	Equity Risk	0	0	2	0	-1
	Property Risk	0	0	1	0	-1
	Spread Risk	0	0	3	-1	-2
	Concentration Risk	0	0	0	0	0
	Currency Risk	0	0	3	-1	-2
	SCR_{mkt} Pre-Div	0	0	12	-3	-5
	SCR_{mkt} Div Credit	0	0	-3	1	1
	SCR_{mkt} Post Div	0	0	9	-2	-4
Counterparty Default Risk	Type 1 Risk	0	0	3	1	1
	Type 2 Risk	0	0	0	0	0
	SCR_{def} Pre-Div	0	0	3	1	1
	SCR_{def} Div Credit	0	0	0	0	0
	SCR_{def} Post Div	0	0	3	1	1
Operational Risk	GEP year previous	0	0	0	0	0
	GEP 2 years previous	0	0	0	0	0
	Operational Risk	0	0	0	0	0

Figure 6.11

Figure 6.12 shows the impact on the P&L. As can be observed the movement in the gross incurred and net incurred claim numbers during CY 2023 are the same as the gross and net reserve deteriorations in Figures 6.8 and 6.9.

Difference					
P&L	**2022**	**2023**	**2024**	**2025**	**2026**
Gross Written Premium	0	0	0	0	0
Ceded Written Premium	0	0	0	0	0
Net Written Premium	**0**	**0**	**0**	**0**	**0**
Gross Earned Premium	0	0	0	0	0
Ceded Earned Premium	0	0	0	0	0
Net Earned Premium	**0**	**0**	**0**	**0**	**0**
Gross Claims Incurred	0	-223	0	0	0
Ceded Claims Incurred	0	59	0	0	0
Net Claims Incurred	**0**	**-164**	**0**	**0**	**0**
Gross Acquisition Costs	0	0	0	0	0
Ceded Acquisition Costs	0	0	0	0	0
Net Acquisition Costs	**0**	**0**	**0**	**0**	**0**
Operational expenses	0	0	0	0	0
Other expenses (e.g. foreign exchange)	0	0	0	0	0
Total Expenses	**0**	**0**	**0**	**0**	**0**
Net UW Result	**0**	**-164**	**0**	**0**	**0**
Investment Income	0	0	2	0	-1
Realised Gains / Losses	0	0	0	0	0
Investment expenses	0	0	0	0	0
Net Investment Result	**0**	**0**	**2**	**0**	**-1**
Other Income (Ceding Coom, Broker Fee)	0	0	0	0	0
Operating Result	0	-164	2	0	-1
Finance costs	0	0	0	0	0
Other	0	0	0	0	0
Profit/(loss) before tax	**0**	**-164**	**2**	**0**	**-1**
Tax	0	33	0	0	0
Net Income after Tax	**0**	**-132**	**2**	**0**	**-1**
Net Income Attributed to non-controlling	0	0	0	0	0
Other Net Income	0	0	0	0	0
Net Income before Dividend	0	-132	2	0	-1
Dividend	0	53	-1	0	0
Retained Earnings Change	**0**	**-79**	**1**	**0**	**0**

Figure 6.12

The other impacts during CY 2023 are on the tax paid (a reduction of 33 = 20% x 164) and dividends (a reduction of 53 = 40% x 132. For CY 2024 there is a small increase in the Net Income after Tax due to the investment income on larger Investments and Cash arising from an increase in the Net Cashflow (see later).

This analysis of change is useful for explaining to the Board / Senior Management the reasons for the movements in the financial projections and to identify and subsequently resolve any potential model inconsistencies or errors.

Figure 6.13 shows the impact on the Balance Sheet. As can be observed the change in the (increase in the Investments and the increase in Cash and Reinsurance recoveries) = 144, equals the change in the reduction in the Retained Earnings and the increase in the Gross claims reserves as at YE 2023.

A reserve deterioration during CY 2023 first manifests itself in increased reserve exposures at YE 2023. The Gross and Ceded reserve movements reconcile with Gross and Net reserve changes that we saw in Figures 6.8 and 6.9.

The Investments and Cash change of +85 reflects the +ve Cashflow change during CY 2023 (see Figure 6.14) and the subsequent -ve Cashflow movements the increase in gross and ceded claims payments arising from the increase in reserves.

Difference

Balance Sheet	31/12/22	31/12/23	31/12/24	31/12/25	31/12/26
Assets					
Investments	0	79	-16	-35	-50
- Debt / Fixed Income	0	66	-14	-29	-40
- Equities / Investment Funds	0	11	-2	-5	-7
- Other	0	3	-1	-2	-3
Cash	0	6	-1	-3	-4
Investments and Cash	**0**	**85**	**-18**	**-38**	**-54**
Property, plant and equipment	0	0	0	0	0
Goodwill & Intangibles	0	0	0	0	0
DAC	0	0	0	0	0
Ceded UPR	0	0	0	0	0
Reinsurance recoveries	0	59	22	15	9
Receivables on insurance / reinsurance	0	0	0	0	0
Reinsurance recoveries	**0**	**59**	**22**	**15**	**9**
Deferred tax assets	0	0	0	0	0
Premium Debtors	0	0	0	0	0
Other Assets	0	0	0	0	0
Total Assets	**0**	**144**	**4**	**-24**	**-45**
Liabilities					
Share Capital & Premium	0	0	0	0	0
Retained Earnings	0	-79	-78	-78	-79
Other	0	0	0	0	0
Equity	0	-79	-78	-78	-79
Non-controlling interest	0	0	0	0	0
Total Equity	**0**	**-79**	**-78**	**-78**	**-79**
Gross UPR	0	0	0	0	0
Gross Claims Reserves	0	223	83	55	34
Ceded DAC	0	0	0	0	0
Creditors insurance / reinsurance	0	0	-1	-1	-1
Financial Liabilities	0	0	0	0	0
Deferred tax liabilities	0	0	0	0	0
Other Liabilities	0	0	0	0	0
Liabilities	**0**	**223**	**82**	**54**	**34**
Equity and Liabilities	**0**	**144**	**4**	**-24**	**-45**
Difference: Total Assets - Liabilities	**0**	**0**	**0**	**0**	**0**
Net Cashflow	**0**	**85**	**-103**	**-21**	**-15**

Figure 6.13

Figure 6.14 shows the impact on the Cashflow Statement.

Difference

Cashflow	2022	2023	2024	2025	2026
Gross Premium Received	0	0	0	0	0
Gross Paid Acquisition costs	0	0	0	0	0
Ceded Premium Paid	0	0	0	0	0
Ceded Acquisition costs	0	0	0	0	0
Gross Losses paid	0	0	-141	-28	-21
Ceded Losses received	0	0	37	8	6
Receivables on insurance / reinsurance	0	0	0	0	0
Operational expenses	0	0	0	0	0
Other expenses (e.g. foreign exchange)	0	0	0	0	0
Investment Income Received	0	0	2	0	-1
Other Income (Ceding Coom, Broker Fee)	0	0	0	0	0
Finance costs	0	0	0	0	0
Other	0	0	0	0	0
Tax Paid	0	33	0	0	0
Dividend Paid	0	53	-1	0	0
Increase in Deferred Tax Assets	0	0	0	0	0
Other Cashflows	0	0	0	0	0
Net Cashflow	**0**	**85**	**-103**	**-21**	**-15**

Figure 6.14

7. Risk Discount Rate

7.1 Introduction

One of the methods used to determine the discount rate on risky equity investments in an equilibrium condition is the Capital Asset Pricing Model ("CAPM"). Indeed on reading Equity Analyst reports, and academic books used in BMA / CFA courses the CAPM formula appeared to be the most popular method used to determine the risk discount rate.

It was developed to determine the appropriate discount rate for risky cashflows. However, being a single period model, it only ever solved the problem for a single time step or period e.g. say one year. This is fine if the only cashflow is one payment, but it is commonly used to discount the cashflows from multiple periods. Therein lies its problem.

