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Solvency II Risk Margins and SCR from First Principles

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Introductions

Challenge of SII and approach outline

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Solvency II

Definitions from EU SII Act

- *“The value of technical provisions shall be equal to the sum of a best estimate and a risk margin”*
- *“The best estimate shall correspond to the probability-weighted average of future cash-flows, taking account of the time value of money (expected present value of future cash-flows), using the relevant risk-free interest rate term structure.”*
- *“the risk margin shall be calculated by determining the cost of providing an amount of eligible own funds equal to the Solvency Capital Requirement necessary to support the insurance and reinsurance obligations over the lifetime thereof.”*
- *“The Solvency Capital Requirement shall correspond to the Value-at-Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period.”*

Practical Complications (I)

Inter-dependence of SCR/Risk Margin and One-year risk horizon

“the risk margin shall be calculated by determining the cost of providing an amount of eligible own funds equal to the Solvency Capital Requirement necessary to support the insurance and reinsurance obligations over the lifetime thereof.”

$$RM = CoC \cdot \sum_{t \geq 0} \frac{E(SCR_0(t))}{(1 + r(t+1))^{t+1}}$$

* $SCR_0(t)$ represents SCR for business on balance sheet at time 0

“The Solvency Capital Requirement shall correspond to the Value-at-Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period.”

$$SCR(t) + OpeningTP_t = VaR_{0.995}\left(\frac{(ClosingTP_{t+1} + X_{t+1})}{1 + r(t+1)} \middle| I_t\right)$$

Where I_t refers to the information available at time *t.*

Note we will ignore operational risk and reinsurance credit risk in this presentation

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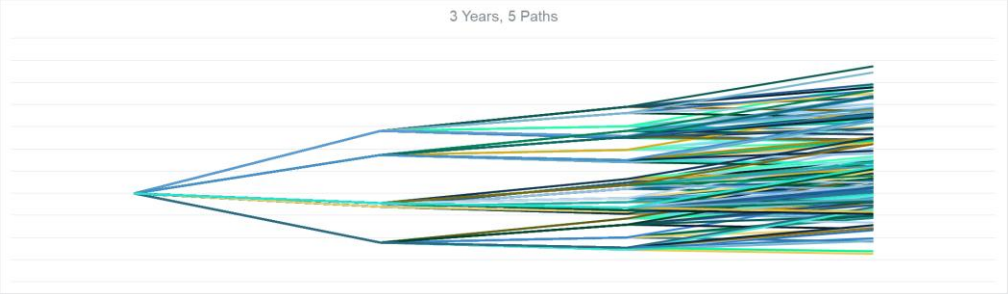
Practical Complications (II)

Inter-dependence of SCR/Risk Margin and One-year risk horizon

- SCR(0) is dependent on distribution of RM(1)
- Each potential observation of RM(1) is dependent on SCR(1) – future SCRs are stochastic in general
- SCR(1) is dependent on distribution of RM(2), conditional on time 1 experience…



In other words:

- To calculate the future values of the SCR and risk margin, we need to condition on experience observed up to that time
- To achieve this for N years and S paths, we need S^N paths in total.

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Some Big Numbers

- 10,000 sims over 9 years:

1,000,000,000,000,000,000,000,000,000,000,000,000 sims

  – That is impossible currently, and for the foreseeable future

  – 1 billion cores processing 1 billion sims per second will take over 30 billion years

- 1,000 sims over 4 years:

1,000,000,000,000 sims

  – With cloud computing and modern vector processors, this is just about feasible

What do we do today?

Common market practices

- Modelling the one-year result
  - Emergence pattern on ultimate risk
  - Re-reserving / Actuary in the box
  - Merz-Wüthrich
- Ignore the effect of the risk margin on $SCR_0(0)$
- Project the initial $SCR_0(0)$ to produce $SCR_0(t)$
  - Run-off patterns based on expected reserves
  - Split of total uncertainty (Merz-Wüthrich)
  - Value at risk / standard deviation of **unconditional** profit/loss
- Other approaches e.g. analytical formula for lognormal/Gaussian copula
- We need to make simplifications to calculate, but we don't know how much of an approximation we are making

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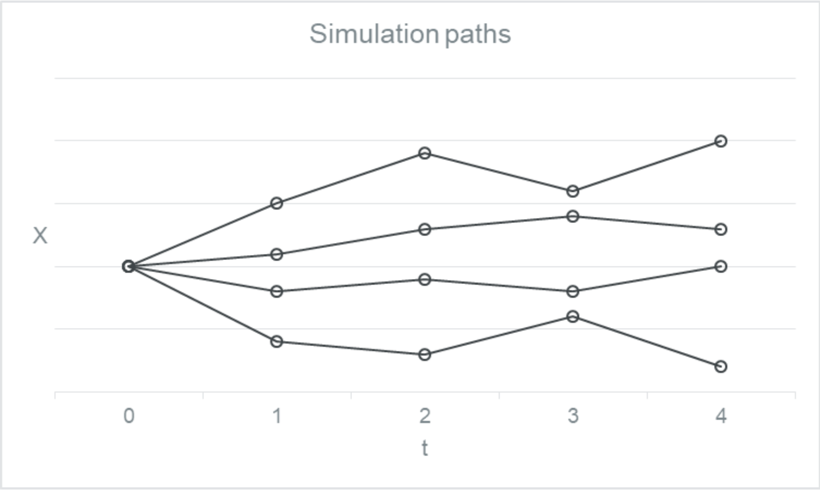
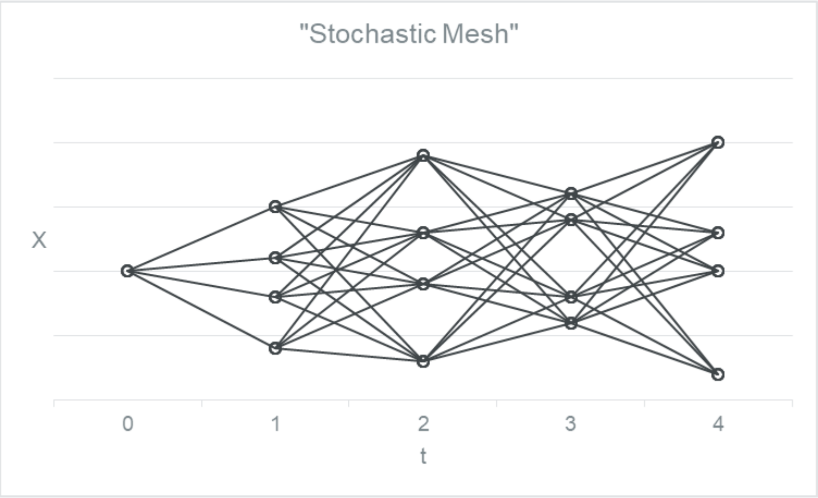
What can we learn from other fields?

