Institute
and Facult
of Actuaries

IFoA Life Conference

Muhammad Amjad

Quantum Computing for Actuaries

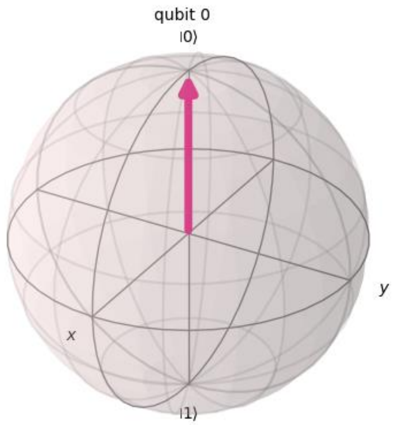
Quantum Computing for Actuaries

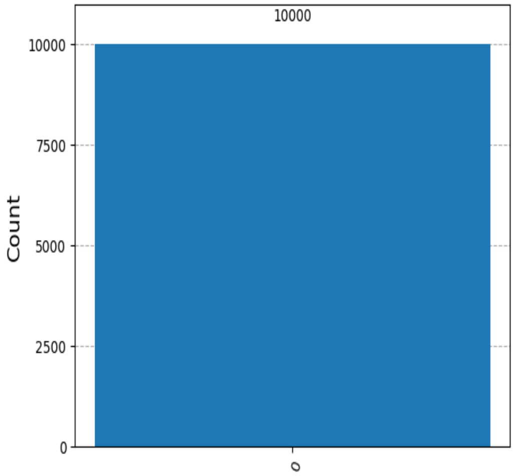
A practical primer

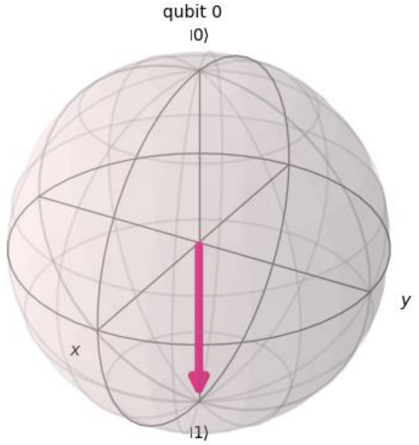
What we will cover

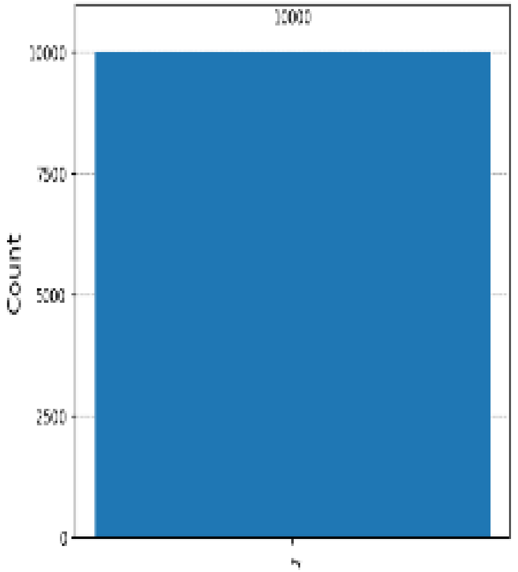
Agenda
Qubits vs Bits – Superposition
Entanglement
Measurement
Encoding
Dense Encoding
Annuity Valuation
Option Valuation
Quadratic Speedup using QAE
Running on real quantum hardware
Wrap up
Q&A