As Sharpe (1964)[3] wrote (page 434, Section III. Equilibrium in the Capital Market):

"However, since the proper test of a theory is not the realism of its assumptions but the acceptability of its implications, and since these assumptions imply equilibrium conditions which form a major part of classical financial doctrine, it is far from clear that this formulation should be rejected-especially in view of the dearth of alternative models leading to similar results."

Note:
This section uses material from a wider paper that I have been writing "Capital Asset Pricing Model – Nice in Theory but not in Practice".

7.2 Capital Asset Pricing Model

Under CAPM the expected return for a share / stock i is given by:

$$E(R_i) = R_f + \beta\left(E(R_M) - R_f\right)$$

where:
R_f = Expected Risk-free rate
$E(R_M)$ = Expected Return on the Market Portfolio
β = a measure of the extent the rate of return of an investment co-varies with the rates of return on a portfolio representing the overall market.

The value of $(E(R_M) - R_f)$ is also known as the Equity Risk Premium, i.e. the margin over the risk-free rate.

The formula above can also be re-written as:

$$E(R_i) = R_f(1 - \beta) + \beta E(R_M)$$

[3] Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk, The Journal of Finance, Vol XIX, Sharpe,W. (1964)

CAPM Assumptions

The following is a list of the assumptions underlying the model. The list is not exhaustive and it is not easy to find a comprehensive list in any papers that I have looked at so I have used a number of different sources and summarised my own understanding:

- Single (not a multi) period model
- Investors are risk-averse and when choosing among portfolios they care about the mean and variance of their one period return
- For any given risk investors prefer the highest expected return
- For any given expected return investors prefer the lowest risk
- Homogeneity of investor expectations i.e. investors agree on the expected values and standard deviations of investments and the correlation coefficients between them
- A common riskless asset exists and all investors agree that it is riskless
- Lending at the risk-free rate is allowed
- Borrowing at the risk-free rate is allowed
- No taxes or transaction costs
- The Capital Market is in Equilibrium i.e. there is no excess supply of, or demand for assets.

Beta [4]

There are two common methods to determine the value for a company:

- Company Beta – The historical share price data of a company can be used to calculate its' Beta. This is achieved via a linear regression of a company's returns against the market returns. They are reported by Bloomberg and other market sources.
- Industry Beta – Due to the volatility caused by changes in a company's risk over time sometimes an industry beta, based on a pool of companies, is used. This has its own inherent challenges e.g. differences in the (i) mix of business between the company and that represented on average by the pool of companies used to calculate the industry beta, (ii) financial leverage caused by issuing debt making cashflows to equity holders riskier.

Capital Market Line

The Capital Market Line ("CML") is the straight line from the risk-free rate R_j through T, the Market Portfolio. This is illustrated in Figure 7.1.[5] The point T represents $E(R_{M.})$, the expected return and volatility of the Market Portfolio.

Points along the line between R_j and T correspond to a position where monies are part invested in the risk-free rate and the Market Portfolio. Points beyond T, the position of borrowing monies at the risk-free rate with the excess invested in the Market Portfolio.

The idea behind the CML is that as everyone has the same assets available to choose from they will all identify the same portfolio of risky assets, the Market Portfolio, represented by Point T. Investors, in order to maximise their return for any given risk, or minimise their risk for any given return will only ever choose an investment which sits along the CML.

[4] CAS Exam 8 Study Note: P&C Insurance Company Valuation, Goldfarb, R. (2005)
[5] The Capital Asset Pricing Model: Theory and Evidence" by Fama and French (2004)

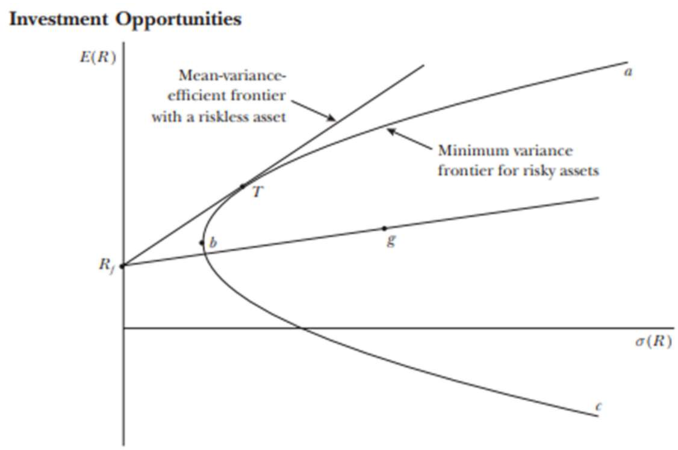

Figure 7.1

Market Portfolio in the real-world
But this brings home one of the issues with CAPM in general in that the Market Portfolio is only assumed and not actually derived from Efficient Frontier data.

We don't need to know what the other values along the Efficient Frontier are because in the standard CAPM proof it is assumed that where the Capital Market Line ("CML") is tangential to the Efficient Frontier we have the Market Portfolio, which all investors have identified as the optimal point. The next step being to use a proxy for it, e.g. the use of Index Funds such as the FTSE 100 or FTSE All-Share indices in the UK or the S&P 500 in the US.

7.3 A Simple Model

To begin matters I decided to build a simple model in Excel so that I could produce a graph that included both the Efficient Frontier and the Capital Market Line ("CML"). I wanted the Efficient Frontier to be not too dissimilar to those typically appearing in text books. The assumptions used are for illustration purposes only. The Efficient Frontier is a plot of 1,000 data points with weights, w and (1-w) varying from 0% to 100% in steps of 0.1%.

I have assumed two assets A and B, each with an expected return and standard deviation of return and a correlation between them. I could have selected three or more assets but then the mathematical exercise would have become rather more complicated, and besides my objective is to explore ideas and illustrate points.

The model is used to derive the values of the expected return and standard deviation of the Market Portfolio for any given value of the risk-free rate and assumptions for A and B.

How this is achieved is explained in Section 10.1, Appendix 1.

7.4 Risk-free rate and the Volatility Adjusted CAPM

When looking at the interaction of the CML and Efficient Frontier in Figure 7.1 the key assumption is that all investors agree on what is the Market Portfolio. However, equally important is that they agree on the value of the risk-free rate of interest.

If you change the risk free rate you change the slope of the CML and the point of intercept on the Efficient Frontier which is the Market Portfolio. This can be seen in Figures 7.2 and 7.3 with different risk-free rate scenarios of 2.0% and 3.0% respectively.

The Market Portfolio $E(R_M)$ = 7.5% and 8.3% respectively and the weights on Asset A:B = 45.5%:54.5% and 36.6%:63.4%. The Equity Risk Premiums are 5.5% (=7.5% - 2.0%) and 5.3% (= 8.3% - 3.0%).

This does not necessarily mean that the Equity Risk Premium will remain constant for different risk-free rate scenarios, e.g. when the risk-free rate is 1.0% the Equity Risk Premium is, according to the model, 6.0%. See Table 7.4 in Section 7.5 for values under different risk-free rate scenarios.