Mark Broadie, Paul Glasserman, A Stochastic Mesh Method for Pricing High-Dimensional American Options, *Journal of Computational Finance* 7 (2004): 35-72.





Paths allowed *between* simulations at neighbouring time steps, with weight $w(t, X_t, X_{t+1})$

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One-year horizon

Formula for Best Estimate, Risk Margin, SCR

Assuming constant risk free rate rf

- Discounted Best Estimate/Reserve:

$$R_t = \frac{\mathrm{E}(X_{t+1} + R_{t+1} | I_t)}{(1 + rf)}$$

- Risk Margin:

$$RM_t = \frac{SCR_t \cdot CoC + \mathrm{E}(RM_{t+1} | I_t)}{(1 + rf)}$$

- SCR:

$$SCR_t = \frac{VaR_{0.995}(R_{t+1} + RM_{t+1} + X_{t+1} | I_t) - R_t(1 + rf) - \mathrm{E}(RM_{t+1} | I_t)}{1 + r}$$

where $r = CoC + rf$

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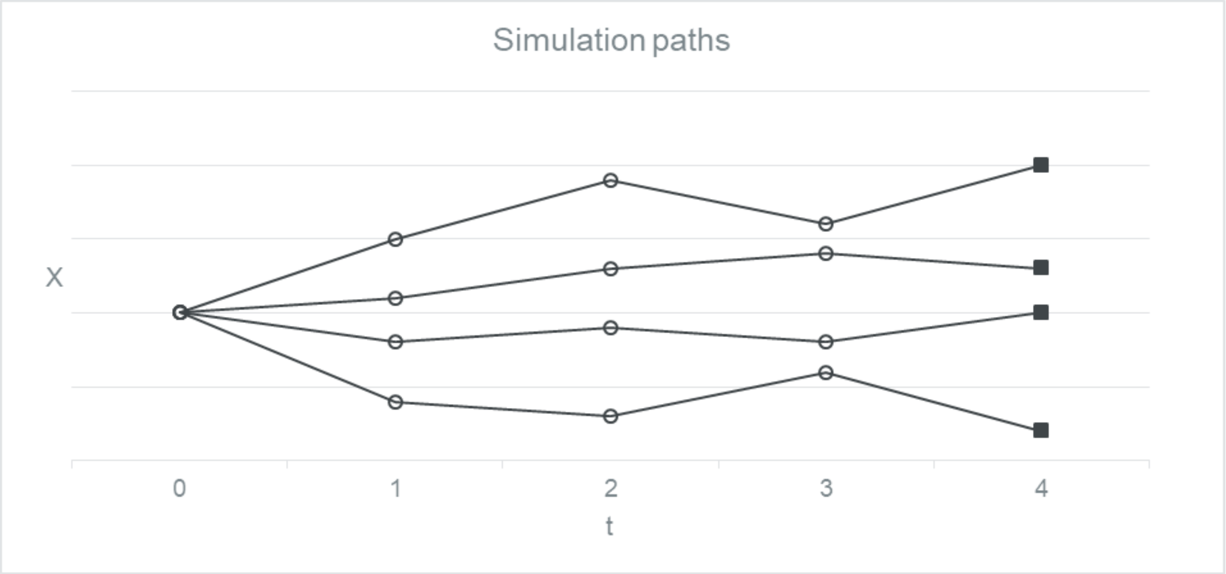
Conditional Re-Weighting

Visualisation

Assume cumulative claims follow a Markov process

Start with a set of simulations of future payments for each period up to maturity

At maturity N, $R_N = RM_N = SCR_N = 0$



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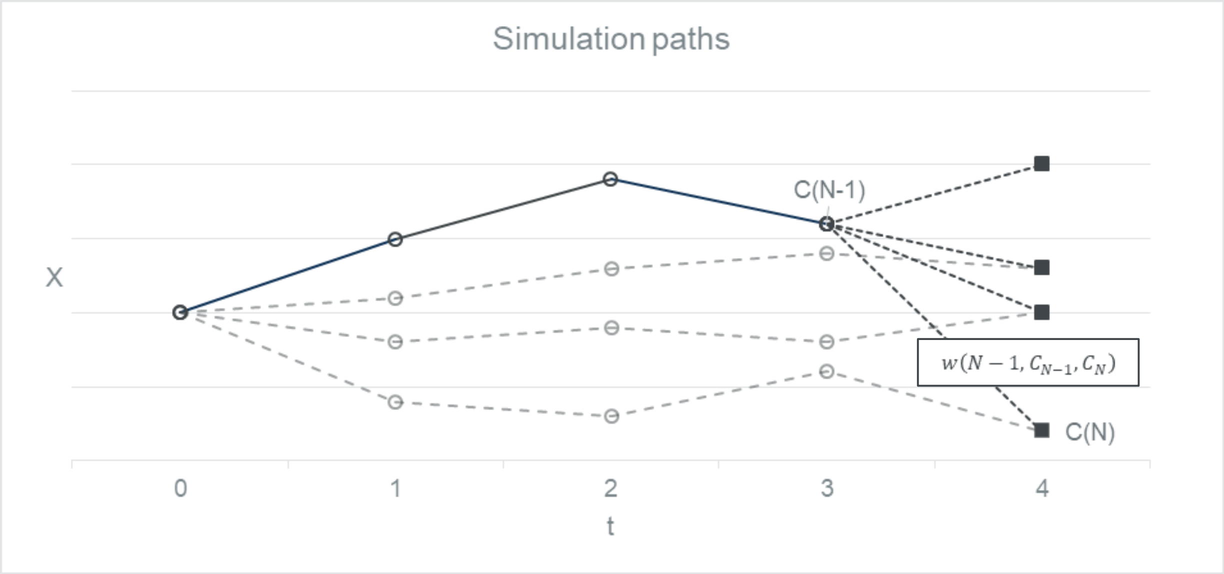
Conditional Re-Weighting

Visualisation

At time N-1:

For each simulation at N-1:

- Determine weights of observing each simulation at N
- Use weights to determine *R*, *RM*, *SCR* at time N-1