Bits



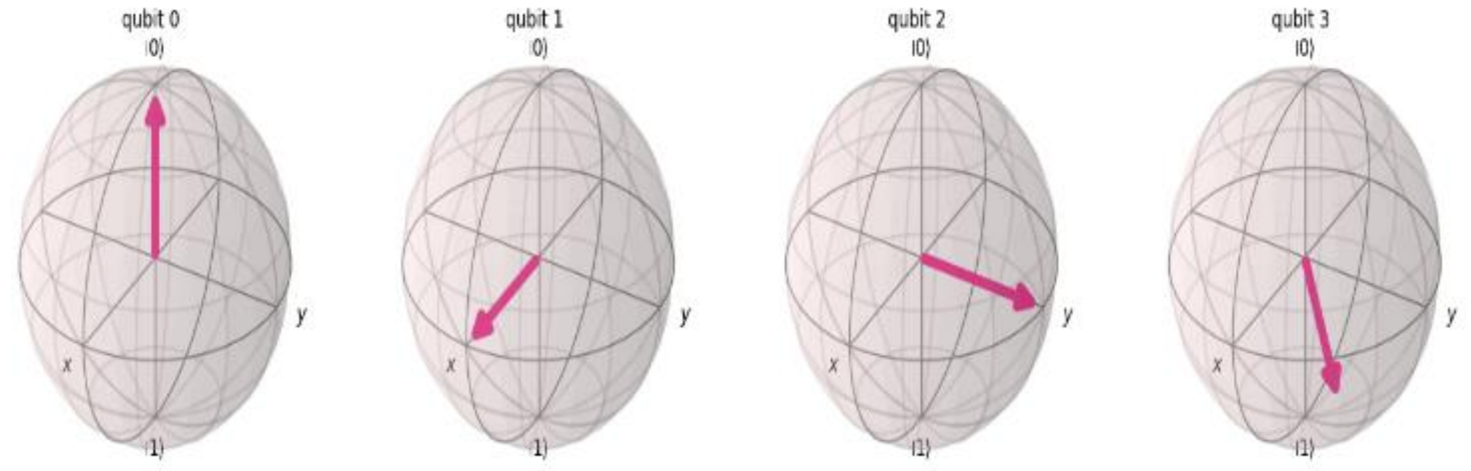


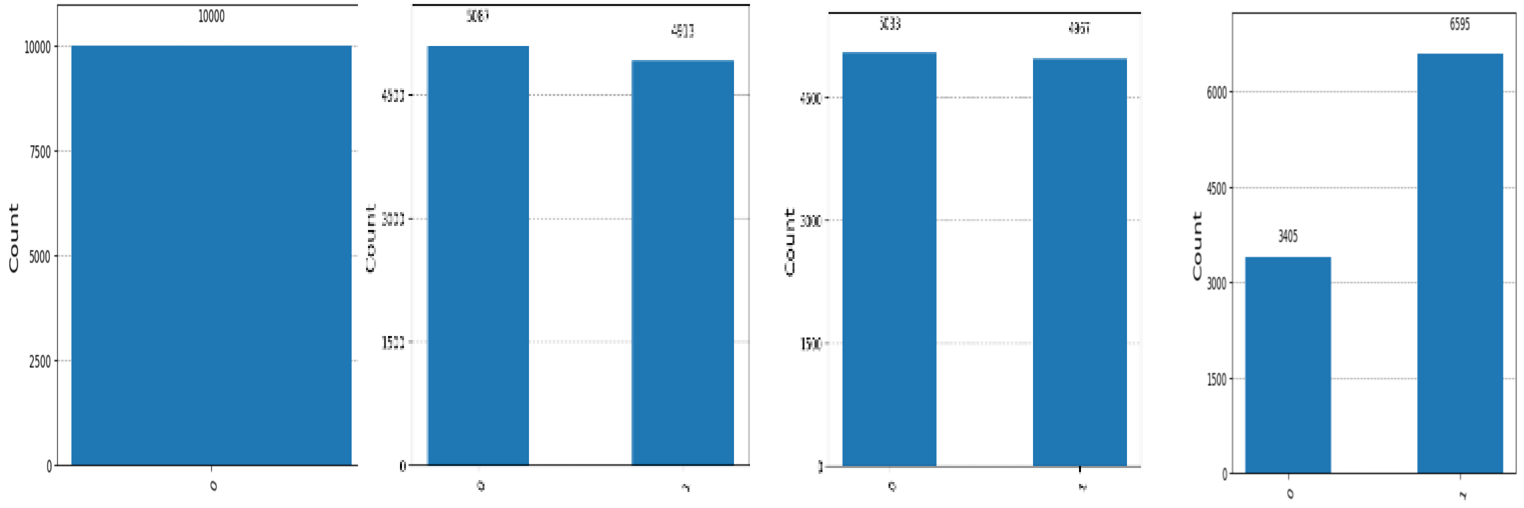




- **All or Nothing – Binary Outcome**
- **Repeated Measurement does not change the outcome**
- **Boolean logic used to manipulate the states to perform computation**
- **N bits can encode 2^N possible bit strings, but only one at a time**