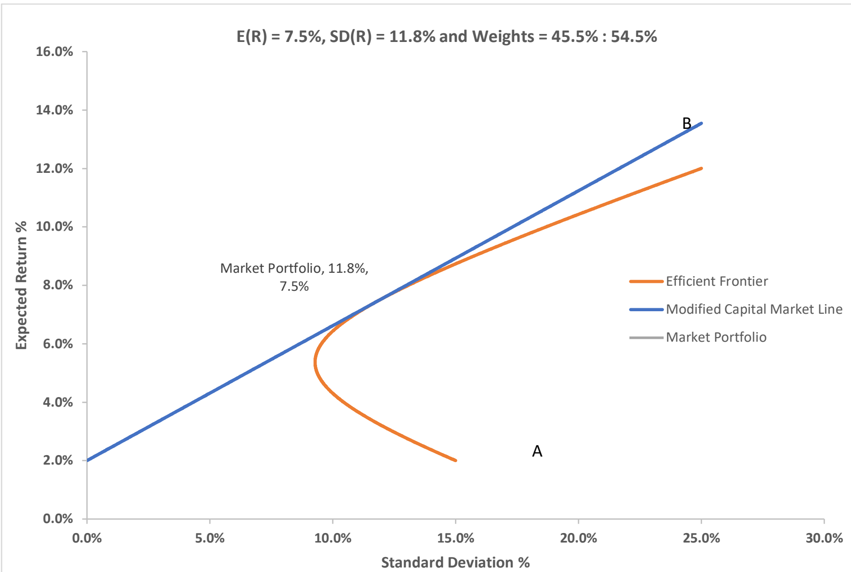

Figure 7.2

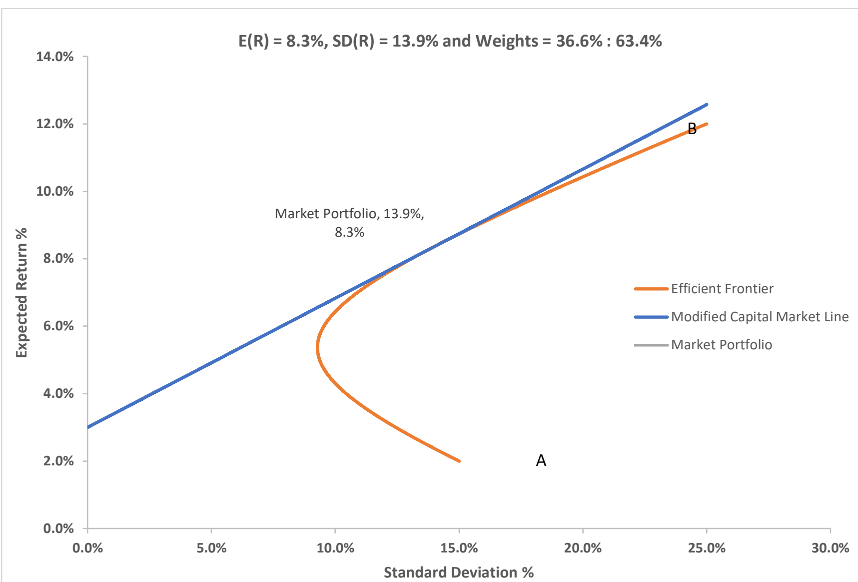

Figure 7.3

As the weights are different one will have two different Market Portfolios, Equity Risk Premiums and β if calculated from a regression against historical data. This has implications if CAPM is to determine the discount rate in future years assuming a varying risk-free rate to reflect the term structure of interest rates; see section 7.5.

Is the Interest-rate volatility zero ?
I find this assumption difficult to always justify as:
1. The assets on the Efficient Frontier and within its boundaries are assumed to be volatile
2. Interest rates are volatile in the real world.

When could the interest rate volatility be zero
I can think of two scenarios where having zero volatility makes sense:
1. Duration of the risk-free asset is equal to the length of the single period, e.g. a one year zero coupon bond in a one-year model
2. The interest rate is guaranteed for each year over N Years.

However if the interest rate instrument is a Zero Coupon Bond ("ZCB") or Treasury Bill and the Duration is not equal to the length of the Single Period then I do not believe the volatility can be zero. This becoming more of an issue for time periods beyond the 1st year.

Let's consider a single period of one year. There are two scenarios:
1. Asset Duration < Single Period – The Asset will mature before the end of the single period resulting in a reinvestment risk, and thus volatility exists
2. Asset Duration > Single Period – the return over one year will depend on the yield curve at the end of that year. For example, suppose that the risk-free investment is a 2-year zero coupon bond and that the 1 and 2-year spot rates are 5% p.a. The return over one year will equal 5% p.a. if the 1-year spot rate at the end of 1 year is 5% p.a. still, however if it decreases to 4% p.a. say then the return over 1 year will equal 6% i.e. volatility is present.

   If we assume a 2-year Zero Coupon Bond = 100, then Value at t = 0 = 100 / 1.05^2 = 90.7. It's value at t = 1 is 100 / 1.04 = 96.2. The return over 1 year = 96.2 / 90.7 – 1 = 6%.

Volatility Adjusted CAPM (Work in Progress)
I have relegated this to Appendix 10.2 as it is paused Work in Progress and I was in two minds whether to include it or not. I had explored what happens if one assumes that the risk-free rate volatility > 0. This led me to develop what I had called the "Volatility Adjusted CAPM" formula involving the "Modified Capital Market Line".

Volatility Adjusted CAPM

$$E(R_i) = \frac{R_f + \beta\big(E(R_M) - R_f\big) - V_R\, E(R_M)}{(1 - V_R)}$$

where,

$$V_R = \frac{\sigma_f}{\sigma_M}$$

As V_R approaches zero the formula approaches the standard CAPM formula. Key values are the volatility of the risk-free rate (σ_f) and the volatility of the Market Portfolio (σ_M).

Modified Capital Market Line
The above formula follows the standard CAPM formula proof except that the CML is replaced by the "Modified Capital Market Line" ("MCML")

$$MCML = \frac{\left(E(R_M) - R_f\right)}{\left(\sigma_M \ - \sigma_f\right)}$$

However having done all of this work and produced a draft Actuarial Essay [6] on the said topic it was pointed out to me by a fellow Actuary that the MCML will only be a straight line if the correlation between the risk-free rate and the market portfolio is 100%.

A discussion of, numerical example and proof of the Volatility Adjusted CAPM is given in Section 10.2, Appendix 2

<u>Risk-free rate vs Market Portfolio correlation</u>
So the question then becomes what is the correlation between the risk-free rate and the Market return. In the real world it will not be 100% however that is not what matters. The key question is <u>what is the implied correlation given the model construct</u> and the way the CAPM formula is derived. This is very different.

In my simple model it hasn't gone unnoticed that if the risk-free rate increases so does the value of the market return, and vice-versa. It is straightforward to calculate pairs of values for small increments in the risk-free rate and compute the correlation between the resultant data.

The correlation will depend on the assumed shape of the Efficient Frontier. For example using the assumptions in construction of the Efficient Frontier the correlation is 90%. Keeping the expected return and volatility the same for assets A and B and only varying the correlation between the two I get 92% and 97%. There is limit to how one can take this exercise as each time I require the Efficient Frontier to be also not unreasonable.

For now I have left in the appendix and will think about this at a later date when completing my wider paper.

<u>What does this mean for the Capital Market Line</u>
The CAPM formula assumes that the risk-free rate volatility is zero. This is important for the CAPM formula proof as the shape of the Capital Market Line (a straight line, am ignoring the word line here) does not depend on the correlation between the interest rate and the market portfolio and the interest rate as two out of the three variance-covariance terms in the derivation of the weighted average volatility are zero.

7.5 Multi-Year CAPM Discount Rates

There are number of points to consider when moving from a single period, say one year, to a multi-period view, two years or more.

This section is limited to a discussion of the use of the standard CAPM model in a multi-period setting. It does not consider other models that exist e.g. models based off the work of Merton (1973)[7] etc.

Standard textbooks, and indeed Equity Analyst reports often assume the discount rate derived using CAPM is the annualised rate for future durations. This section expands on that idea.

[6] "The Volatility Adjusted CAPM and CAPM Issues (Part I)" of 4/7/24.
[7] An intertemporal Capital Asset Pricing Model, Merton, R. (1973)

CAPM Formula Variations
In theory one could argue that there are three potential variations of the CAPM formula:
1. Use the same rate for all durations
2. Risk-free rate varies by term with the same value of the Equity Risk Premium
3. Risk-free rate varies by term with the same value of the Market Portfolio Return,
   - The Equity Risk Premium will also change too.