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Conditional Re-Weighting

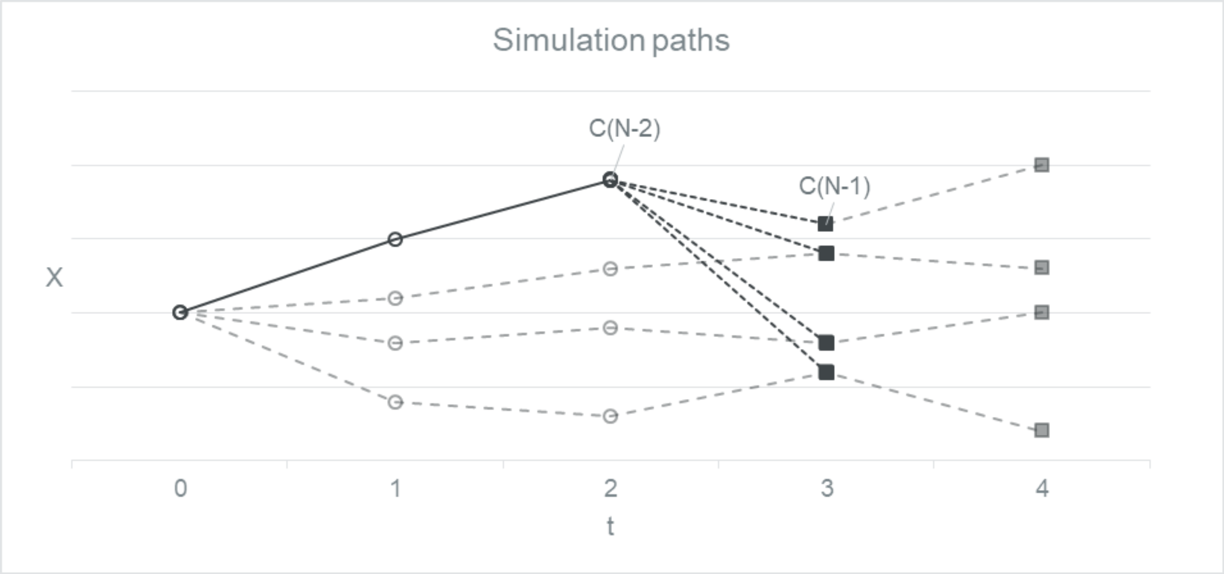
Visualisation

After *R*, *RM*, *SCR* is determined for each simulation at N-1:

For each simulation at N-2:

- Determine weights of observing each simulation at N-1
- Use weights to determine R, RM, SCR at time N-2

And repeat for all previous time steps up to time 0.



Conditional Re-Weighting

Calculating the weights

- Using Broadie and Glasserman approach, weights are ratio of conditional to unconditional probability densities

$$w(N-1, C_{N-1}, C_N) \propto \frac{f(C_N|C_{N-1})}{f(C_N|C_0)}$$

- **Computational effort for above is of the order $\boldsymbol{N * S^2}$.**
- **Avoids $\boldsymbol{S^{\mathrm{N}}}$ paths, recycles available simulations**
- A different though related method using a maximum entropy approximation was explored in England and Czernuszewicz, GIRO 2009

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Illustration with Actuarial Model

Mack's Additive Model

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Mack's Additive Model
(aka Incremental Loss Ratio Model)

Model of incremental claims X_{ij} in origin period i and development period j

- $E[X_{ij}] = E_i\beta_j$, $Var[X_{ij}] = \phi_j E_i \beta_j$
- X_{ij} independent
- E_i are known volume factors for the origin year (exposure, premiums)
- β_j are unknown parameters giving the expected burning cost emerging in the development period

β_j estimated by $\widehat{\beta}_j = \frac{S_j}{\sum_{i=0}^{N-j+1} E_i}$ where $S_j = \sum_{i=0}^{N-j+1} X_{ij}$ are the column sums

THOMAS MACK (1997): Schadenversicherungsmathematik. Sonderauflage von Heft 28 der Schriftenreihe Angewandte Versicherungsmathematik der Deutschen Gesellschaft fur Versicherungsmathematik e.V. Verlag Versicherungswirtschaft e.V. Karlsruhe, 1997. IISN 0178-8116, ISBN 3-88487-582-5

Generating a Stochastic Mesh

In Mack's additive model, the cumulative column sums $S_j(\mathrm{t})$ are a Markov process

Given an initial claims triangle $\boldsymbol{X_0}$, generate simulations of future payments (e.g. through bootstrapping or Bayesian methods), hence column sums $S_j(\mathrm{t})$. This generates the nodes of the mesh.

Given new simulated claims $S_j(\mathrm{t})$ after *t* periods, we want to calculate the conditional distribution of $S_j(t+1)$, from the unconditional simulations.

We need two types of re-weighting *(details in appendix)*:

1. To capture the reduction in parameter uncertainty due to new data

$$w^{(1)}(t, S(t)) \propto \prod_{j=t+1}^{N} \frac{L\left(\beta_j \middle| S_j(t)\right)}{L\left(\beta_j \middle| S_j(0)\right)}$$

2. To calculate the likelihood of observing the unconditional simulation of the next column sums $S(t+1)$, given the new data and parameters at time *t*

$$w^{(2)}\big(t, S(t), S(t+1)\big) \propto \prod_{j=t+1}^{N} \frac{f\left(S_j(t+1) \middle| S_j(t), \beta_j\right)}{f\left(S_j(t+1) \middle| \beta_j\right)}$$

The total weights are then the product of the individual weights

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Numerical Example

Results and Observations

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Example – Paid Triangle

Accident Year	Exposure	Dev Yr 1	2	3	4	5	6	7	8	9	10
2009	381,364	11,825	24,673	27,270	29,097	32,388	13,530	20,240	43,325	13,061	7,675
2010	444,663	9,240	24,982	29,384	23,282	19,885	13,658	11,502	5,641	23,814	
2011	536,238	10,965	31,188	47,734	42,687	62,449	75,356	46,503	11,822		
2012	563,501	10,591	42,384	66,881	92,102	48,806	24,568	20,178			
2013	586,200	20,438	61,917	66,616	50,885	40,036	32,817				
2014	629,365	29,901	55,092	65,339	65,731	34,430					
2015	596,429	22,823	51,469	74,413	34,128						
2016	510,807	17,570	58,787	80,207							
2017	527,781	22,567	51,593								
2018	586,881	23,145									

Implementation details

- 500,000 simulations
- 9 future time periods
  - (2 trillion weights to calculate!)
- Constant dispersion parameter
- No new business
- Over-dispersed Poisson distribution for claims
- rf=0