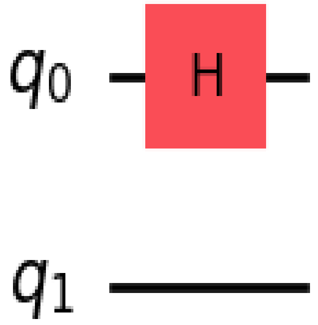
Qubits - Superposition

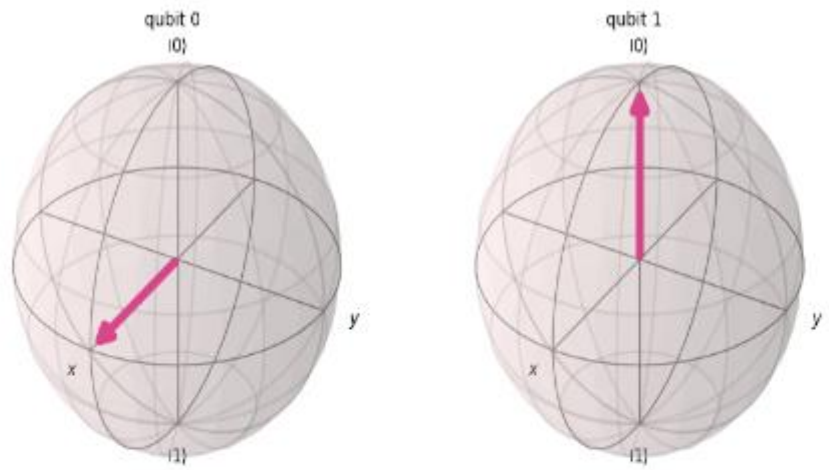


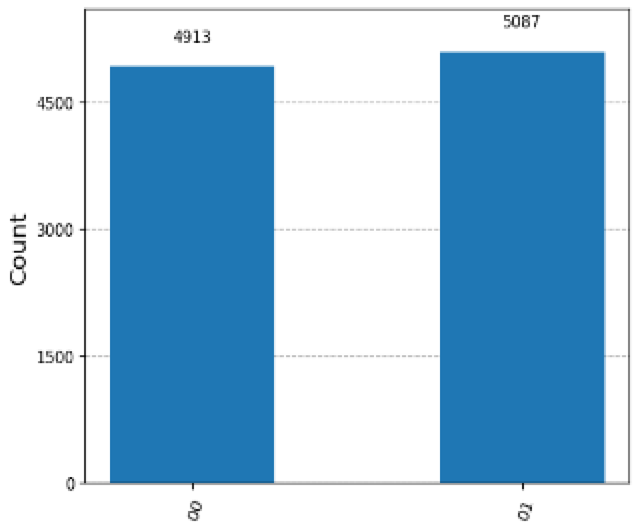


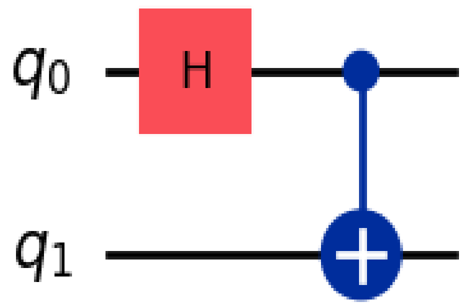
- Can exist anywhere on the surface
- Spherical coordinates (θ, φ) on the Bloch Sphere, directly analogous to latitude and longitude
- Repeated Measurement does not yield the same outcome
- Quantum logic used to manipulate the states to perform computation
- N qubits can encode 2^N possible bit strings, and can be encoded simultaneously with respective probability amplitudes