Assuming options 2. or 3. can lead to either under estimation or overestimation of the risk discount rate as there is not a straightforward relationship between values of the risk-free rate, the market portfolio return and the equity risk premium. This can be seen in Table 7.4 which shows the calculated values of E(R) and the Equity Risk Premium for values of the risk-free rate from 0.5% to 4.5%.

The implied Equity Risk Premium is not constant and under the CAPM framework its value is a function of the value of the Risk-free rate. This raises interesting questions on how the Equity Risk Premium is parameterised.

Looking at historical data and comparing the difference between historical values (either arithmetic or geometric returns) of the Market Return and the appropriate Risk-free rate, be it Short term or Long term rates implicitly assumes that the expected value of the Equity Risk Premium is invariant to the level of the Risk-free rate. Table 7.4 would indicate otherwise.

Risk-free rate	E(R)	E(R) - R_f	σ	A Weight	B Weight
0.5%	6.8%	6.3%	10.6%	51.9%	48.1%
1.0%	7.0%	6.0%	10.9%	50.2%	49.8%
1.5%	7.2%	5.7%	11.2%	48.2%	51.8%
2.0%	7.5%	5.5%	11.8%	45.5%	54.5%
2.5%	7.8%	5.3%	12.6%	41.8%	58.2%
3.0%	8.3%	5.3%	13.9%	36.6%	63.4%
3.5%	9.1%	5.6%	16.1%	28.7%	71.3%
4.0%	10.5%	6.5%	20.3%	14.9%	85.1%
4.5%	12.0%	7.5%	25.0%	0.0%	100.0%

Table 7.4

Dealing with each of these methods.

Level Discount Rate

Advantages:
1. Simple
2. It also avoids the issue that in theory when one changes the risk-free rate there will be new values for both the Equity Risk Premium and the Market Portfolio Return.

Disadvantages:
1. Risk-free rate will not be level in the real world
2. The 12-month forward discount rates are identical for each future year.
   - When discounting cashflows in year N the discount rate is assumed to be $1/(1+ r\%)^N$ where r% is the level discount rate. However this implicitly assumes that all of the forward 12-month risk discount rates are the same e.g. the 12-month rate as viewed from year end 4 is also r% and so on.

- This approach ignores the uncertainty in the estimation of the risk premium that will increase with time, if in a simplistic view of the world the risk-free rate is assumed to be constant over all durations.

The subtlety of point 2. will not necessarily be noticed because in present value terms $1/(1+r\%)^{N+1} < 1/(1+r\%)^{N}$ i.e. the present value is lower for longer durations.

Equity Risk Premium / Market Portfolio Return

Advantages:
- Simple
- Reflects the term structure of interest rates

Disadvantages:
- A different risk-free rate will imply a new Market Portfolio (and Return), Equity Risk Premium value of β if calculated from a regression against the Market
- This will require knowledge of the underlying Efficient Frontier, but does this exist
- Deciding which of the two bases is more appropriate. Each responds differently according to the shape of the yield curve.

7.6 CAPM Parameterisation inconsistencies

If one follows through step by step the CAPM formula proof [8] there are two key assumptions:
1. Risk-free volatility is zero
2. Risk-free rate and the Equity Risk Premium are consistent with each other i.e. the same single period in the same diagram.

If either or both of these conditions do not hold then using the CAPM formula to set the risk discount rate would appear to no longer be valid. Looking at each of these points in turn.

1. Selected Risk-free rate instrument

In the parameterisation of the CAPM model it is important that the selected risk-free rate instrument does not contradict the assumption that the risk-free rate volatility is zero. From what I have read whether in reports, papers or books this does not always appear to be the case. See Section 7.4 on the duration point.

For example in the paper by Goldfarb (2005) he cites having observed the following Risk-free rate instruments, (i) 90-Day T-Bills, (ii) Maturity Matched T-Notes (a security with a term equal to that of the average maturity of the cash flows and (iii) T-Bonds (Yields on 20-year T-Bonds representing the estimate of the long run average short-term yields. A term premium between the long-term and short-term yields being netted out.

Another example comes from Copeland (2000) where 10-year Treasury Bonds are recommended, with 30-year Treasury Bonds and Treasury Bills being reasonable alternatives.

[8] This can be seen in the first three steps of the formula in Appendix 10.2 where the only departure being step 4 where the derivative is set equal to the CML rather than the MCML as I have done.

2. Risk-free rate and the Equity Risk Premium consistency

The Risk-free rate and Equity Risk Premium are assumed to be consistent with each other, i.e. if one chooses a value for the Risk-free rate its value is also used to derive the Equity Risk Premium. Again I do not find this to be always so especially in studies where the Equity Risk Premium is derived from historical averages which is very common.

To combine an Equity Risk Premium based on averages with a risk-free rate based on current market values, even if the same instrument is used for both, would again appear to invalidate the CAPM proof assumptions and steps.

This is not to say that the CAPM formula cannot be used, however it imposes on the Analyst / Modeller the need for additional justification of the resultant Risk Discount rate.

7.7 Discount Rate and DDM / EVA Equivalence

My own preference is for the discount rate at any point in time N to be considered as the product of 12-month forward rates.

Let's further define K_t:

$$(1 + K_t)^t = \prod_{i=1}^{t}(1 + k_i)$$

where k_t = varying cost of equity for the time period (t-1,t) and not the same value of k. i.e. similar to how one would define, K_t, if it was a spot rate of interest for term t.

What's interesting is that the equivalence of the different methods does not depend on the discount rate being the same. I first wrote about this in a draft paper [9] during the summer of 2023

Proof

In the standard modelling it is assumed that future 12-month Risk Discount rates, k say, are unchanged as the Discount Factor = 1/ $(1+k)^n$ for the period (n-1,n), i.e. the Discount Factor decreases with time as one would expect.

With reference to the proof in section 3.12.

$$EVA\ Value = NAV_0 + \sum_{t=1}^{\infty}\frac{EP_t}{(1+k)^t}$$

$$EP_t = \mathrm{NI_t} - k_t \times NAV_{t-1}$$

$$\mathrm{NI}_t = \mathrm{D_t} + NAV_t - NAV_{t-1}$$

$$\mathrm{EP}_t = \mathrm{D_t} + NAV_t - (1 + k_t) \times NAV_{t-1}$$

$$(1 + K_t)^t = \prod_{i=1}^{t}(1 + k_i)$$

[9] "Non-Life Insurer Equity Valuation Top Down vs Ground Up Models" of 11/6/23

A key relationship for the purposes of this proof is:

$$(1 + K_t)^t = (1 + K_{t-1})^{t-1} \times (1 + k_t)$$

The formula now becomes:

$$\sum_{t=1}^{\infty} \frac{EP_t}{(1+K_t)^t} = \sum_{t=1}^{\infty} \frac{D_t}{(1+K_t)^t} + \sum_{t=1}^{\infty} \frac{NAV_t}{(1+K_t)^t} - (1 + k_1) \times \frac{NAV_0}{(1+K_1)^1} - (1 + k_2) \times \frac{NAV_1}{(1+K_2)^2} - (1 + k_3) \times \frac{NAV_2}{(1+K_3)^3} \cdots\cdots\cdots -(1 + k_n) \times \frac{NAV_{n-1}}{(1+K_n)^n} \cdots\cdots\cdots$$

$$\sum_{t=1}^{\infty} \frac{EP_t}{(1+K_t)^t} = \sum_{t=1}^{\infty} \frac{D_t}{(1+K_t)^t} + \sum_{t=1}^{\infty} \frac{NAV_t}{(1+K_t)^t} - NAV_0 - \frac{NAV_1}{(1+K_1)^1} - \frac{NAV_2}{(1+K_2)^2} \cdots\cdots\cdots - \frac{NAV_{n-1}}{(1+K_{n-1})^{n-1}} \cdots\cdots\cdots$$

As before the various terms $\frac{NAV_k}{(1+K_n)^k}$ cancel each other out and we are left with:

$$\sum_{t=1}^{\infty} \frac{EP_t}{(1+K_t)^t} = \sum_{t=1}^{\infty} \frac{D_t}{(1+K_t)^t} - NAV_0$$

$$\mathrm{NAV}_0 + \sum_{t=1}^{\infty} \frac{EP_t}{(1+K_t)^t} = \sum_{t=1}^{\infty} \frac{D_t}{(1+K_t)^t}$$

i.e. EVA = DDM

Numerical Example
A numerical example is provided in Figure 7.5.