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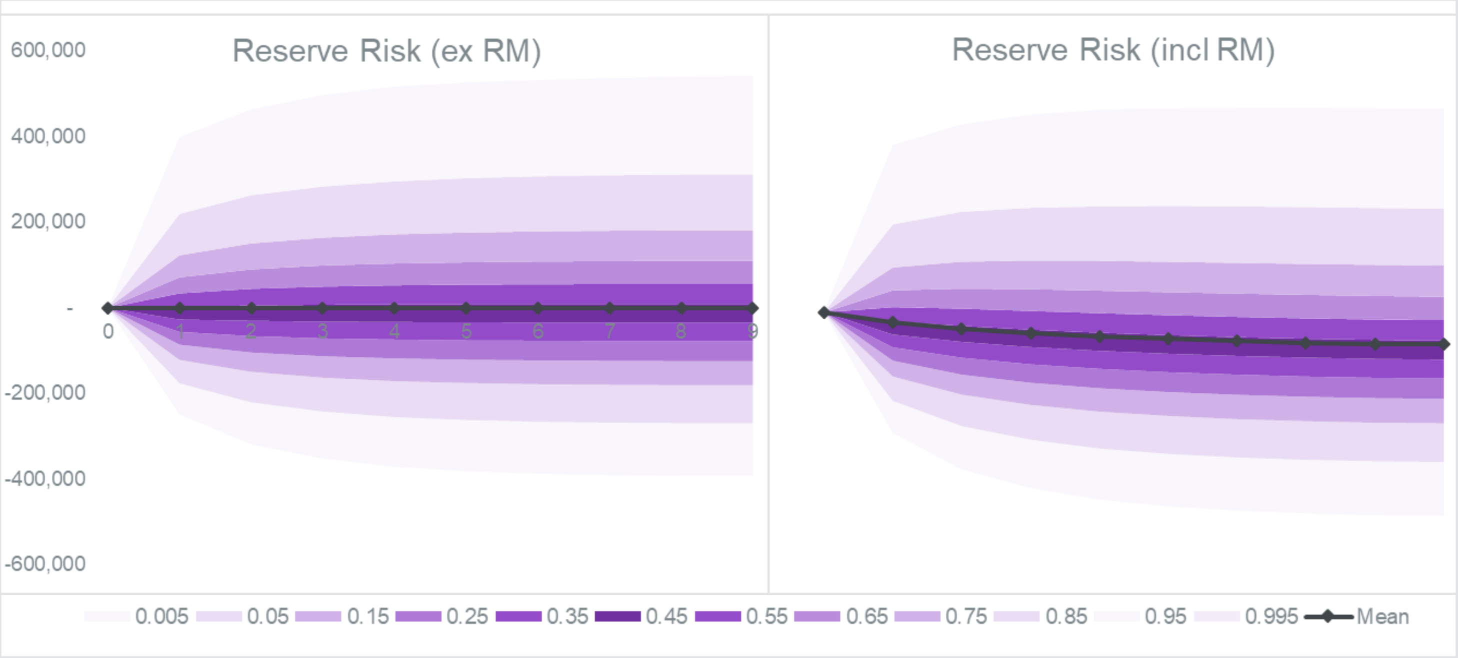
One-Year vs Ultimate Reserve Risk



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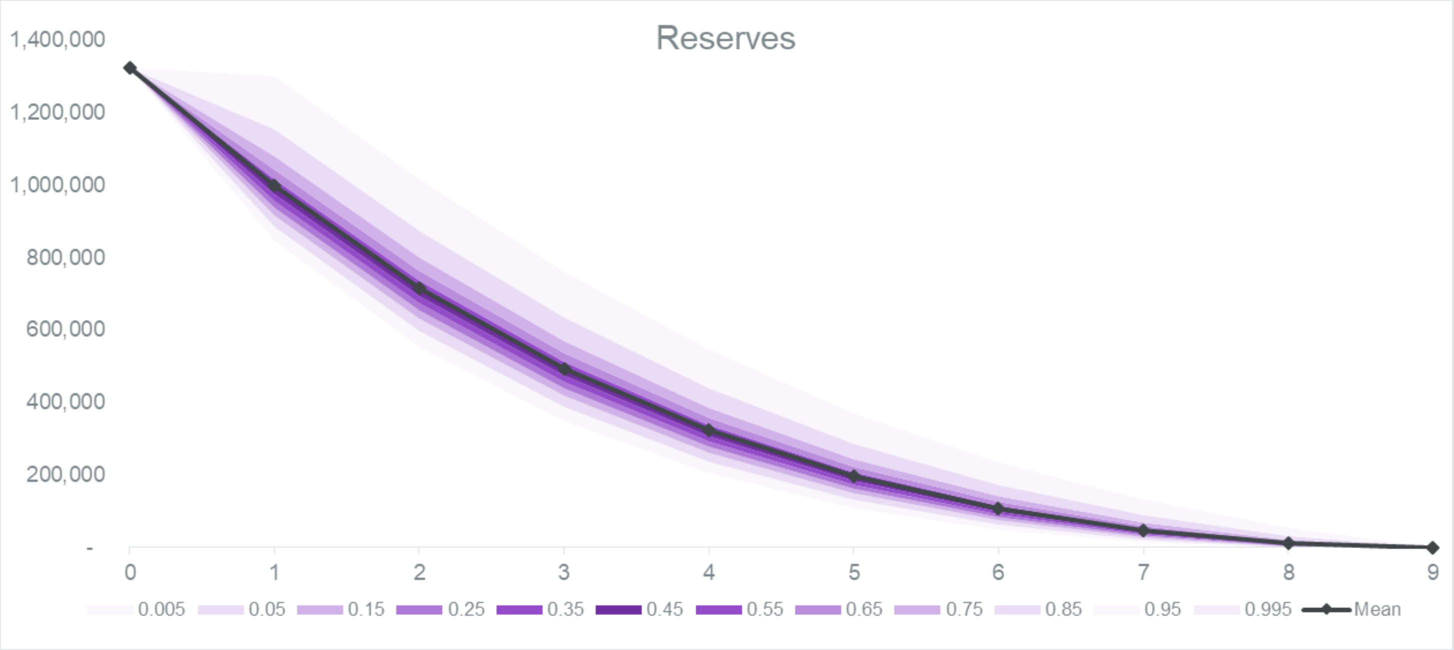
Reserves Projection Results



