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LCP powering possibility

Parameterising capital modelling volatility: allowing for changes in volume

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21 January 2025

Agenda

- **Specifying the problem:** what are we trying to achieve, and why?
- **Developing the solution:**
  - Bucket analysis of historical data
  - Bootstrapping
  - Survey data
- **Monitoring the results:** use cases and next steps

Specifying the problem

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Some case studies

Reserve risk

- CoVs parameterised on 2023 year-end data
- Projected to 2024 year-end based on Q2 data
- *Mismatch between parameterised CoV and modelled reserves*

Validation

- Sense check selected CoVs against benchmarks
- Market typically is much larger than a single firm, and hence less volatile
- *Mechanism is needed to ensure a fair comparison*

Sensitivity/scenario testing

- Eg: stretch view of business plan volumes
- All else being equal, this implies lower volatility
- *Unlikely the capital team have scope or appetite to re-parameterise from scratch*

New classes of business

- Insufficient scope to parameterise small classes, or those with no data
- *Assumed the class behaves similarly to a class with "known" volatility, but which is much larger*

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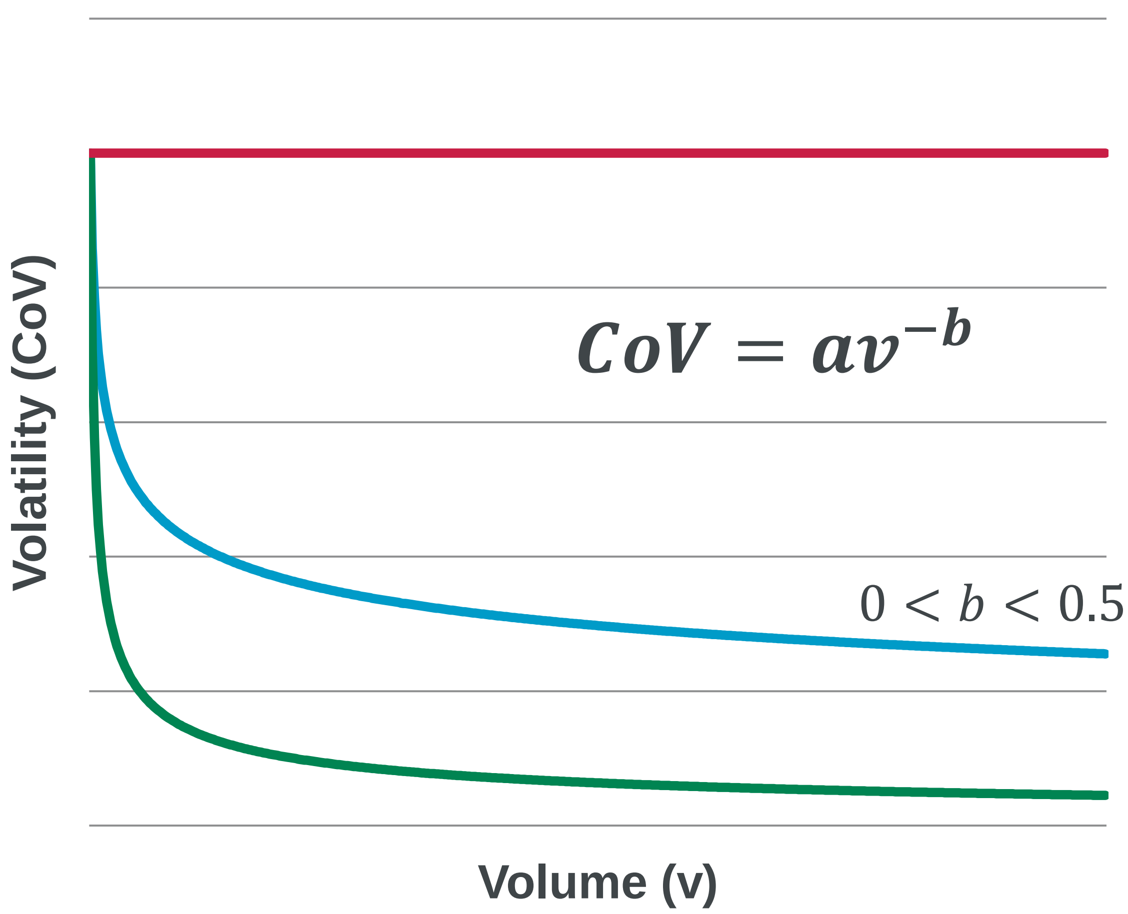
Model



- Power curve
- "a" parameter does not affect calculations: focus is on fitting "b"
- **Core concept**: b takes non-negative values:
  - Zero implies no sensitivity: volatility is invariant to volume (unlikely – pure systemic risk)
  - 0.5 implies risks are all independent (also unlikely – pure specific risk)
  - In practice, we expect a result somewhere in the middle
  - Higher values of b imply increasing volume has greater effect on volatility, ie the business has more specific risk
  - Conversely, lower values of b imply more systemic risk

Some maths

Equation 1: $CoV_T = av_T^{-b}$

Equation 2: $CoV_R = av_R^{-b}$

Where CoV_T and v_T are the volatility and volume of the target distribution, and CoV_R and v_R are the volatility and volume of the reference distribution

Eq 1 divided by Eq 2 gives: $\frac{CoV_T}{CoV_R} = \frac{av_T^{-b}}{av_R^{-b}} \Longrightarrow CoV_T = CoV_R \left(\frac{v_T}{v_R}\right)^{-b}$

or, equivalently: $CoV_T = CoV_R \left(\frac{v_R}{v_T}\right)^{b}$

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Approach

Biggest and most comprehensive dataset we have access to is Schedule P of the National Association of Insurance Commissioners (NAIC) return

Two high-level approaches:

Bucket analysis of one-year reserve movements observed historically

Bootstrapping of reserve triangles and analysis of the calculated CoVs

Additional analyses:

Additional factors: class of business and/or cohort

LCP capital benchmarking data

Data from APRA (Australian Prudential Regulation Authority)

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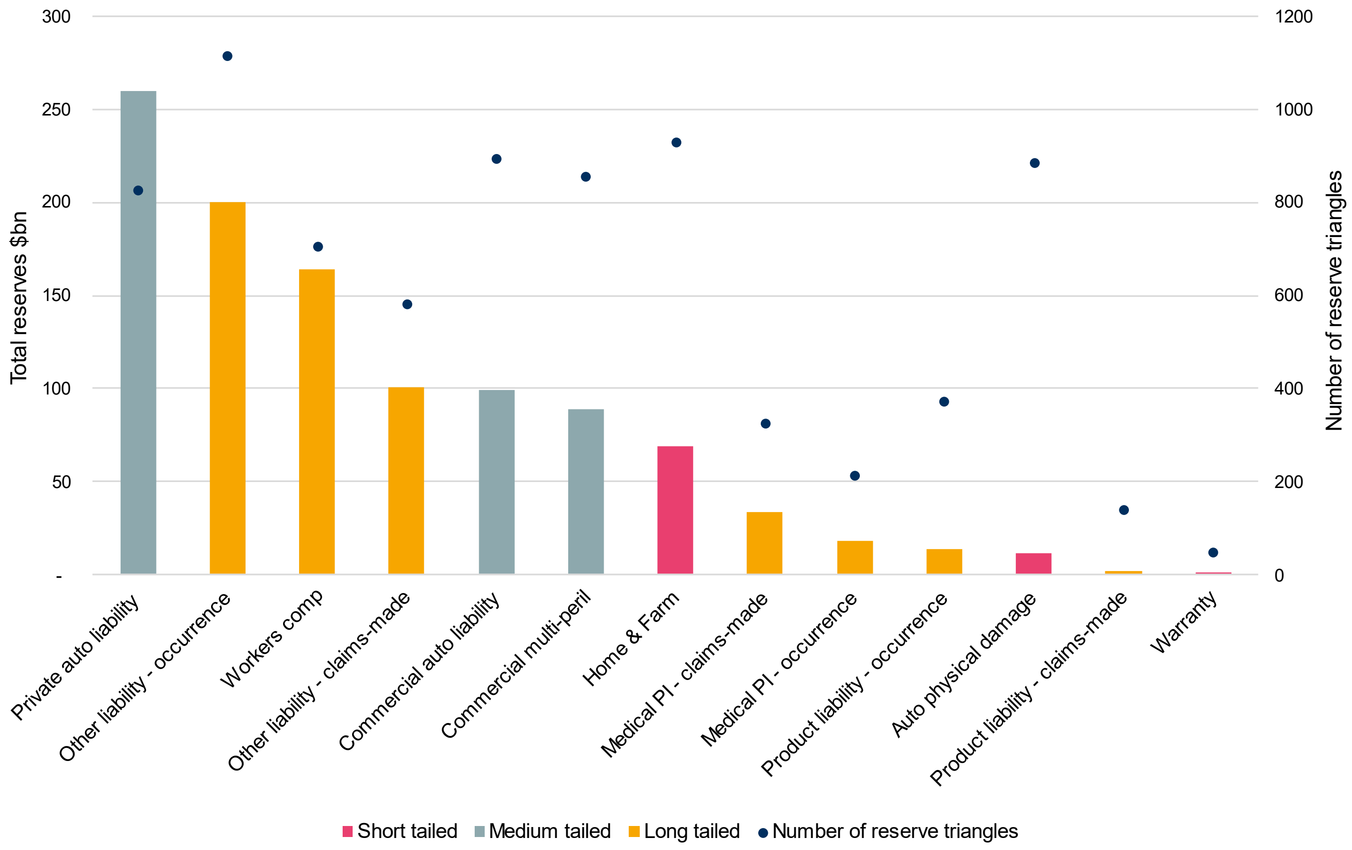
Schedule P dataset