The assumptions are the same as those in section 3.10 except that the initial 12-month forward discount rate of 11.0% increases by 0.15% for each subsequent year, e.g. the rate is 11.0%, 11.1%, 11.2% etc. for years 1, 2 and 3. For year n = 11.0% + 0.1% x (n -1).

Even through there is a pattern of increasing rates by duration the same equivalence arises for a series of random discount rates for each year.

EVA vs Dividend Discount Model

Input

ROE Basis	Period	ROE	Dividend %	Dividend	NAV Growth	ROE - k
1	1 - 5	18.0%	50.0%	9.0%	9.0%	7.0%
2	6 - 15	14.0%	50.0%	7.0%	7.0%	3.0%
3	TV (16+)	12.5%	50.0%	6.3%	6.3%	1.5%

1st Period	5
k initial	11.0%
k increment	0.10%
NAV_0	1,000

Output

Period	t = 0	1 - 5	6 - 15	TV	Total	Price / NAV	PE
EVA	1,000	295	140	0	1,434	1.43	8.0
DDM	0	390	469	576	1,434	1.43	8.0
EVA %	70%	21%	10%	0%	100%		
DDM %	0%	27%	33%	40%	100%		

Comparison	
EVA - DDM	0
as % DDM	0.0%

	NAV Undisc	ROE	Dividend %	Net Income Undisc	Dividend Undisc	NAV Change Undisc	k COE	EP Undisc	EVA Disc	DDM Disc	Discount Rate	Discount Factor
Period t									1,434	1,434		
0	1,000								1,000	0		
1	1,090	18.0%	50%	180	90	90	11.0%	70	63	81	111.0%	111.0%
2	1,188	18.0%	50%	196	98	98	11.1%	75	61	80	111.1%	123.3%
3	1,295	18.0%	50%	214	107	107	11.2%	81	59	78	111.2%	137.1%
4	1,412	18.0%	50%	233	117	117	11.3%	87	57	76	111.3%	152.6%
5	1,539	18.0%	50%	254	127	127	11.4%	93	55	75	111.4%	170.0%
6	1,646	14.0%	50%	215	108	108	11.5%	38	20	57	111.5%	189.6%
7	1,762	14.0%	50%	230	115	115	11.6%	40	19	54	111.6%	211.6%
8	1,885	14.0%	50%	247	123	123	11.7%	41	17	52	111.7%	236.3%
9	2,017	14.0%	50%	264	132	132	11.8%	41	16	50	111.8%	264.2%
10	2,158	14.0%	50%	282	141	141	11.9%	42	14	48	111.9%	295.7%
11	2,309	14.0%	50%	302	151	151	12.0%	43	13	46	112.0%	331.1%
12	2,471	14.0%	50%	323	162	162	12.1%	44	12	44	112.1%	371.2%
13	2,644	14.0%	50%	346	173	173	12.2%	44	11	42	112.2%	416.5%
14	2,829	14.0%	50%	370	185	185	12.3%	45	10	40	112.3%	467.7%
15	3,027	14.0%	50%	396	198	198	12.4%	45	9	38	112.4%	525.7%
TV = 16+		12.5%	50%		3,027		12.5%	0	0	576		
Discounted Value to t = 15												
16+	3,225			378	189	189		0				

Figure 7.5

The reason for the zero Terminal value is because the EP in year 16 is zero as the ROE = COE = 12.5% for that period.

7.8 Varying Risk Discount Rate

The CAPM discounting is based on expected values of future cashflows / earnings. What if a scenario is run say assuming a much riskier business plan based on higher but more uncertain growth rate expectations, other things being equal.

Common sense would appear to dictate that as the cashflows / earnings are more risky they should be discounted using a higher risk discount rate, however with CAPM the Risk Discount rate will remain the same.

Indeed this very point was made by Ryan (1990) in Section 3.9.
"A company with high growth expectations, which are more uncertain, will likely warrant a higher discount rate and so what one finds is that the margin between the numbers is more stable than the variables modelled."

This is because the risk-free rate and equity risk premium are fixed in value and the only potential parameter that may vary is the value of β. If its' value is based on a calculation using retrospective information on a company's price history/return compared to an appropriate equity index or from use of a benchmark then this won't respond to changes in the perceived riskiness of business plans.

7.9 Recommendations

I am of the opinion that one should never treat CAPM as a "Proven" formula as though that somehow excuses the user in not justifying the key inputs and more importantly the value of the resultant risk discount rate.

There will always remain the need to justify:

- Expected value of the risk-free rate
- Why the risk-free rate volatility should be zero
- Expected value of the Equity Risk Premium or Market Return
- Discount rate for the initial single period and its' risk margin over the risk-free rate
- The basis and values for the discount rate in future years.

8. Equity Valuation – Bringing it all together

8.1 Introduction

This section describes a Ground Up calculation using the financial projection outputs from Section 5 together with different modelling approaches for time periods beyond the first five future years, the latter being discussed in Section 8.4.

It also shows how outputs from the Ground Up approach can be reconciled with Top Down ROE modelling approaches, in particular deriving relevant Top Down ROE model inputs.

8.2 Assumptions

The following is a list of the key assumptions:

1. Modelling Time Horizon = 20 years and TV for years 21+ (as after 20 years TV will be very small). There is nothing special about 20 years, it could be for fewer year e.g. 10 or 15 years but then the TV as % of the total will be higher.
2. First 5 years – individual years are modelled as per Section 5.
3. Years 6 to 20 - individual years are not modelled but instead distinct periods are decided where assumptions are assumed to be the same for each year within the period:
   - I have chosen three distinct periods – 5 years / 5 years / 5 years
   - One could choose one, two or four or more periods
   - The length of the three periods is a variable; could be 8 / 4 / 3 - the Top Down ROE Model outputs averaging will reflect the length of the periods.
4. The Top Down ROE Model outputs e.g. ROE, NAV growth etc are derived for the same period lengths

8.3 Model Outputs

The following is a list of the key model outputs:

1. Valuation Summary – Key Value / Price summary with high level summary of sources of EVA Economic Profit outputs making up the valuation
2. Investor Outputs – Values of ROE / NAV growth metrics for each of the modelling periods as described in section 8.2.
3. Economic Value Added calculations for the first 10 years
4. P&L for the first 10 years

Valuation Summary

In the standard Valuation Summary the Price / NAV and Price / Earnings ratios are shown together with the implied dividend yield based on the estimated target price and CY 2022 forecast dividend. The outputs are shown in Figure 8.1.

The Price Earnings Ratio uses a prospective view of Net Income after Tax (“NI”) for CY 2022:

SEVA Model						
Entity	**Foxes Capital**					
BS Date	**31/12/2021**					
First AY	**2022**					
Currency / Units	**£ m**					
Valuation Summary						
		Total Value £ m	**per Share £**			
Equity Value		1,395	279.05			
NAV		806	161.10			
Price / NAV Ratio		**1.73**	**1.73**			
Earnings per Share £		89.25	17.85			
Price Earnings Ratio			**15.63**			
Dividend per Share £		49.17	9.83			
Dividend Yield			3.5%			
Period		**1**	**2**	**3**	**TV**	**Total**
No. of years		5	5	5		
Period Start - CY	**2022**	**2027**	**2032**	**2037**	**2042**	
Period End - CY	**2026**	**2031**	**2036**	**2041**		
EVA - Undiscounted	305	276	259	233	749	
EVA - Discounted	220	129	79	47	115	
% of Equity Value £ 1395 m in each period	16%	9%	6%	3%	8%	42%
Number of Shares YE 2021	5 m					

Figure 8.1

The bottom section shows the Economic Value Added on both undiscounted and discounted bases for each of the periods shown. The discounted value of the EVA for each of these periods is expressed as a percentage of the Total Value.