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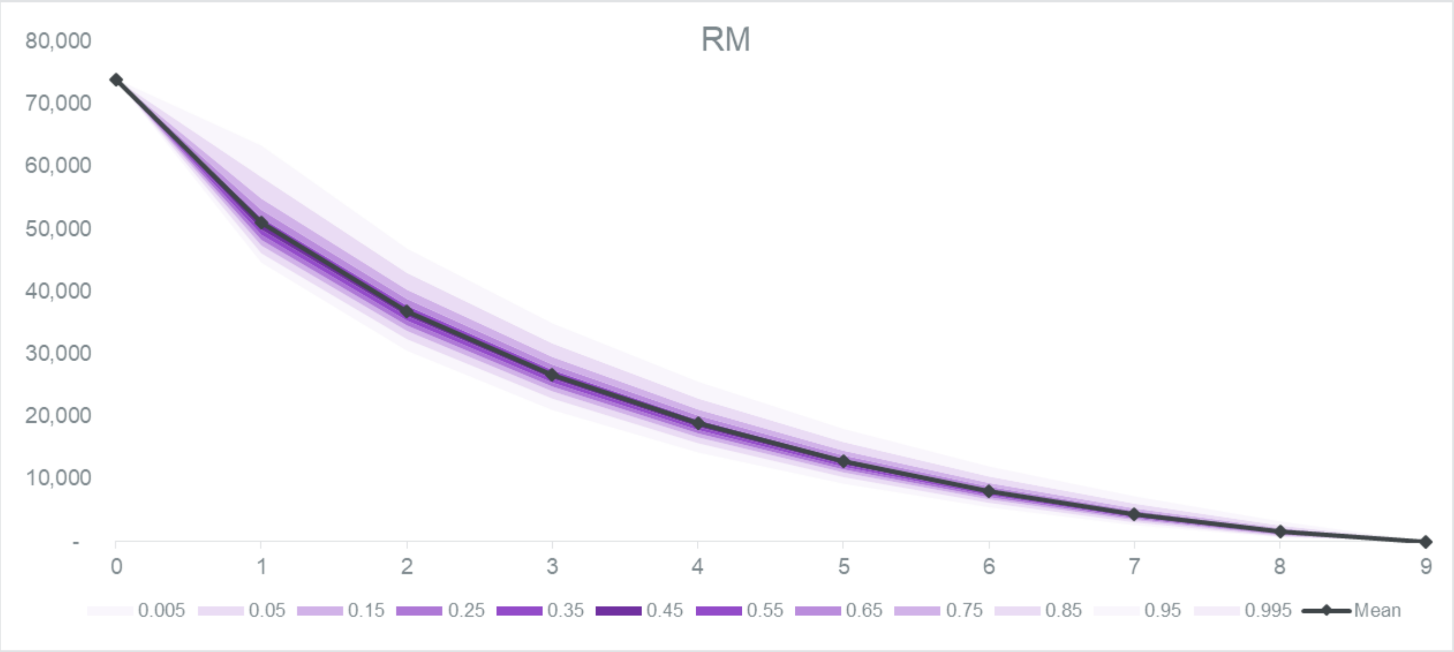
Risk Margin Projection Results



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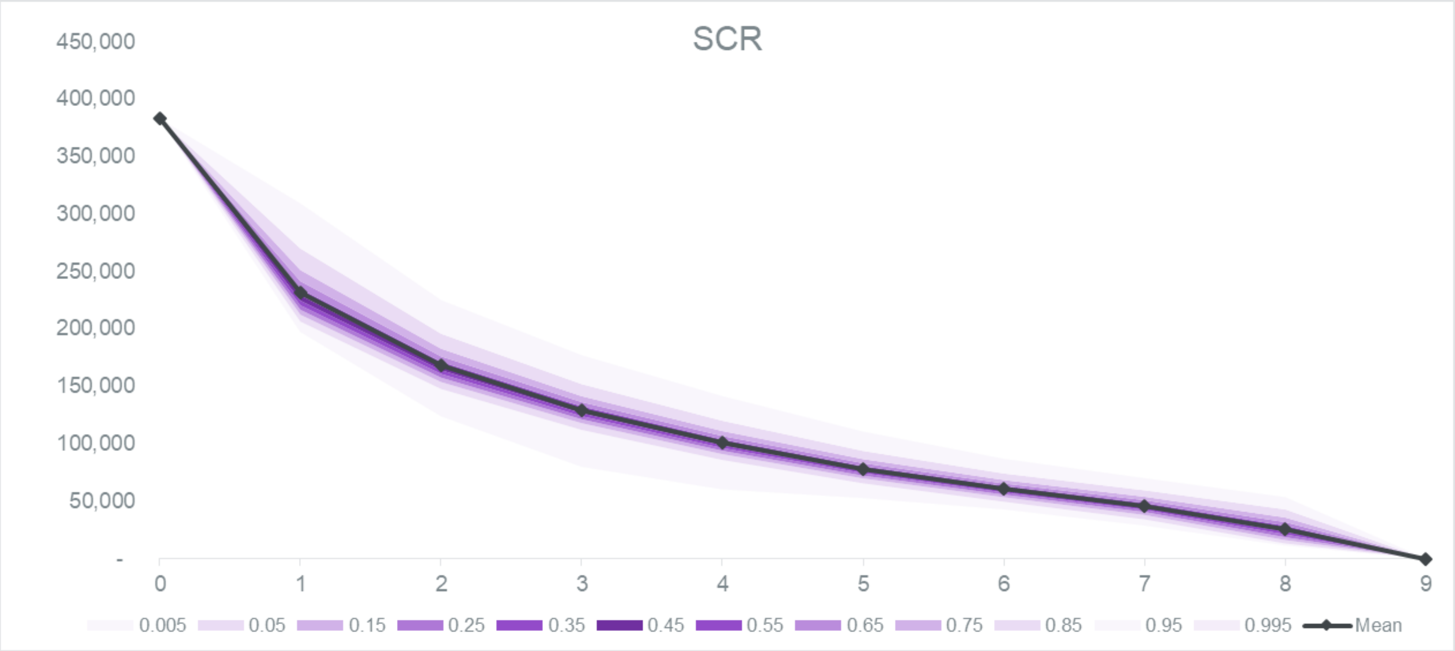
SCR Run-Off Results



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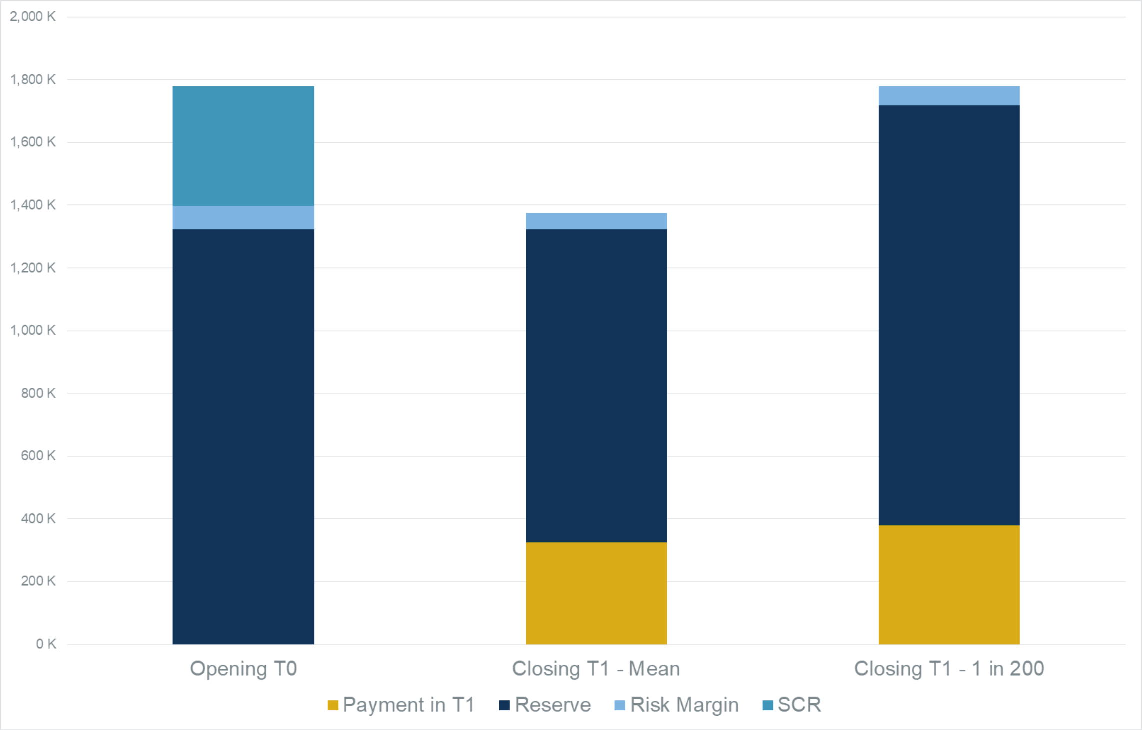
Consistent SII Run Off