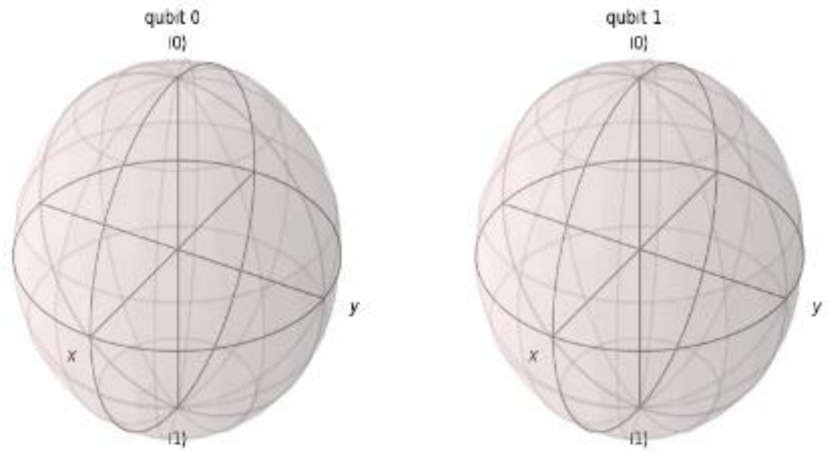
Entanglement

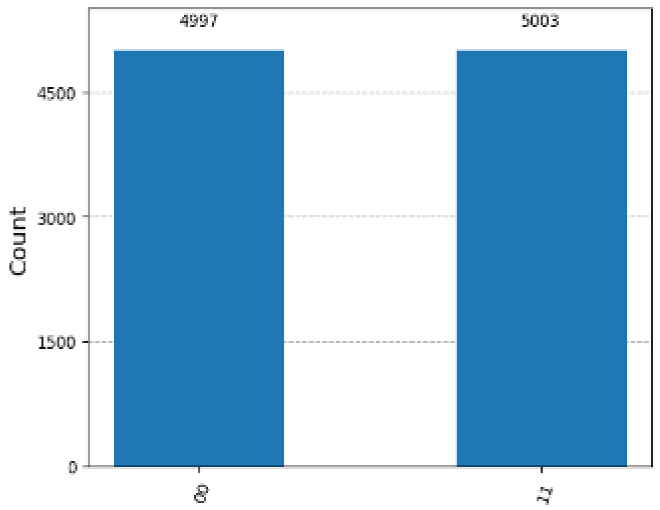












Encoding

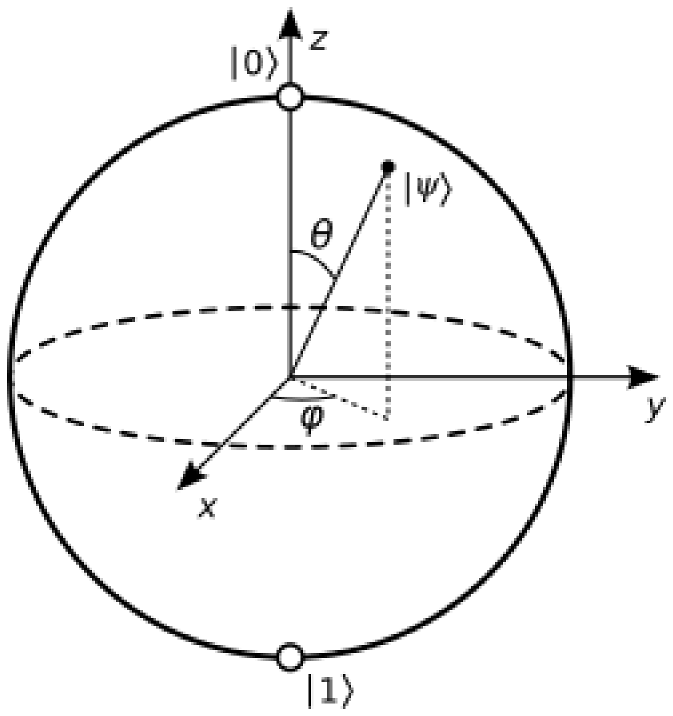
A single qubit state is a superposition of basis states $|0\rangle$ and $|1\rangle$:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

where $\alpha, \beta \in \mathbb{C}$ (complex numbers) and must satisfy the **normalisation condition**:

$$|\alpha|^2 + |\beta|^2 = 1$$

This ensures the total probability of measuring $|0\rangle$ or $|1\rangle$ is 1.