In this example we can see that 42% of the overall valuation comes from projections for 2022 and later with 58% representing the portion made up by the opening NAV.

The proportions of the overall valuation in the 3x 5 year intervals (2027 – 2031), (2032 – 2036) and (2037 – 2042) are 9%, 6% and 3% respectively.

Investor Outputs

The Key Outputs are shown in Table 8.2

Investor Outputs						
Period Start - CY	**2022**	**2027**	**2032**	**2037**	**2042**	
Period End - CY	**2026**	**2031**	**2036**	**2041**	**TV**	**Averages**
ROE	13.5%	11.5%	10.6%	9.9%	9.5%	Weighted
NAV Growth p.a.	9.8%	8.0%	7.1%	6.5%	6.0%	Compound
NEP Growth pa	4.6%	5.0%	5.0%	5.0%		Compound
Combined Ratio	90.7%	90.7%	90.7%	90.7%		Weighted
Dividends	267	349	460	600		Sum
Dividend Growth pa	5.3%	5.9%	5.6%	5.4%		Weighted

Assumptions	
Cost of Capital	8.9%
Riskfree rate	2.3%
Equity Risk Premium	6.0%
Beta	1.10

Table 8.2

The Cost of Capital has been derived using CAPM and the assumptions shown. The same 12-month forward rate has been assumed to be true for each future year.

<u>Calculation of % changes</u>
For the period CY 2022 to 2026 a number of averages or growth rates p.a. are shown. These are calculated using a (i) Compound function or (ii) Weighted averages. The only exception to this is Dividends which is straightforward sum of the values.

The two calculations are described as follows for the CY time interval i = 1 to T:

$$Compound\ for\ X = \left(\frac{X(T)}{X(0)}\right)^{\left(\frac{1}{T}\right)}$$

$$Weighted\ for\ \frac{Y}{X} = \frac{\sum_{i=1}^{T} Y(i)}{\sum_{i=1}^{T} X(i)}$$

For all CY time intervals the ROE and NAV Growth p.a. are calculated values using either of these two approaches.

For example let's look at the period 2022 to 2026:
- NAV Growth p.a. – this is based on balance sheet values so X(0) will be the NAV at YE 2021 and X(5) the NAV at YE 2026.
- NEP Growth p.a. – this is based on P&L values so X(0) will be the NEP at for CY 2021 and X(5) the NEP for CY 2026.

Assumptions
The Key Assumptions are shown in Table 8.3.

Assumptions					
Cost of Capital	8.9%				
Riskfree rate	2.3%				
Equity Risk Premium	6.0%				
Beta	1.10				
Period Start - CY	**2022**	**2027**	**2032**	**2037**	**2022 - 2026**
Period End - CY	**2026**	**2031**	**2036**	**2041**	**Averages**
Net Loss Ratio (x)	55.0%	55.0%	55.0%	55.0%	Weighted
Net Expense Ratio (x)	35.7%	35.7%	35.7%	35.7%	Sum
Net Acquisition Costs Ratio (x)	18.0%	18.0%	18.0%	18.0%	Weighted
Operational Expenses Ratio (x)	17.7%	17.7%	17.7%	17.7%	Weighted
GWP Growth pa	5.0%	5.0%	5.0%	5.0%	Compound
NWP / GWP	73.0%	73.0%	73.0%	73.0%	Weighted
Investments / (NAV + Net Reserves)	1.20	1.23	1.23	1.23	Weighted
Other Income / NAV	0.03	0.03	0.03	0.03	Weighted
Finance Costs / NAV	0.03	0.03	0.03	0.03	Weighted
Investment return rate	2.2%	2.2%	2.2%	2.2%	Weighted
Premium Earnings pattern		1st year	2nd year		
		55%	45%		
Dividend					
Basis	% Net Income				
Fixed Amount		40.0	40.0	40.0	
% Net Income After Tax		40%	40%	40%	

Table 8.3

Some other points of note:

- Operational Expenses – separately specified rather than use an overall Expense ratio as they may not always be a function of the NEP, see below
- Dividend – based on either a fixed monetary amount or a % Net Income After Tax.

<u>Separating Acquisition Costs and Operational Expenses</u>

Expenses have been separated out into (i) Acquisition Costs and (ii) Operational Expenses for projection purposes. This is done to reflect the different dynamics of the two in particular how operational expenses respond to changes in premium volumes. In a steady growth or decline situation then projecting total expenses, the sum of the two, is perhaps not unreasonable.

However, there are issues if there is a rapid change in premium volumes, for example if the premium volume between one year and the next dropped by 50% then it is highly unlikely that operational expenses would reduce by anywhere near this amount. The limiting case when premium volumes are zero in a run-off situation will not result in zero operational expenses given there still will be operational costs e.g. management costs, non-claims related salaries, computer costs, premises costs etc.

<u>Investments / (NAV + Net Reserves) ratio</u>

I have assumed that the Investment assets increase in line with the increase in the NAV plus Net Reserves As the NAV and Net Reserves increase over time using a fixed ratio might be thought to be too conservative in which case a lower ratio could be assumed for future years.

Economic Value Added
The calculations are shown in Tables 8.4 and 8.5.

I have shown two separate time periods, (i) CY 2022 to 2026 and (ii) CY 2027 to 2031.

EVA (CY 2022 to 2026)

Economic Value Added						
	YE	31/12/21	31/12/22	31/12/23	31/12/24	31/12/25
	CY	2022	2023	2024	2025	2026
NAV		806	895	986	1,082	1,180
ROE - Calculated		17.2%	15.9%	15.0%	14.3%	13.7%
NAV Growth p.a.		11.1%	10.2%	9.6%	9.1%	8.7%
EVA - Undiscounted		67	63	60	58	56
EVA - Discounted		56	49	43	38	34
NEP growth p.a.		2.9%	5.0%	5.0%	5.0%	5.0%

Table 8.4

EVA (CY 2027 to 2031)

Economic Value Added						
	YE	31/12/26	31/12/27	31/12/28	31/12/29	31/12/30
	CY	2027	2028	2029	2030	2031
NAV		1,284	1,391	1,506	1,626	1,752
ROE - Calculated		13.2%	13.0%	12.6%	12.3%	12.0%
NAV Growth p.a.		8.4%	8.2%	8.0%	7.7%	7.5%
EVA - Undiscounted		55	57	56	55	54
EVA - Discounted		30	29	26	23	21
NEP growth p.a.		5.0%	5.0%	5.0%	5.0%	5.0%

Table 8.5

For a constant Net Loss Ratio:
- If the NAV growth p.a. > NEP growth p.a. then the ROE will reduce over time
- If the NAV growth p.a. < NEP growth p.a. then the ROE will increase over time.

Profit & Loss
The calculations are shown in Tables 8.6 and 8.7.

I have shown two separate time periods, (i) CY 2022 to 2026 and (ii) CY 2027 to 2031.