- Total reserves of $937 bn
- 13 reserving classes
- Data over the period 2011 - 2022
- Data cleaning:
  - Removed outlier reserve deteriorations
  - Removed negative reserves

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Designing the solution

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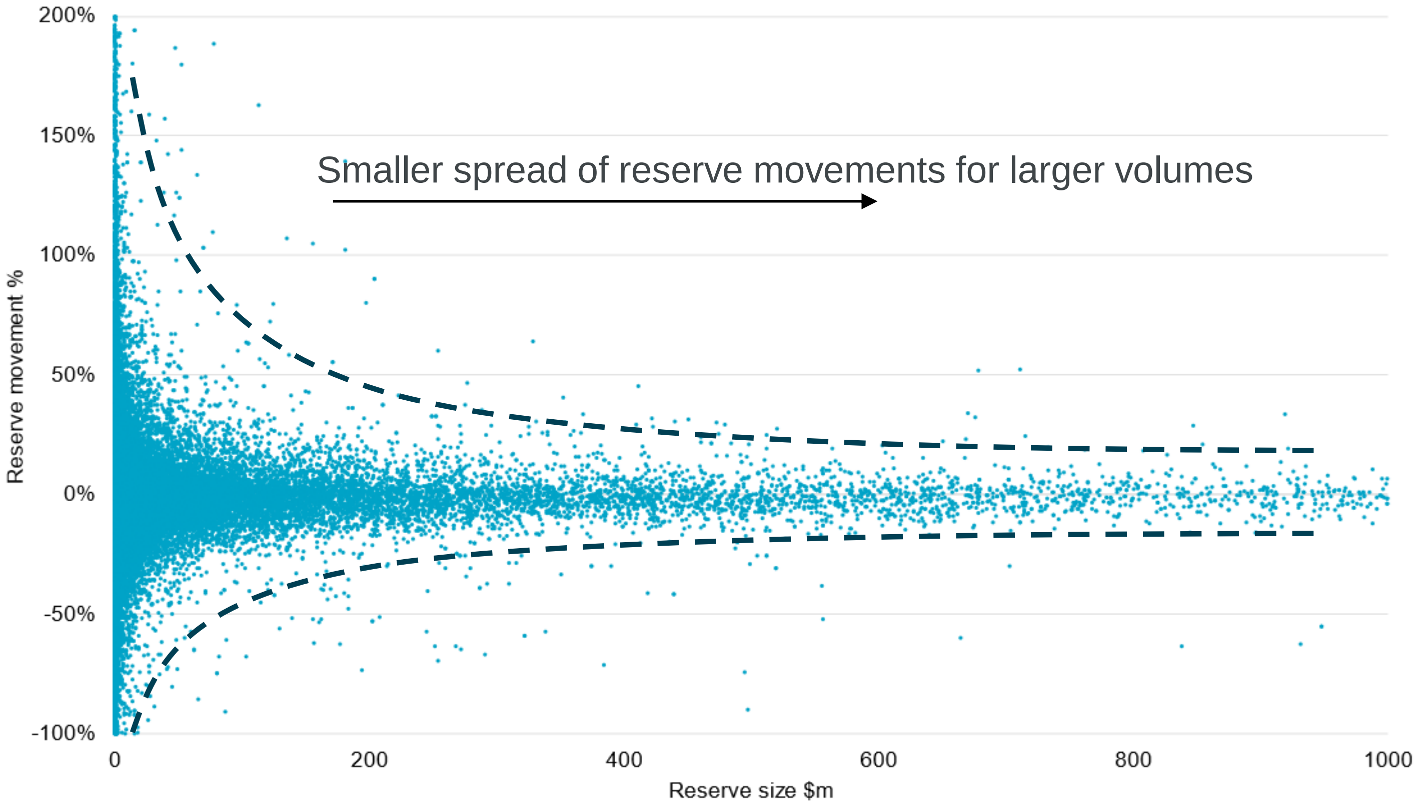
Bucket analysis

Calculate reserve movements % → Order reserve movements by reserve size → Partition data into buckets of uniform reserve size → Calculate volatility of reserve movements in each bucket → Bucket volatility vs Bucket mean reserve size



Bucket analysis

Calculate reserve movements %

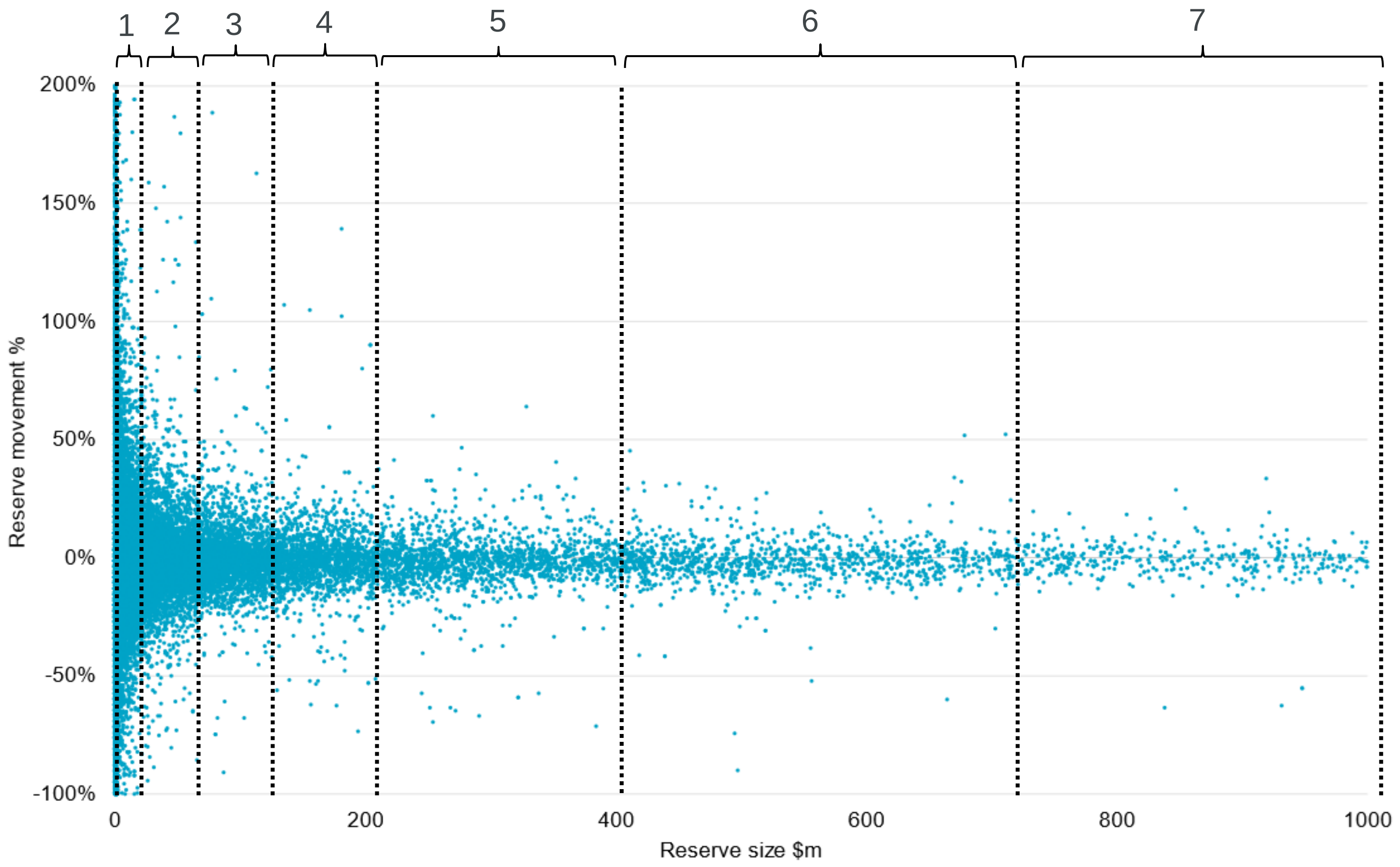
→ Order reserve movements by reserve size