- Reserve at T0 matches Reserve at T1 + Payment in T1 at mean level
- Risk Margin at mean level reduced by 22k = 6% * SCR
- 1-in-200 Payment + TP at T1 matches SCR + TP at T0

Item	Opening T0	Closing T1 - Mean	Closing T1 - 1 in 200
Payment in T1		326 K	380 K
Reserve	1,323 K	998 K	1,338 K
Risk Margin	74 K	51 K	63 K
SCR	383 K		
Total	**1,780 K**		**1,780 K**

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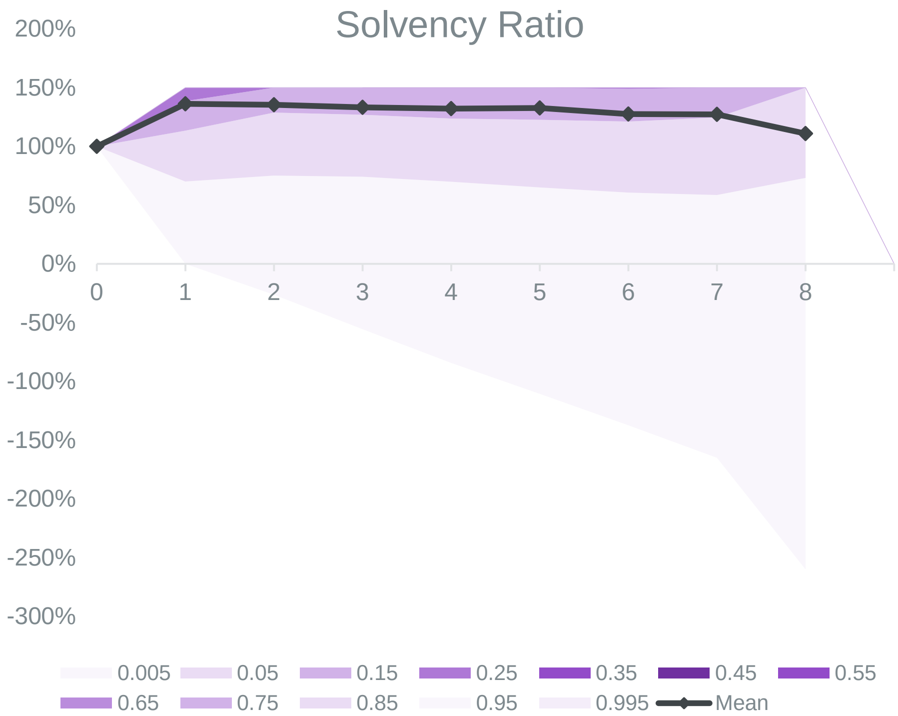
Example use of capital projection



- Can integrate capital management strategies based on future SCR
  - i.e. capital release if Solvency Ratio > threshold
- Could be used to analyse contingent convertible capital structures (CoCo's)
  - Subordinated debt where the principle is converted to equity if Solvency Ratio < threshold

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Lessons Learned

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Underestimation of Proxy SCR by excluding risk margin

$$ProxySCR_0 = \frac{VaR_{0.995}(R_1 - R_0 + X_1)}{1 + CoC} \quad \text{vs} \quad SCR_0 = \frac{VaR_{0.995}\left(R_1 - R_0 + X_1 + \left(RM_1 - E(RM_1)\right)\right)}{1 + CoC}$$

	Mean	SD
Reserves and Payment	1,323,282	121,927
Risk Margin	51,012	3,775

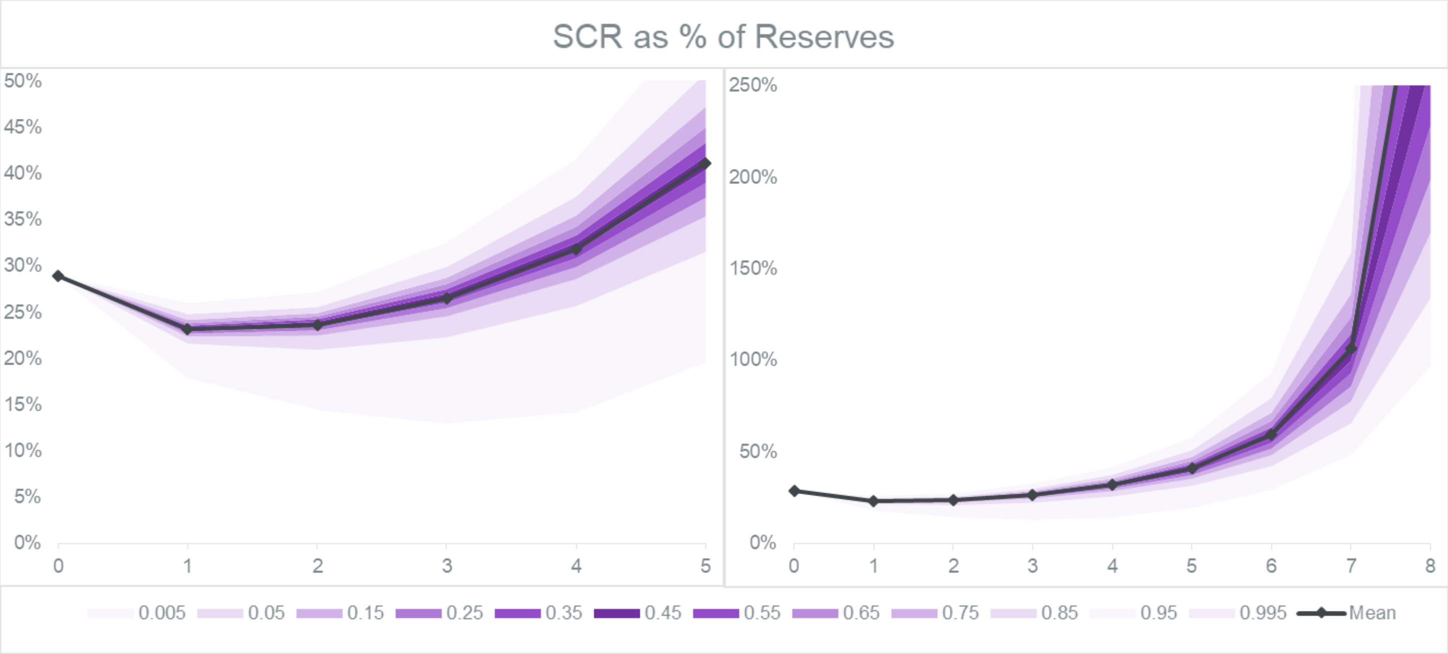
Correlation matrix	Reserves and Payment	Risk Margin
Reserves and Payment	100%	74%
Risk Margin	74%	100%