Profit & Loss (CY 2022 to 2026)

Profit & Loss					
CY	2022	2023	2024	2025	2026
GWP	1,260.0	1,323.0	1,389.2	1,458.6	1,531.5
NWP	919.8	965.8	1,014.1	1,064.8	1,118.0
NEP	900.1	945.1	992.3	1,042.0	1,094.1
Net Claims Incurred	-495.0	-519.8	-545.8	-573.1	-601.7
Net Acquisition Costs	-162.0	-170.1	-178.6	-187.6	-196.9
Operational / Other expenses	-159.3	-167.3	-175.6	-184.4	-193.6
Net UW Result	**83.72**	**87.91**	**92.30**	**96.92**	**101.77**
Investment return	66.5	67.3	69.8	72.9	76.5
Other Income	28.4	29.8	31.3	32.9	34.5
Operating Result	**178.6**	**185.0**	**193.5**	**202.7**	**212.8**
Finance Costs	-25.0	-26.3	-27.6	-28.9	-30.4
Profit Before Tax	**153.6**	**158.7**	**165.9**	**173.8**	**182.4**
Tax	-30.7	-31.7	-33.2	-34.8	-36.5
Net Income after Tax	**122.9**	**127.0**	**132.7**	**139.0**	**145.9**
Other Net Income	15.5	15.5	15.5	15.5	15.5
Net Income before Dividend	**138.4**	**142.5**	**148.2**	**154.5**	**161.4**
Dividend	-49.2	-50.8	-53.1	-55.6	-58.4
Retained Earnings	**89.2**	**91.7**	**95.1**	**98.9**	**103.0**

Table 8.6

For each of these years the insurance modelling is Gross, Ceded and Net.

Profit & Loss (CY 2027 to 2031)

Profit & Loss					
CY	2027	2028	2029	2030	2031
GWP	1,608.1	1,688.5	1,772.9	1,861.6	1,954.7
NWP	1,173.9	1,232.6	1,294.3	1,359.0	1,426.9
NEP	1,148.8	1,206.2	1,266.5	1,329.8	1,396.3
Net Claims Incurred	-631.8	-663.4	-696.6	-731.4	-768.0
Net Acquisition Costs	-206.8	-217.1	-228.0	-239.4	-251.3
Operational / Other expenses	-203.3	-213.5	-224.2	-235.4	-247.1
Net UW Result	**106.85**	**112.20**	**117.81**	**123.70**	**129.88**
Investment return	80.4	89.4	94.9	100.8	107.0
Other Income	37.5	40.7	44.0	47.5	51.2
Operating Result	**224.7**	**242.2**	**256.7**	**272.0**	**288.1**
Finance Costs	-33.0	-35.8	-38.8	-41.8	-45.1
Profit Before Tax	**191.7**	**206.4**	**218.0**	**230.2**	**243.0**
Tax	-38.3	-41.3	-43.6	-46.0	-48.6
Net Income after Tax	**153.4**	**165.1**	**174.4**	**184.1**	**194.4**
Other Net Income	15.5	15.5	15.5	15.5	15.5
Net Income before Dividend	**168.9**	**180.6**	**189.9**	**199.6**	**209.9**
Dividend	-61.3	-66.1	-69.8	-73.7	-77.8
Retained Earnings	**107.5**	**114.6**	**120.1**	**126.0**	**132.1**

Table 8.7

For these and later years, the insurance modelling is Net only except for GWP. Net Reserves (and not Gross Reserves) are only projected too.

8.4 Modelling Bases

I have adopted a number of different modelling bases. These vary by time horizon and are summarised in Table 8.8:

Period CYs	Gross	Net	Gross AY Reserves	Net AY Reserves	Net Reserve Approximation	BS / CS Modelling	Investments Approximation
2022 - 2026	Yes	Yes	Yes	Yes	No	Yes	No
2027 - 2031	No	Yes	No	Yes	No	No	Yes
2032 - 2036	No	Yes	No	Yes	No	No	Yes
2037 - 2041	No	Yes	No	No	Yes	No	Yes
TV (2042)	No	Yes	No	No	Yes	No	Yes

Table 8.8

The granularity of modelling decreases over time. It would have been easier to have modelled each of the 20x CYs using the same basis as used during the initial time interval (2022 – 2026) however the purpose of the variation is to illustrate the different possible options.

Modelling an additional 15 CYs in the same way would quite simply have meant the extension of the integrated P&L, balance sheet and cashflow projections from 6 years to 20 years. This would have only involved the insertion of 15 more columns in each of the P&L, balance sheet and cashflow projection worksheets plus 15 more CY columns and 15 more AY rows in the Reserving worksheet to enable the projection of Gross and Net AY reserves.

<u>Net Reserve Approximation</u>
For Net Reserves there are two bases:
1. AY Projection of Net Reserves (not Gross Reserves)
2. Approximation of Net Reserves from other data

The Net Reserves approximated for later years, not the earlier years, are estimated by applying Net Reserves / NEP ratios, to future NEPs, based on the trended ratio values to date. This will be for those years where this approximation appears valid.

This approximation is better suited to a company with a reasonably stable future business plan than one with projected material changes over time. The year in which the Net Reserves / NEP ratio will have appeared to have stabilised is often related to the length of the payment pattern. The longer the pattern, the longer the time period until the method should be used.

<u>Investment Approximation</u>
The Investments for future years are estimated by applying Investments / (NAV + Net Reserves) ratios, to future projections of (NAV + Net Reserves). Ratios for future years can be estimated from analysis of the ratios in each year of the first 5 years and observations of the future business profile. The Net Reserves are calculated using either the AY projection method or the Net Reserve Approximation.

9. Conclusions

On review of the different methods used to estimate the "theoretical" value of a company's share price, taking into account the advantages and disadvantages of each, the Economic Value Added valuation approach is the one that I find most appealing. This is because it identifies the sources of economic profit / loss over time and the overall value is not based on 100% of prospective assumptions which would be the case with the Dividend Discount model, my next preference.

What the work has shown is that there can be unanswered questions in people's analyses. Two main areas stand out:

1. One is the use of Top Down ROE models using assumptions where I feel more justification is needed. The eight examples discussed in section 4 give an illustration of what some of these are.
2. The second area is the justification of the risk discount rate, not only the method used e.g. CAPM and the values of the underlying parameters but the value itself and the basis of the discount rate for all future years beyond the initial single period.

This all becomes very important as some types of model are can be very sensitive to small changes in key input assumptions.

The key takeaway for me is the need for people to justify more, not only in the methods they have selected but the key assumptions too. The following is a non-exhaustive list.

1. Justification for the selected method
   - Not just we have used this

2. Methodology and Assumptions used for the following Projections
   - Gross and Net reserves (if shown)
   - Future Required Capital, Target Capital and Solvency Ratios (if shown)

3. Top Down ROE Models
   - All key assumptions

4. Risk Discount Rate justification
   - Value for initial period and basis for all future years

   and in if CAPM is used:
   - Expected value of the risk-free rate
   - Why the risk-free rate volatility should be zero
   - Expected value of the Equity Risk Premium or Market Return
   - Discount rate for the initial single period
   - The basis and values for future years.

10. Appendices

10.1 Appendix 1 – CAPM Simple Model

A simple portfolio (P) is constructed with weights of w and (1-w) on each of the two assets.

Then:
$E(R_P) = wE(R_A) + (1-w)E(R_B)$

$\sigma_P^2 = w^2\sigma_A^2 + (1-w)^2\sigma_B^2 + 2\rho w(1-w)\sigma_A\sigma_B$

where:
- $E(R_A)$ and $E(R_B)$ are expected returns for assets A and B
- σ_A and σ_B are standard deviations of returns for assets A and B
- ρ = correlation of returns between A and B

We need to find the value of w representing the Market Portfolio. This is where the Capital Market line is tangential to the Efficient Frontier. This point is where the value of the gradient of the Efficient Frontier is equal to that of the CML.

<u>Solving for w:</u>
The formula for the Capital Market Line for any value of w is:

$$CML = \frac{\left(E(R_P) - R_f\right)}{\sigma_P}$$

The values of $E(R_P)$ and σ_P and hence the CML can be calculated for each value of w.

Let R = $E(R_p)$ and σ_P is as before.

$$Slope = \frac{dR}{d\sigma_P} = \frac{dR/_{dw}}{d\sigma_P/_{dw}}$$

$$\frac{dR}{dw} = E(R_i) - E(R_M)$$

$$\frac{d\sigma_P}{dw}2\sigma_P = 2w\sigma_i^2 - 2(1-w)\sigma_M^2 + 2\rho(1-2w)\sigma_i\sigma_M$$

From the two formulas above and the value of σ_P we also have a mathematical equation for the Gradient in terms of w, from which the value of the gradient can be calculated for each value of w.