→ Partition data into buckets of uniform reserve size

→ Calculate volatility of reserve movements in each bucket

→ Bucket volatility vs Bucket mean reserve size

Put data into buckets based on reserve size*



*Actual number of buckets is 300

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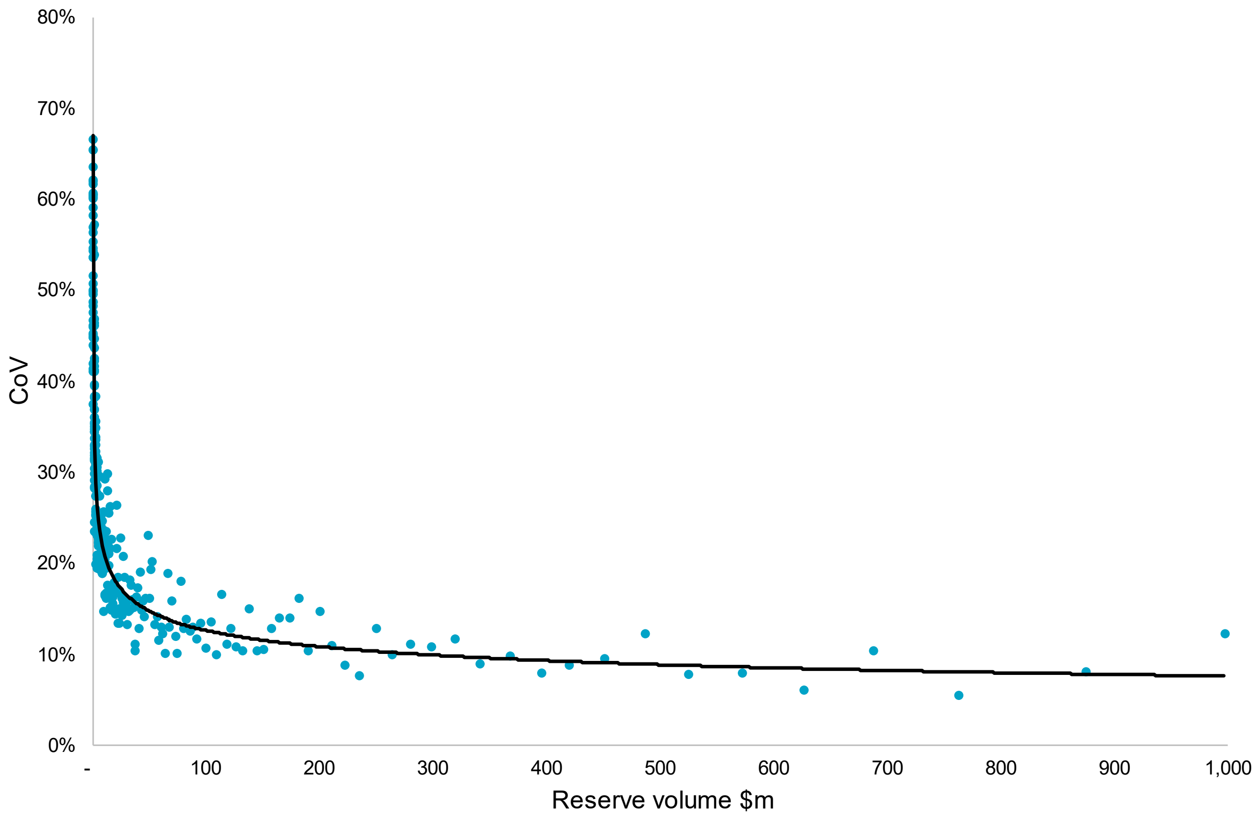
Bucket analysis – results

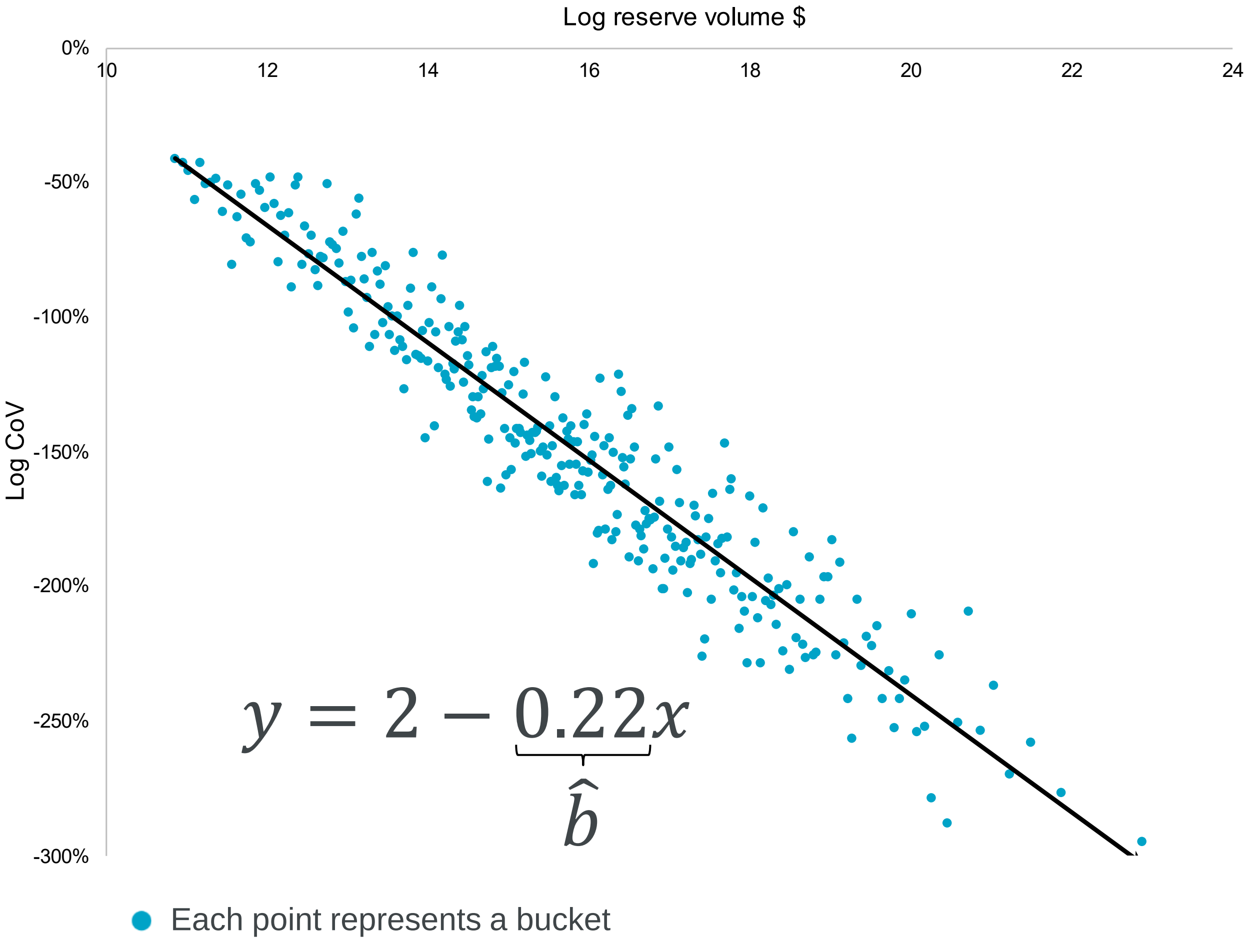


$$CoV = av^{-b}$$

$$\Rightarrow \log CoV = \log a - b * \log v$$

- Each point represents a bucket

Bucket analysis – results



- **Strong log-linear relationship between reserve volume and volatility**
- Implies the relationship $CoV = av^{-b}$ holds, with $\boldsymbol{\hat{b} = 0.22}$
- Observed r^2 value of 91% – great model fit!
- Further attempted to fit model $CoV = av^{-b} + \gamma$, where γ can be interpreted as undiversifiable volatility
- Findings: $\gamma = 0$ provided the best model fit

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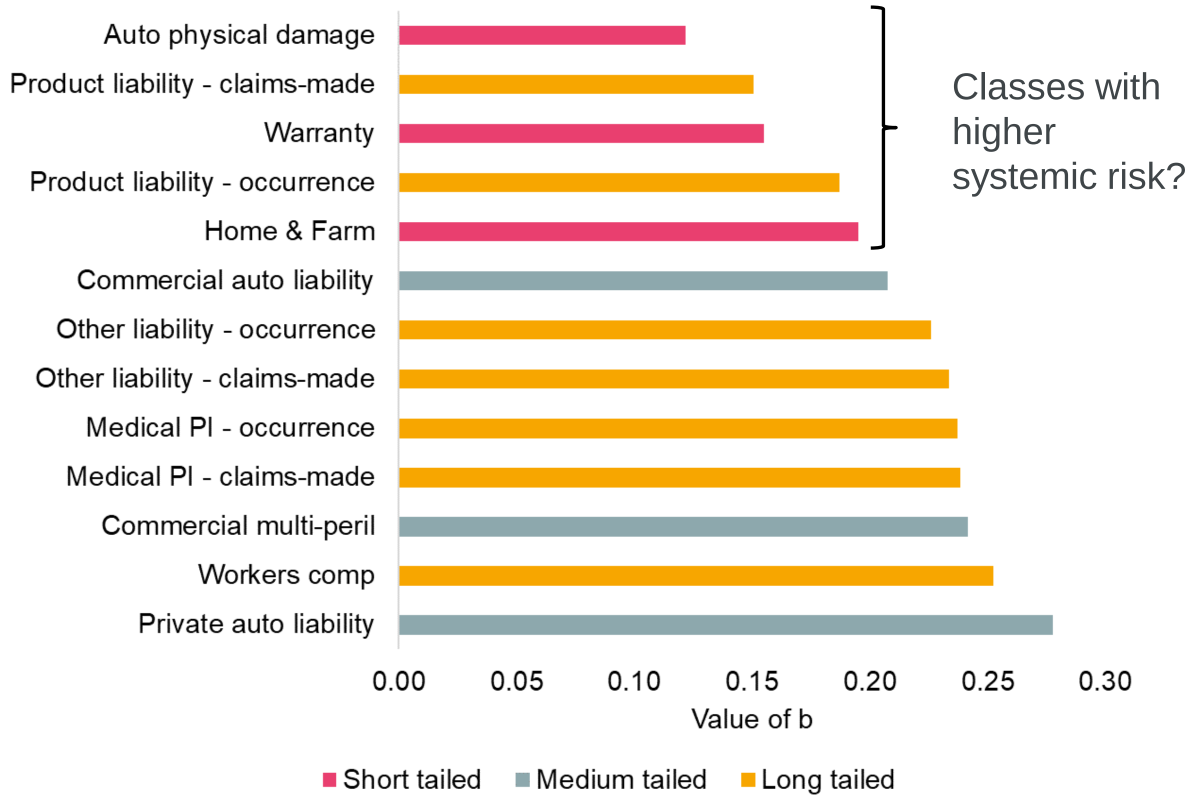
Bucket analysis – granular level results