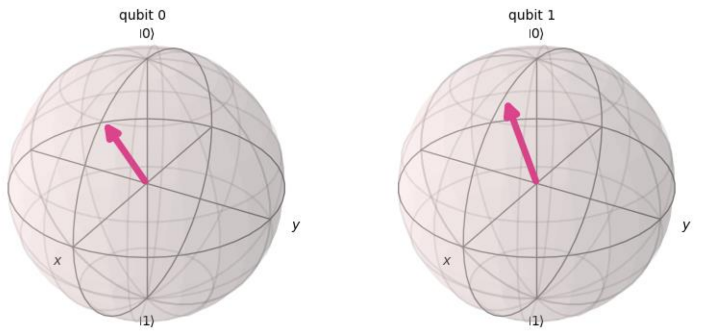
$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

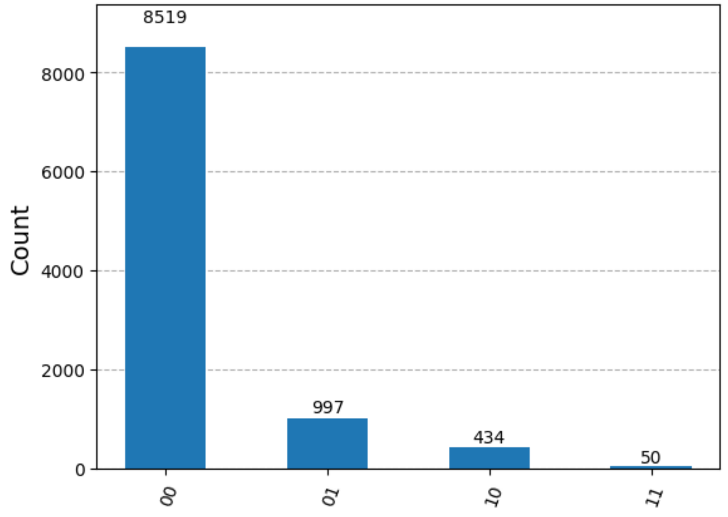
$$\alpha = \cos\left(\frac{\theta}{2}\right)$$