Proxy SCR	SCR	Underestimation	Risk Margin SD as % of Reserve SD
375,789	382,991	-1.9%	3.1%

As risk margin is stochastic and correlated to reserves (and payment) at time 1, this approximation will tend to underestimate opening SCR.

Size of underestimation is dependent on relative volatility of the components.

SCR decreases as % of reserves initially



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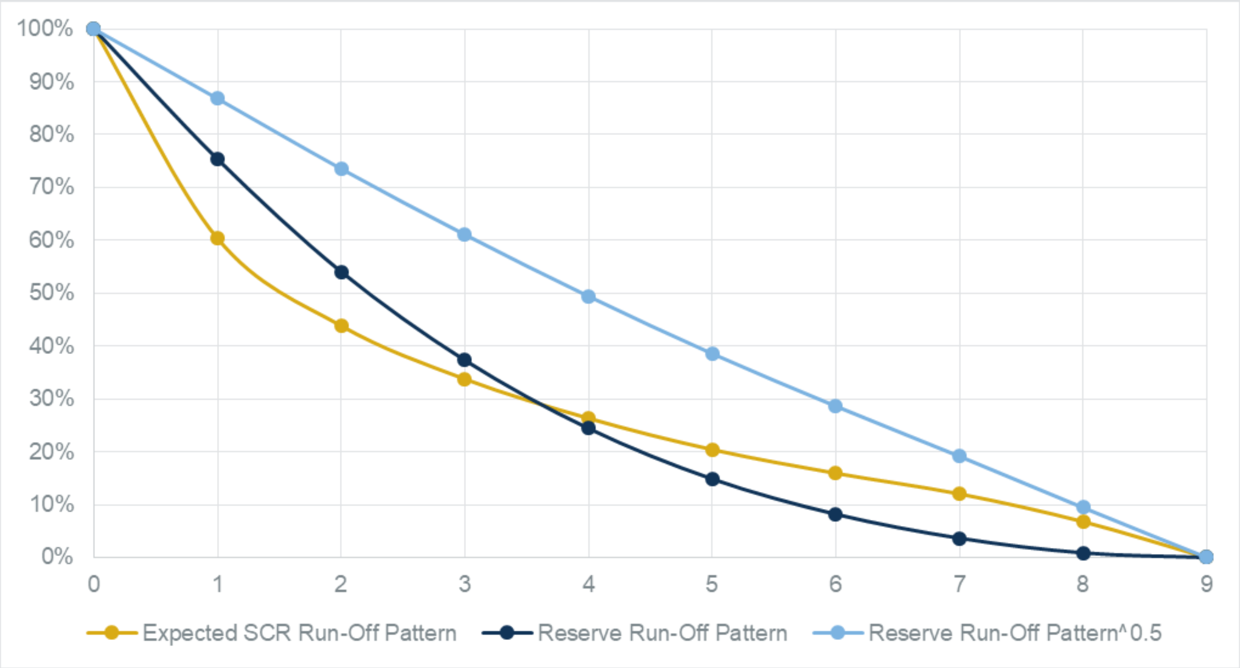
Sqrt Reserve Run-Off Pattern not a good proxy for SCR Run-Off Pattern



- Sqrt Reserve Run-Off Pattern overestimates opening RM by about 46%

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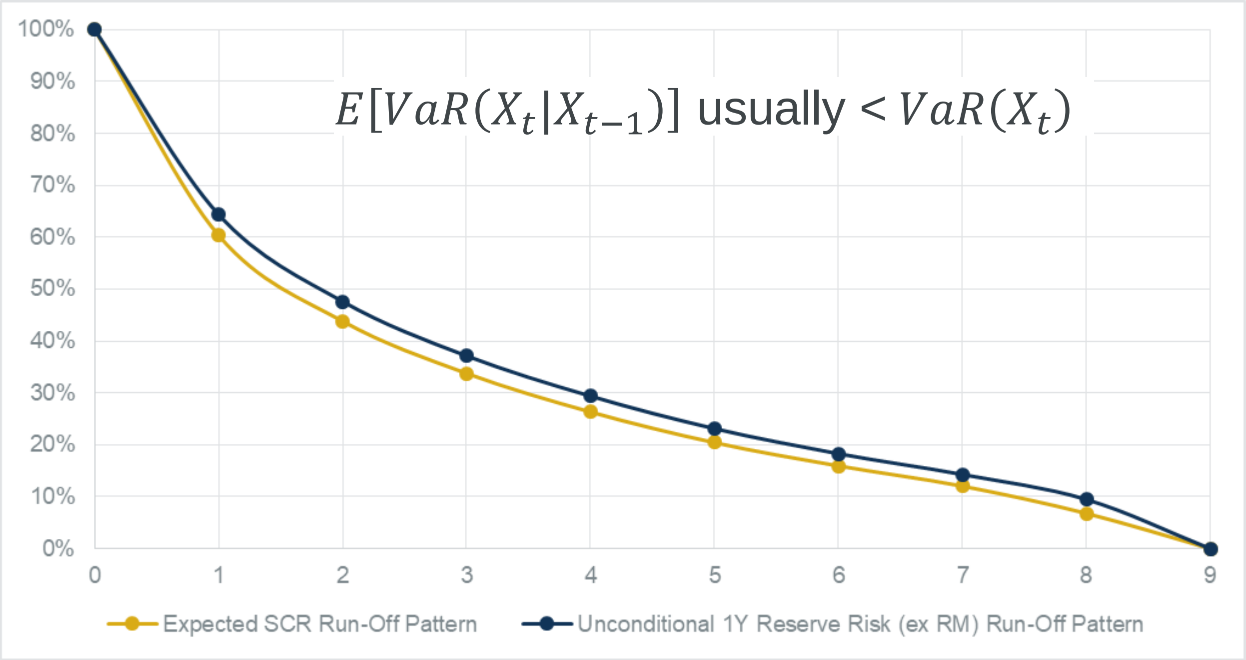
Unconditional One-Year Reserve Risk is a much better proxy for SCR Run-Off Pattern