Model		$\hat{b}$	R^2	# parameters
Base model	$CoV = av^{-b}$	**0.22**	**91%**	**2**
Class model (regular)	$CoV = a_i v^{-b}$	0.22	90%	12
Class model (advanced)	$CoV = a_i v^{-b_i}$	0.12-0.28	60%	26
Duration model (regular)	$CoV = a_i v^{-b}$	0.22	91%	4
Duration model (advanced)	$CoV = a_i v^{-b_i}$	0.18-0.23	90%	6



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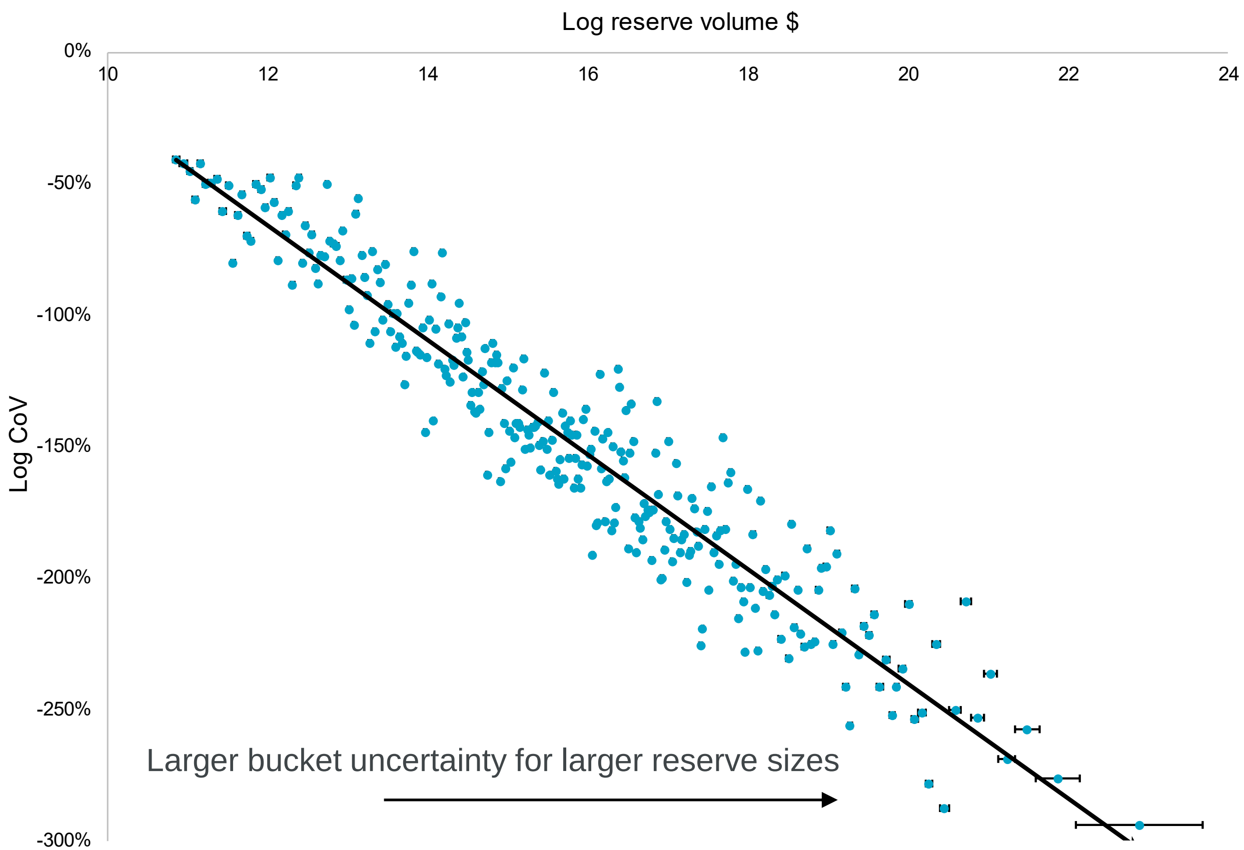
Bucket analysis – uncertainty



Horizontal error bars visualise range of reserve sizes within a bucket

Parameter uncertainty

Uncertainty in estimating $\hat{b}$ using a linear model

Materiality: very low - measured $se(\hat{b}) = 0.004$

Bucket uncertainty

Uncertainty in quantifying reserve size for each bucket

Materiality: low – performed a range of stability tests

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Bootstrapping analysis

Fit a GLM to the incremental claims triangle → Calculate residuals → Resample the residuals to create a pseudo triangle → Refit the GLM to produce a new estimate of the reserves → Repeat *n* times to derive a distribution of reserve estimates

Benefits

- Purely data driven – not reliant on reserving actuary's estimates
- Over 5,000 triangles
- 10,000 simulations used

Limitations

- Usual bootstrapping limitations
- Data:
  - Market data
  - High residuals and therefore CoVs
- No tail factor used

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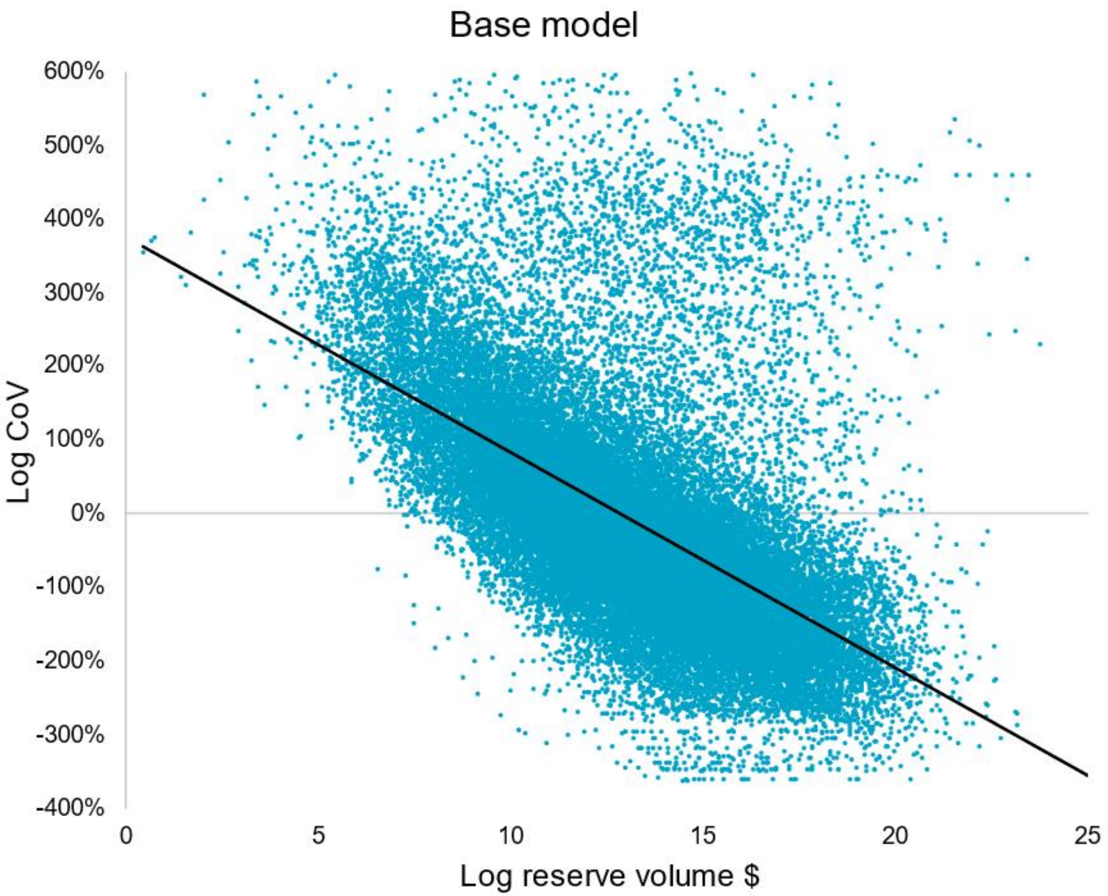
Bootstrapping analysis results



- Each point represents a cohort

Model		$\hat{b}$	$se(\hat{b})$	r^2	# parameters
Bootstrap base model	$CoV = av^{-b}$	0.29	0.002	33%	2
Class model	$CoV = a_i v^{-b}$	0.28	0.002	40%	12
Cohort model	$CoV = a_j v^{-b}$	0.26	0.003	35%	3
Class and cohort model	$CoV = a_{ij} v^{-b}$	0.25	0.002	42%	13