$$\theta = 2 * \cos^{-1}(\alpha) \quad Or\ \theta = 2 * \sin^{-1}(\beta)$$

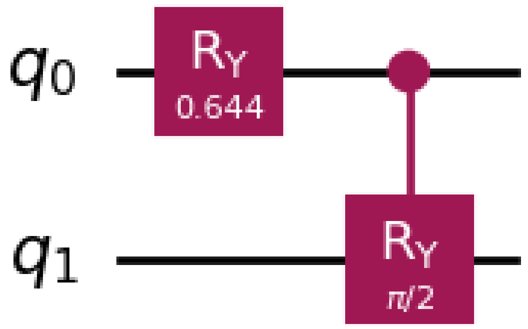
$$\theta = 2 * \sin^{-1}(\sqrt{p})$$

Alice and Bob

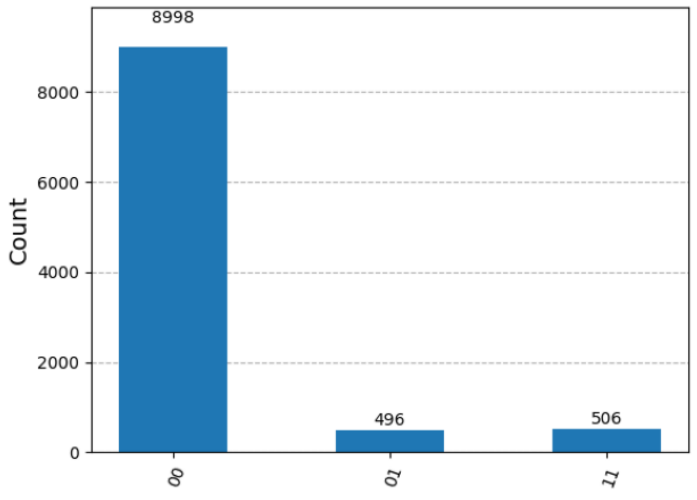




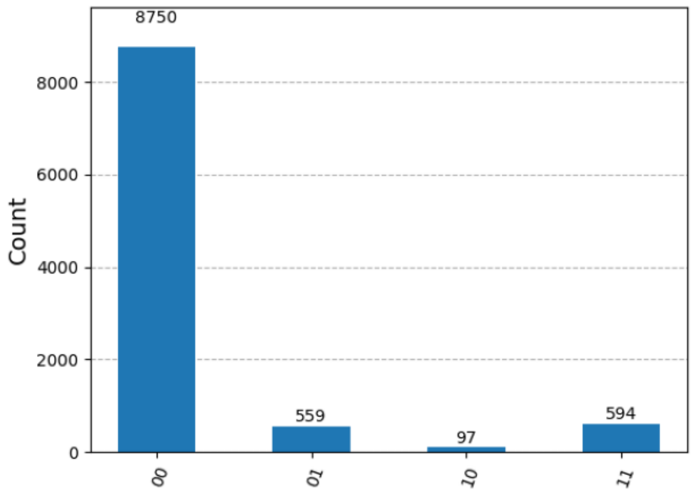
- Independent lives
- Joint probabilities obey product law
- Individual qubit states distinct and well define



Simulated Result



Quantum Hardware Result



- Entangled Lives
- Joint probabilities obey Bayes Rule
- Two-way causality

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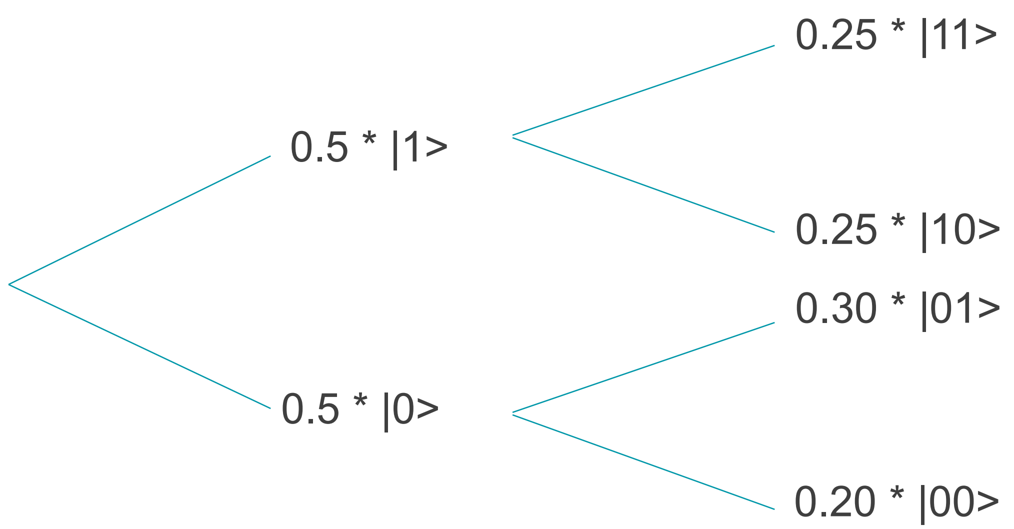
Dense Encoding

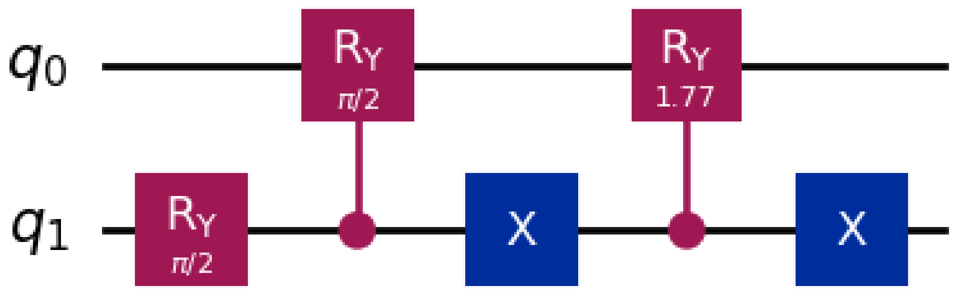
Loading any distribution onto a set of qubits