- Unconditional One-Year Reserve Risk Run-Off Pattern overestimates opening RM by about 8%

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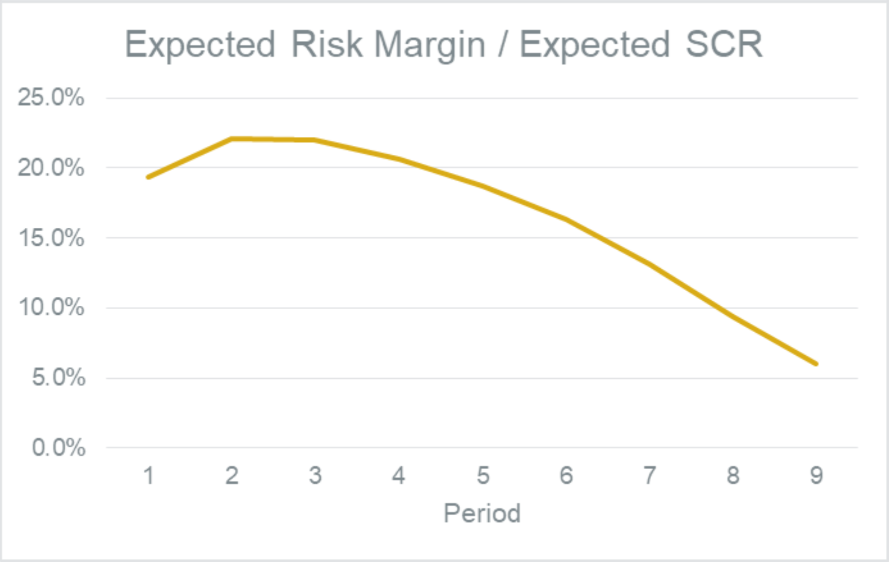
Risk Margin not a constant proportion of SCR or Reserve



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Conclusion

Conclusion

- Calculating a risk margin and SCR consistently with Solvency 2 principles is a considerable challenge!
- Ideas from option pricing can make it technically possible
- Computationally intensive, but can give useful insights
  - Square-root reserve run-off pattern is not a good proxy SCR pattern
  - Ignoring the risk margin in the calculation of $SCR_0(t)$ (*small understatement*)
  - Using unconditional VaR, rather than conditional VaR, for $SCR_0(t)$ (*small overstatement*)
- Results only sensible if model assumptions are reasonable

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Future Enhancements

- Apply to other models (e.g. chain-ladder based Mack)
- Incorporating new business – ORSA
- Reinsurance (e.g. ADC)
- Inflation and other external drivers
- Simulation error – choice of weights
- IFRS17 risk adjustment

Questions

Comments

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Appendix

Details for conditional re-weighting on Mack's Additive Model

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Generating Future Claims with Mack's Additive Model

- Given a historical claims triangle, Mack's additive model can be bootstrapped, or use Bayesian statistics to get a distribution of future payments
- Bayes approach taken here (little difference in results)
  - Assume $X_{ij} \sim \phi_j Poisson\left(\frac{E_i\beta_j}{\phi_j}\right)$ (i.e. an over-dispersed Poisson)
  - Assume uniform prior for lnβ_j
  - Use plug-in estimate for dispersion:

$$\widehat{\phi_j} = \frac{1}{n_j - 1}\sum_i \frac{\left(X_{ij} - E_i\widehat{\beta_j}\right)^2}{E_i\widehat{\beta_j}}, \text{ or } \hat{\phi} = \frac{1}{N(N-1)/2}\sum_{i,j}\frac{\left(X_{ij} - E_i\widehat{\beta_j}\right)^2}{E_i\widehat{\beta_j}}$$

  - Then $\beta_j|\mathrm{X} \sim \mathrm{Gamma}\left(S_j/\phi_j, \sum_{i=0}^{N-j+1} E_i/\phi_j\right)$, where $S_j = \sum_{i=0}^{N-j+1} X_{ij}$ are the column sums
  - Generate simulations of β_j to incorporate parameter error, then, given the parameters generate future payments from an over-dispersed Poisson

Conditional Re-Weighting – Parameter Error

- From the original triangle $\boldsymbol{X_0}$, we have

$$\beta_j|\boldsymbol{X_0} \sim \text{Gamma}\left(\frac{S_j(0)}{\phi_j}, \frac{\sum_{i=0}^{N-j+1} E_i}{\phi_j}\right)$$

- At time t, we will have observed new data $\boldsymbol{X_t}$ and will have

$$\beta_j|\boldsymbol{X_t} \sim \text{Gamma}\left(\frac{S_j(t)}{\phi_j}, \frac{\sum_{i=0}^{N-j+1+t} E_i}{\phi_j}\right)$$

- Rather than generating new simulations, we can just reweight the original simulations of β_j by the likelihood of the new observations

$$w_j^{(1)}(t, S_j(t)) \propto \frac{f(\beta_j|\boldsymbol{X_t})}{f(\beta_j|\boldsymbol{X_0})} \propto \beta_j^{(S_j(t)-S_j(0))/\phi_j} e^{-\beta_j \sum_{i=N-j+2}^{N-j+1+t} E_i/\phi_j}$$

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Conditional Re-Weighting - Second Step

- In Mack's additive model, every quantity is a function of the column sums $S_j(t) = \sum_{i=0}^{N-j+1+t} X_{ij}$ (they are a *sufficient statistic*)
- The conditional probability of observing a given column sum at time $t+1$, given the value at t

$$f\left(S_j(t+1) \middle| S_j(t), \beta_j\right) = \frac{\left(\frac{E_i\beta_j}{\phi_j}\right)^{\Delta S_j(t+1)} e^{-\frac{E_i\beta_j}{\phi_j}}}{\Delta S_j(t+1)!}$$

- The unconditional probability is

$$f\left(S_j(t+1) \middle| \beta_j\right) = \frac{\left(\frac{\sum E_i\beta_j}{\phi_j}\right)^{S_j(t+1)} e^{-\frac{\sum E_i\beta_j}{\phi_j}}}{S_j(t+1)!}$$

- The second set of weights are then $w_j^{(2)}\left(t, S_j(t), S_j(t+1)\right) \propto \frac{f\left(S_j(t+1) \middle| S_j(t), \beta_j\right)}{f\left(S_j(t+1) \middle| \beta_j\right)}$

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