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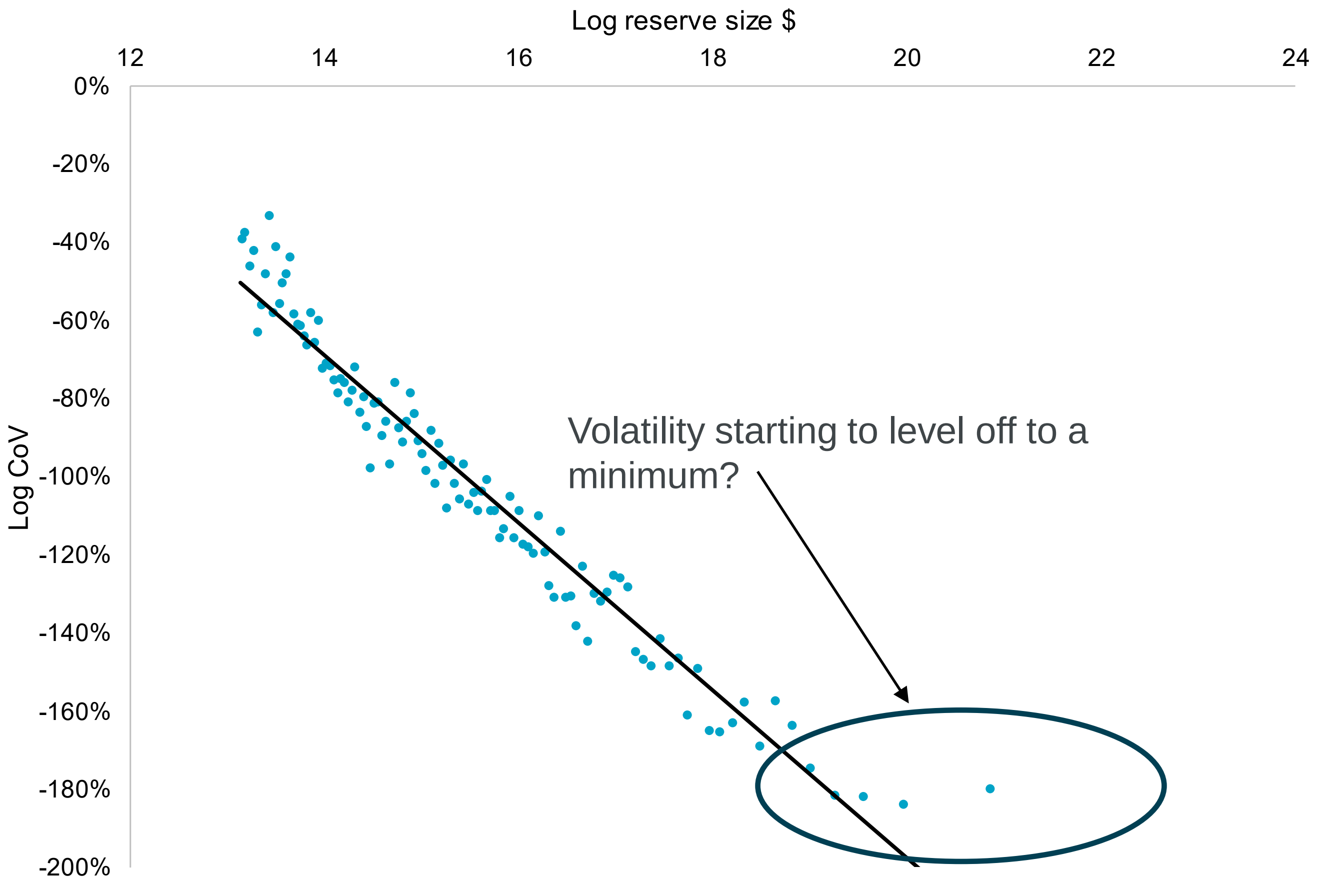
Bucketing the bootstrapping results



- What happens when we combine bucketing and bootstrapping?
- Obtain value $\hat{b} = 0.21$ – very close to base model!
- Observed r^2 value of 95%
- Evidence of undiversifiable volatility?

LCP capital benchmarking survey

- 37 respondents across the London market during April 2024
- Collected data on reserve volume and CoVs for each respondent's classes of business

$$CoV = av^{-b}$$

Model fit to LCP benchmarking data:

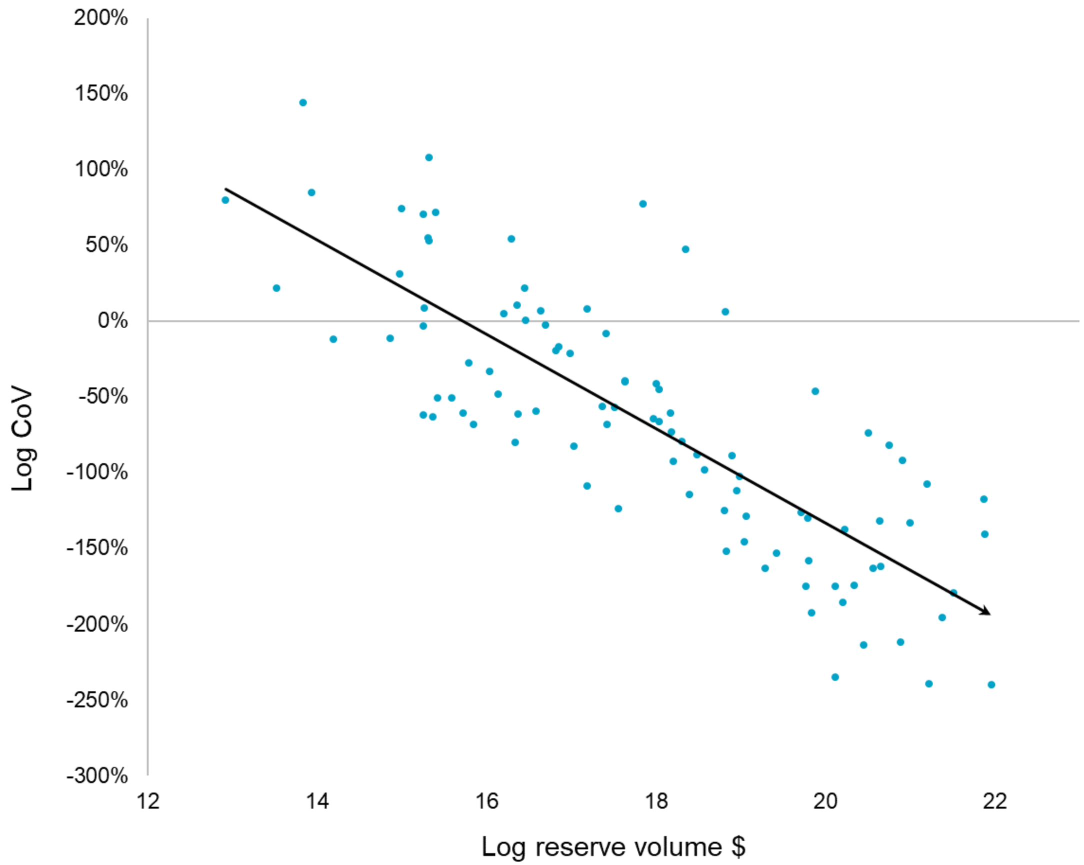
- Obtained value of $\hat{b} = 0.11$
- Low sensitivity of parameterised CoVs to changes in reserve volume

Other market datasets: APRA

- Australian equivalent to Schedule P data
  - Total reserves < $50bn (AUD)
  - 16 classes of business

$$CoV = av^{-b}$$

- Obtain value of $\hat{b} = 0.31$
- R-squared value of 66% - significantly better model fit than Schedule P



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Summary of results

	Dataset	Model name	$\hat{b}$	r^2	# parameters
Bucket analysis	**Schedule P**	**Base model**	**0.22**	**91%**	**2**
	Schedule P	Category model (regular)	0.22	91%	4
	Schedule P	Category model (advanced)	0.18-0.23	90%	6
	Schedule P	Class model (regular)	0.22	90%	12
	Schedule P	Class model (advanced)	0.12-0.28	92%	22
Bootstrap	Schedule P	Bootstrap base model	0.29	33%	2
	Schedule P	Bootstrap class model	0.29	40%	12
	Schedule P	Bootstrap cohort model	0.26	45%	3
	Schedule P	Bootstrap class/cohort model	0.25	42%	13
	Schedule P	Bootstrap bucket model	0.21	95%	2
	APRA	Bootstrap base model	0.31	66%	2
Survey data	2023 LCP Benchmarking	Base model	0.13	12%	2
	2024 LCP Benchmarking	Base model	0.11	12%	2

Monitoring the solution

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Uses

Reserve risk

- CoVs parameterised on 2023 year-end data
- Projected to 2024 year-end based on Q2 data
- *Adjust CoVs for movement in reserves between year-ends*

Validation

- Sense check selected CoVs against benchmarks
- Market typically is much larger than a single firm, and hence less volatile
- *Scale down market benchmarks to compare on like-for-like basis with model*

Sensitivity/scenario testing

- Eg: stretch view of business plan volumes
- All else being equal, this implies lower volatility
- *Adjust selected parameters to allow for proposed changes*

New classes of business

- Insufficient scope to parameterise small classes, or those with no data
- *Scale up existing distributions to allow for additional volatility on smaller book(s)*

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Worked examples

Reference volume	Target volume	Reference CoV	Target CoV
100	10	30%	$30\%\left({}^{10}/_{100}\right)^{-0.22} = \mathbf{49.8\%}$
100	25	30%	$30\%\left({}^{25}/_{100}\right)^{-0.22} = \mathbf{40.7\%}$
100	50	30%	$30\%\left({}^{50}/_{100}\right)^{-0.22} = \mathbf{34.9\%}$
100	75	30%	$30\%\left({}^{75}/_{100}\right)^{-0.22} = \mathbf{32.0\%}$
100	125	30%	$30\%\left({}^{125}/_{100}\right)^{-0.22} = \mathbf{28.6\%}$
100	250	30%	$30\%\left({}^{250}/_{100}\right)^{-0.22} = \mathbf{24.5\%}$
100	500	30%	$30\%\left({}^{500}/_{100}\right)^{-0.22} = \mathbf{21.1\%}$
100	1,000	30%	$30\%\left({}^{1,000}/_{100}\right)^{-0.22} = \mathbf{18.1\%}$

Note: method assumes the risk profiles of target and reference distribution are the same!