For each value of w, in increments of 0.1%, the Gradient and CML are calculated and the absolute differences noted. The point at which the difference is the smallest will be the Market Portfolio. I do also check visually that this is the case, 0.1% is sufficiently granular for this process to work.

10.2 Appendix 2 – Volatility Adjusted CAPM

The CAPM formula does not hold true if the risk-free rate volatility is > 0. Allowing for risk-free rate volatility results in a slightly different formula:

$$E(R_i) = \frac{R_f + \beta\left(E(R_M) - R_f\right) \; - V_R E(R_M)}{(1 - \; V_R)}$$

where V_R = Volatility ratio is given by the formula:

$$V_R = \frac{\sigma\left(R_f\right)}{\sigma(R_M)}$$

So, what happens if the volatility > 0. Two things happen:

1. The Market Portfolio will be different as the Capital Market Line will be tangential to the Efficient Frontier at a lower point (both expected return and volatility)
2. The CAPM formula no longer holds true. I have derived an alternative formula which I have called the "Volatility Adjusted CAPM".

The value of $E(R_i)$ is not a function of the usual CAPM formula i.e. it is not

$$E(R_i) = \frac{CAPM\ Formula \; - V_R E(R_M)}{(1 - \; V_R)}$$

The reason for this is that Market Portfolio identified by "M" is different to that assumed in the usual CAPM formula. Other points to mention:

1. Value of β - I have assumed that the β associated CAPM and Volatility Adjusted CAPM are identical. This is not unreasonable.
2. There is a need to assign values to both the expected return and standard deviation of the Market Portfolio, which is perhaps not a bad thing, instead of subjectively assigning a value for the Equity Risk Premium, say.

<u>Example</u>
The following is an example using my simple model.
As you can see there difference in the value of $E(R_i)$ = 0.3%.

The graph in Figure 10.1 shows a CML where it is assumed the volatility of the risk-free rate is not zero. I have called this line the "Modified Capital Market Line" ("MCML").

$$MCML = \frac{\left(E(R_M) - R_f\right)}{\left(\sigma_M \; - \sigma_f\right)}$$

This is a continuation of the numerical examples shown in Section 7.2.

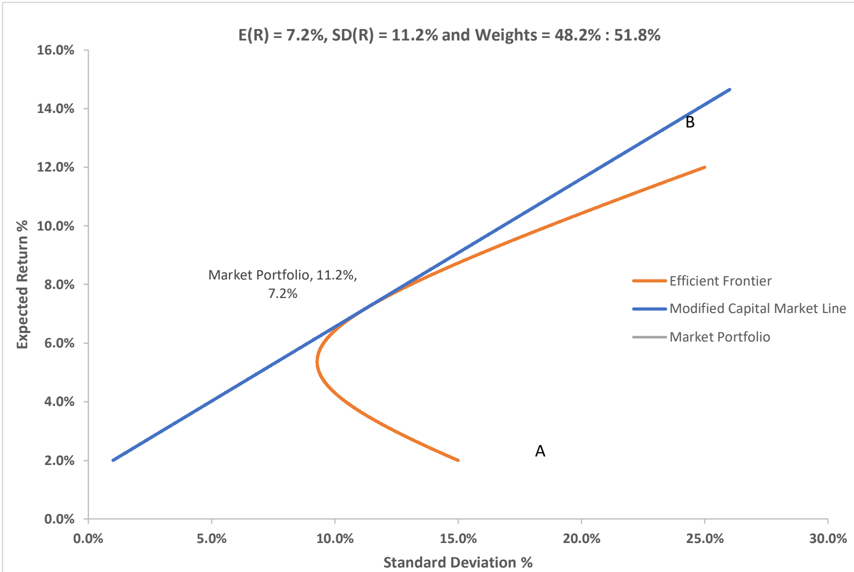

Figure 10.1

The difference will depend on the assumptions underlying the Efficient Frontier and those of the Risk-free rate. The purpose of this example is to illustrate a calculation. The calculations are shown in Table 10.2.

Parameters	CAPM	VA CAPM
R_f	2.0%	2.0%
σ_M	0.0%	1.0%
$E(R_M)$	7.5%	7.2%
$SD(R_M)$	11.8%	11.2%
β	1.20	1.20
V_R		8.9%
CAPM $E(R_i)$	**8.6%**	8.2%
VA CAPM $E(R_i)$		**8.3%**

Table 10.2

Derivation of the Volatility Adjusted CAPM

The following is taken from Shaw (1993) [10] as far as part 4. after which the MCML is substituted for the CML.

Consider a Portfolio consisting of Security i and the Market Portfolio M

1.

$$E(R_P) = wE(R_i) + (1-w)E(R_M)$$

$$\sigma_P^2 = w^2\sigma_i^2 + (1-w)^2\sigma_M^2 + 2\rho w(1-w)\sigma_i\sigma_M$$

$$Slope = \frac{dR}{d\sigma_P} = \frac{dR/dw}{d\sigma_P/dw}$$

[10] Optimum Portfolio Selection Methods, Shaw, R. Institute of Actuaries, (1993)

2.

$$\frac{dR}{dw} = E(R_i) - E(R_M)$$

$$\frac{d\sigma_P}{dw} 2\sigma_P = 2w\sigma_i^2 - 2(1-w)\sigma_M^2 + 2\rho(1-2w)\sigma_i\sigma_M$$

when w = 0 we have $\sigma_P = \sigma_M$

$$\frac{d\sigma_P}{dw} = (\rho\sigma_A\sigma_B - \sigma_M^2) \Big/ \sigma_M$$

3.

$$Slope = \frac{dR}{d\sigma_P} = \frac{dR/dw}{d\sigma_P/dw} = \sigma_M \left(E(R_i) - E(R_M)\right) \Big/ (\rho\sigma_i\sigma_M - \sigma_M^2)$$

4. (New Step)

Now the slope at w = 0 is equal to the MCML, i.e.

$$\frac{\left(E(R_M) - R_f\right)}{\left(\sigma_M - \sigma_f\right)} = \sigma_M \left(E(R_i) - E(R_M)\right) \Big/ (\rho\sigma_i\sigma_M - \sigma_M^2)$$

$$\beta_i = \frac{\rho\sigma_i}{\sigma_M}$$

$$\frac{\left(E(R_M) - R_f\right)}{\left(\sigma_M - \sigma_f\right)} = \sigma_M \left(E(R_i) - E(R_M)\right) \Big/ \sigma_M^2(\beta - 1)$$

$$\frac{\left(E(R_M) - R_f\right)}{(1 - V_R)} = \left(E(R_i) - E(R_M)\right) \Big/ (\beta - 1)$$

$$V_R = \frac{\sigma_f}{\sigma_M}$$

$$\left(E(R_M) - R_f\right)(\beta - 1) = \left(E(R_i) - E(R_M)\right)(1 - V_R)$$

$$E(R_i)(1 - V_R) = \left(E(R_M) - R_f\right)(\beta - 1) + E(R_M)(1 - V_R)$$

$$E(R_i)(1 - V_R) = R_f + \beta\left(E(R_M) - R_f\right) - V_R\, E(R_M)$$

$$E(R_i) = \frac{R_f + \beta\left(E(R_M) - R_f\right) - V_R\, E(R_M)}{(1 - V_R)}$$

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Richard Shaw was born in Kirby Muxloe, Leicestershire, England and attended Durham University where he studied Mathematics. He has a Master Degree in Finance from London Business School and is a Fellow of the Institute of Actuaries, FIA.

He is married with children. He has played football since the age of 9 and was signed on for 3 years with Leicester City as an associate schoolboy and played representative football before exchanging a life in the limelight for that of an Actuary. He and his children are Leicester City season ticket holders. His hobbies include swimming, reading 20th century history books, travelling, music and collecting memorabilia.