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Conclusions and next steps

- Power curve well describes the relationship between starting reserve volume and reserve volatility
- Suggested exponential parameter: b = 0.22
- Possible evidence of anchoring bias in reserve risk CoV selections?
- Some possible refinements to model: eg to better understand effects of class
- Other bases, eg underwriting risk, and other geographies

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Questions

Comments

Expressions of individual views by members of the Institute and Faculty of Actuaries and its staff are encouraged.

The views expressed in this presentation are those of the presenter.

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Appendix

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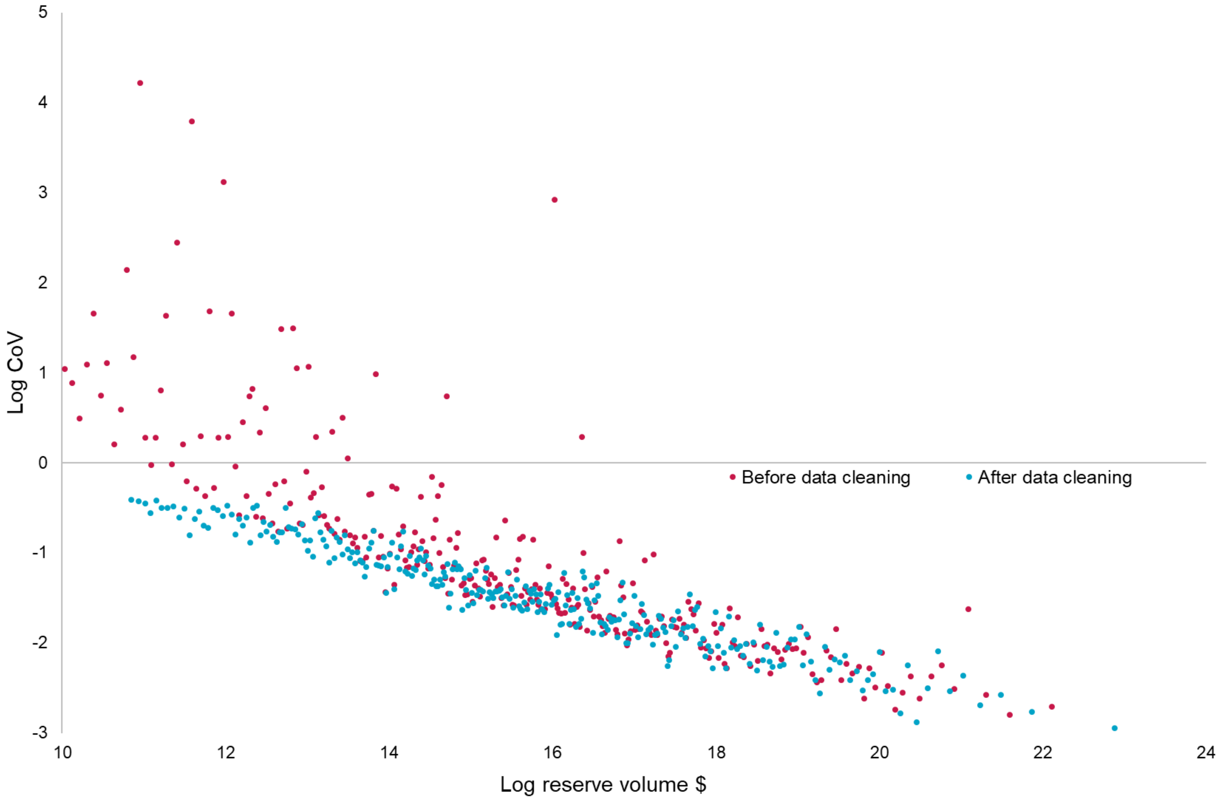
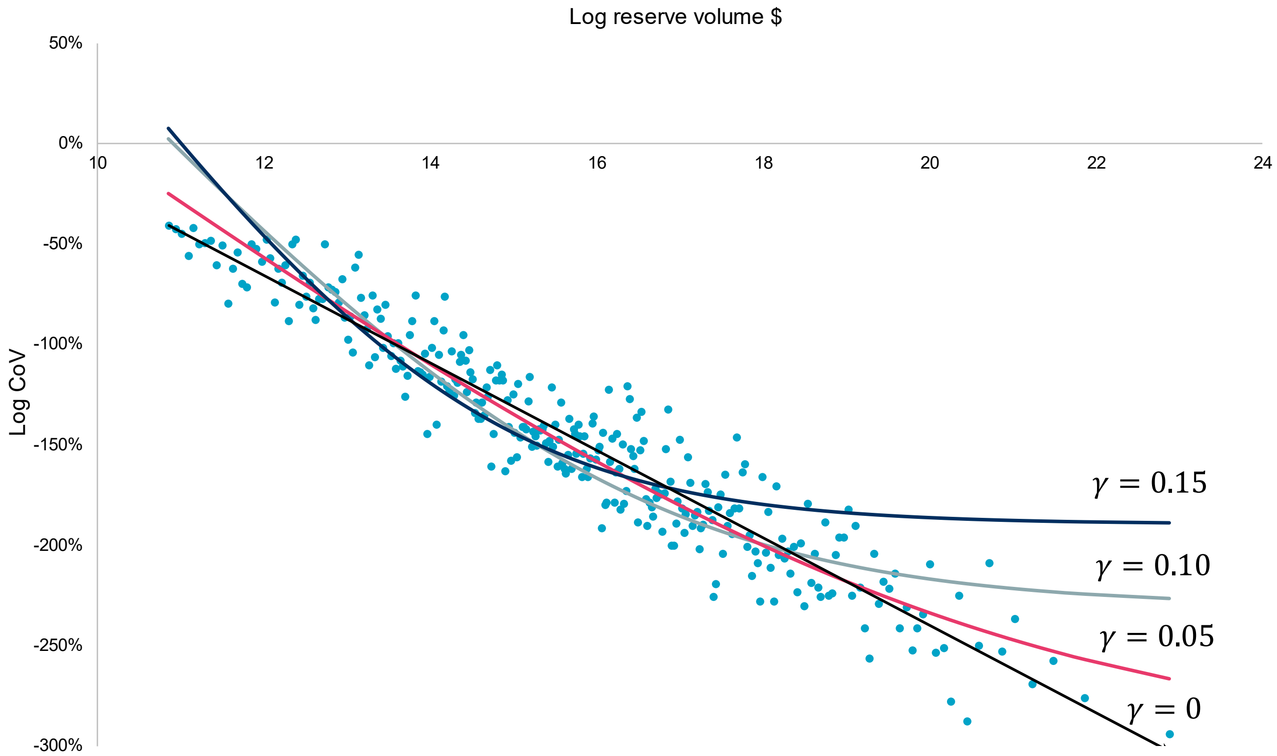
Effect of data cleaning

Section 5.1



Systemic volatility model

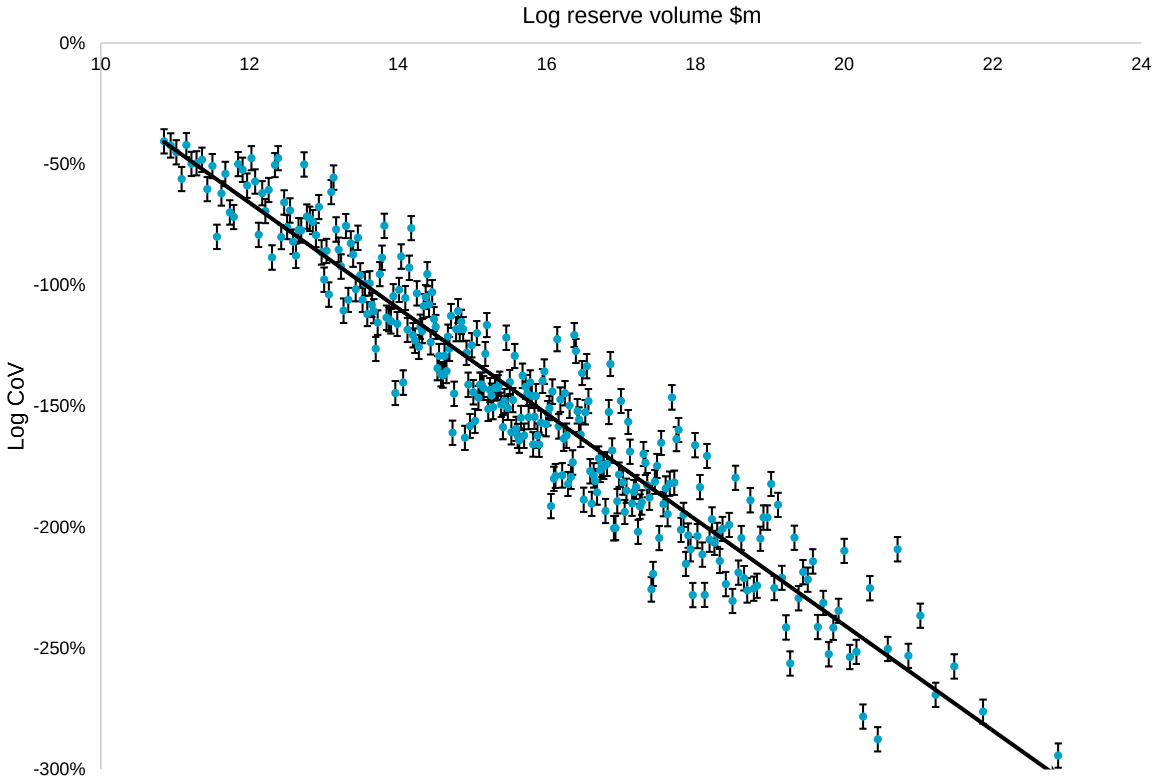
Section 5.2

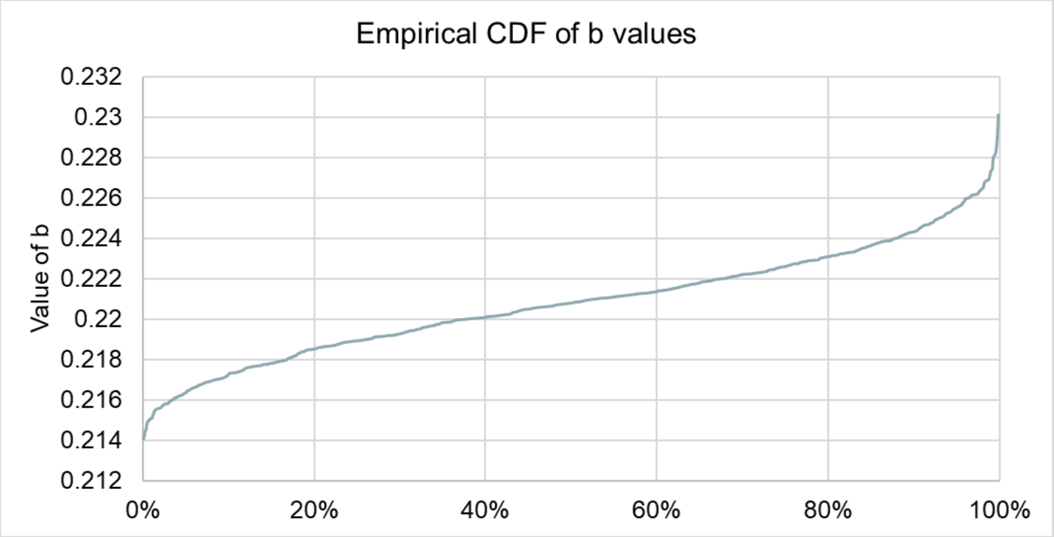


$$\log CoV = \log(av^{-b} + \gamma)$$

Vertical bucket uncertainty

Section 9.4





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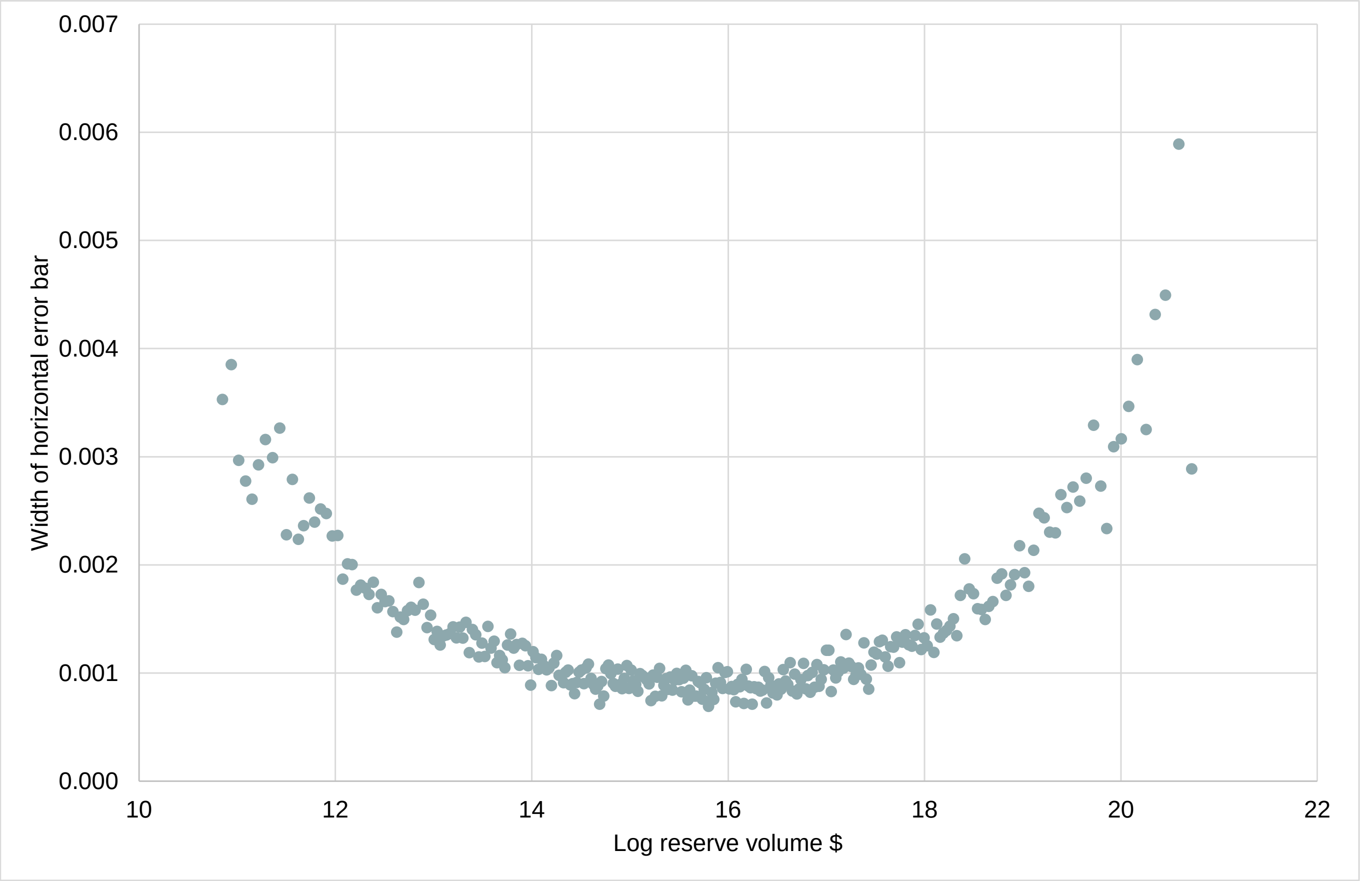
Horizontal bucket uncertainty

Section 9.5



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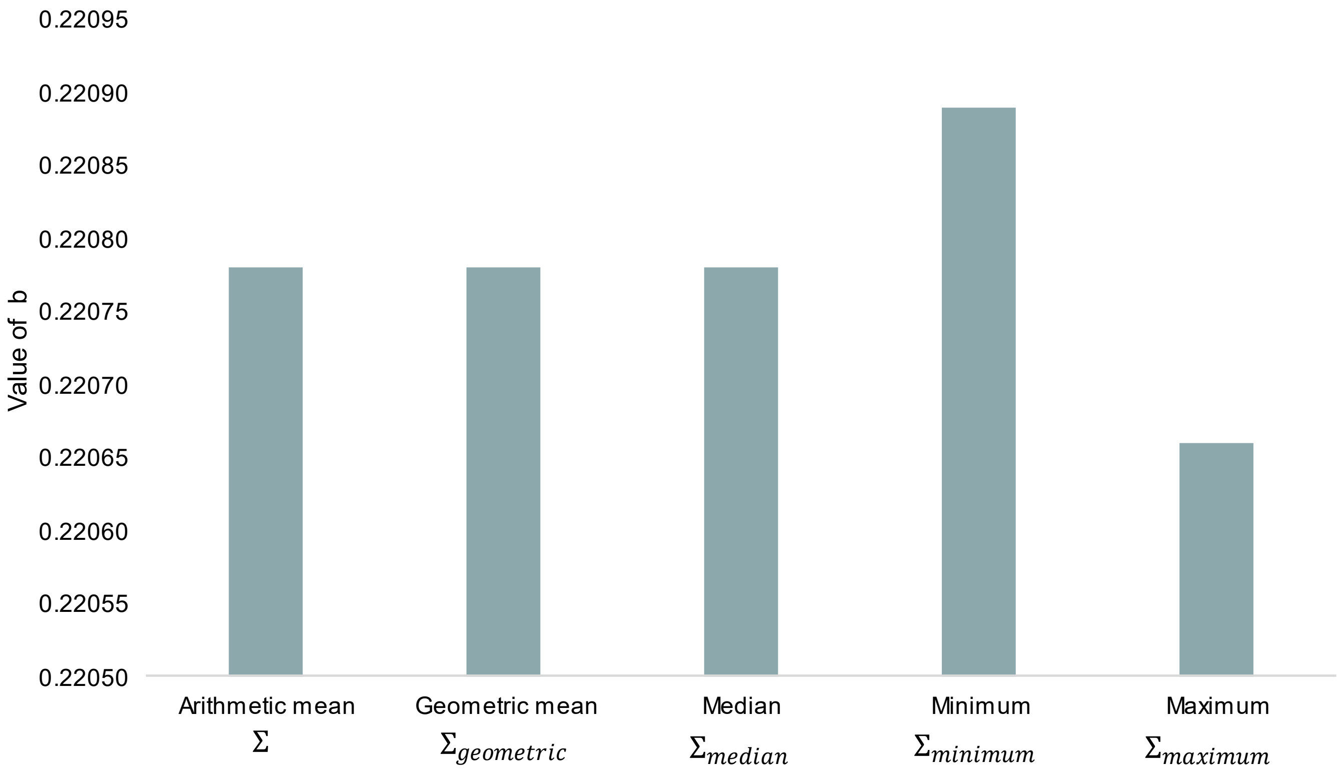
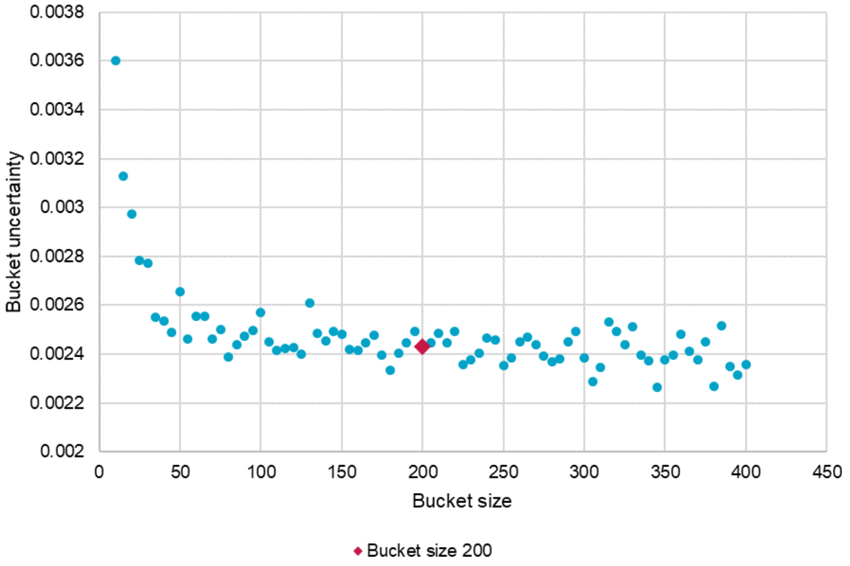
Alternative metrics

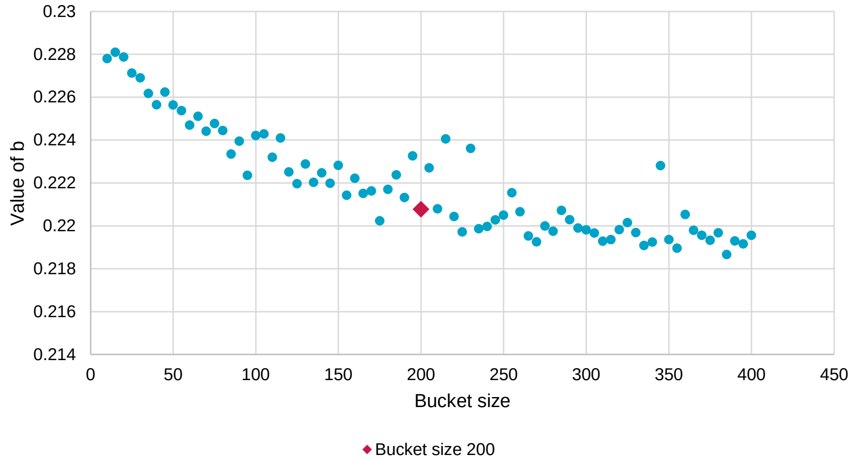
Section 9.5

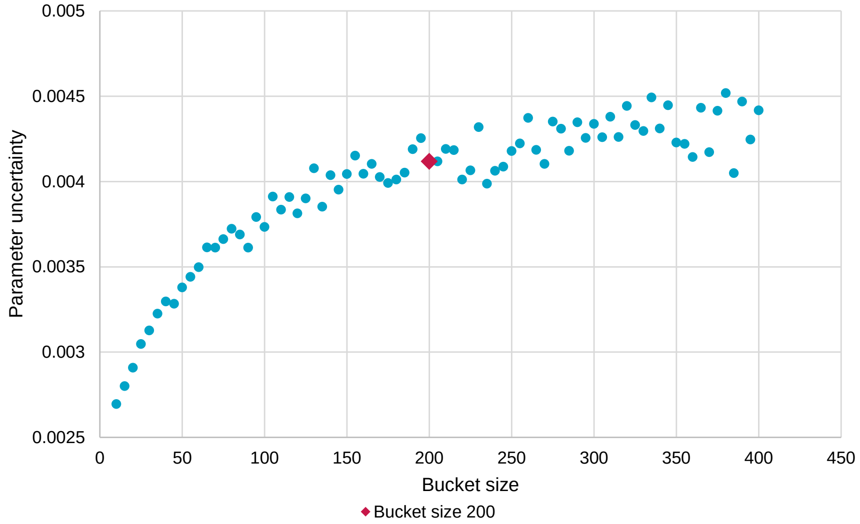


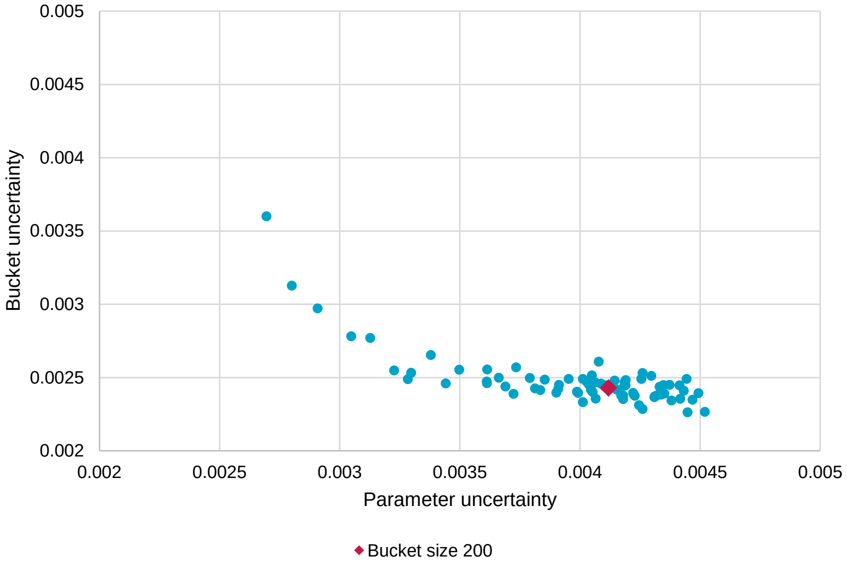
Bucket size selection

Section 9.6



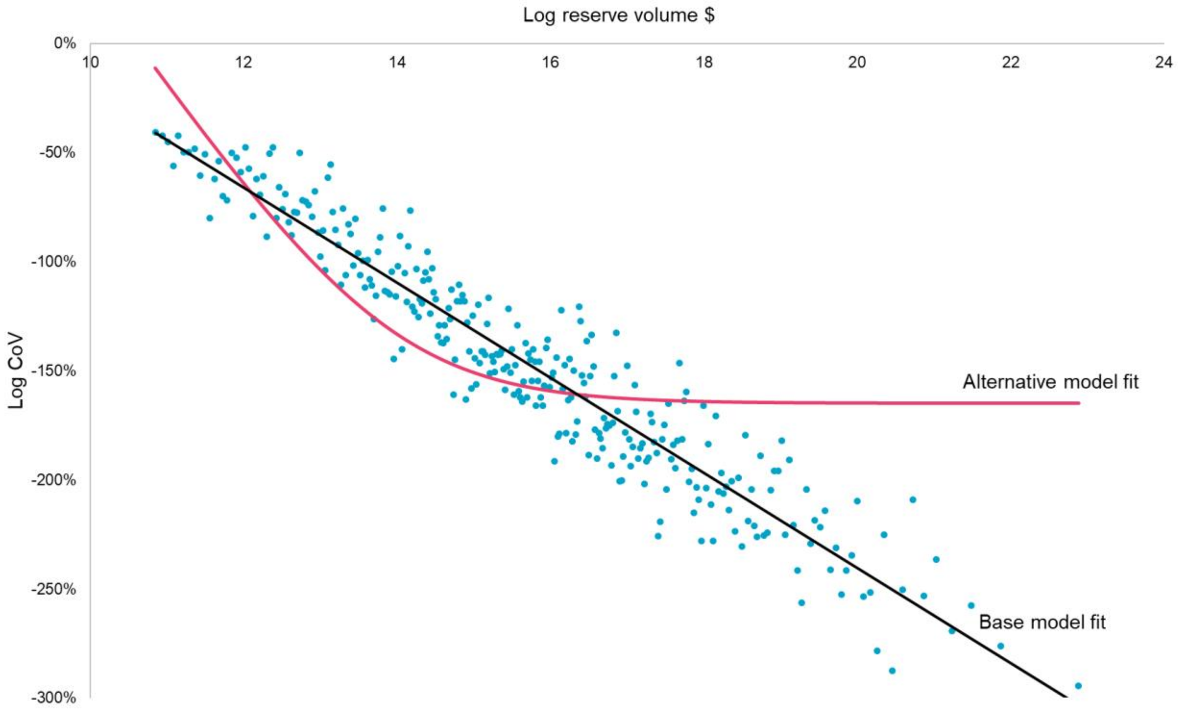






Theoretical model

Appendix 4



$$CoV(S_n) = \frac{\sqrt{Var(S)}}{E[S]} = \frac{\sqrt{n(\sigma^2 + \mu^2) + n^2\rho\sigma^2}}{n\mu}$$

Can be simplified:

$$CoV = \sqrt{c + \frac{d}{v}}$$

$$\lim_{V \to \infty} CoV = \lim_{V \to \infty} \sqrt{c + \frac{d}{V}}$$

$$= \sqrt{c} = \sqrt{0.037} = 19.2\%$$