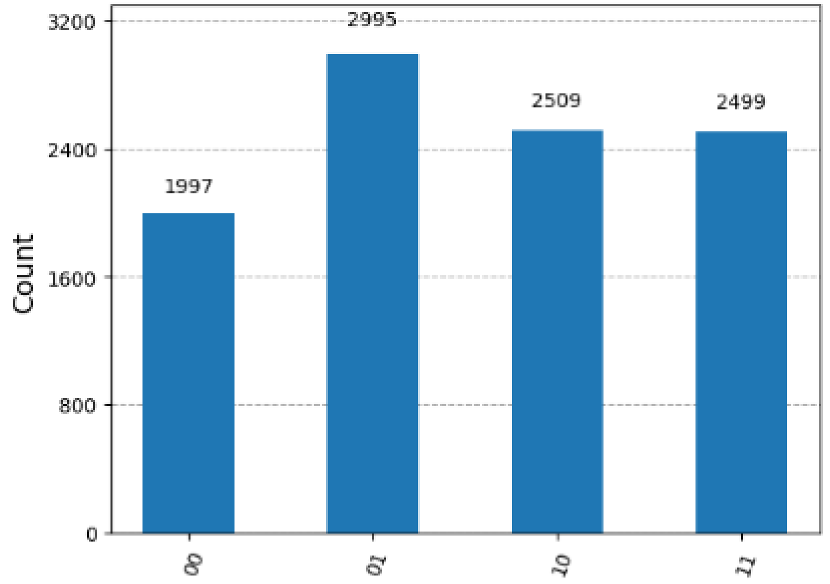
|Ψ>= √0.2|00> + √0.3|01> + √0.25|10> + √0.25|11>



$$p_1 = p_{11} + \rho_{10}$$
$$\theta_1 = 2\sin^{-1}(\sqrt{p_1})$$
$$p_2 = p_{11}/\rho_1$$
$$\theta_2 = 2\sin^{-1}(\sqrt{p_2})$$
$$p_3 = p_{01}/\rho_0$$
$$\theta_3 = 2\sin^{-1}(\sqrt{p_3})$$

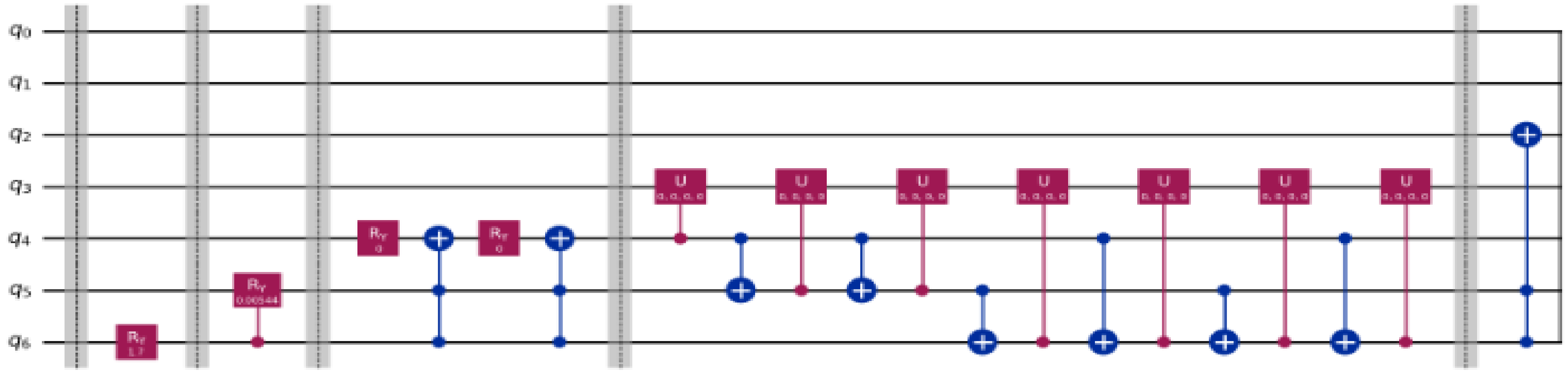
$$p_1 = 0.25 + 0.25 = 0.5$$
$$\theta_1 = 2\sin^{-1}\left(\sqrt{0.5}\right) = \pi/2$$
$$p_2 = 0.25/0.5 = 0.5$$
$$\theta_2 = 2\sin^{-1}\left(\sqrt{0.5}\right) = \pi/2$$
$$p_3 = 0.3/0.5 = 0.6$$
$$\theta_2 = 2\sin^{-1}\left(\sqrt{0.6}\right) = 1.772154$$

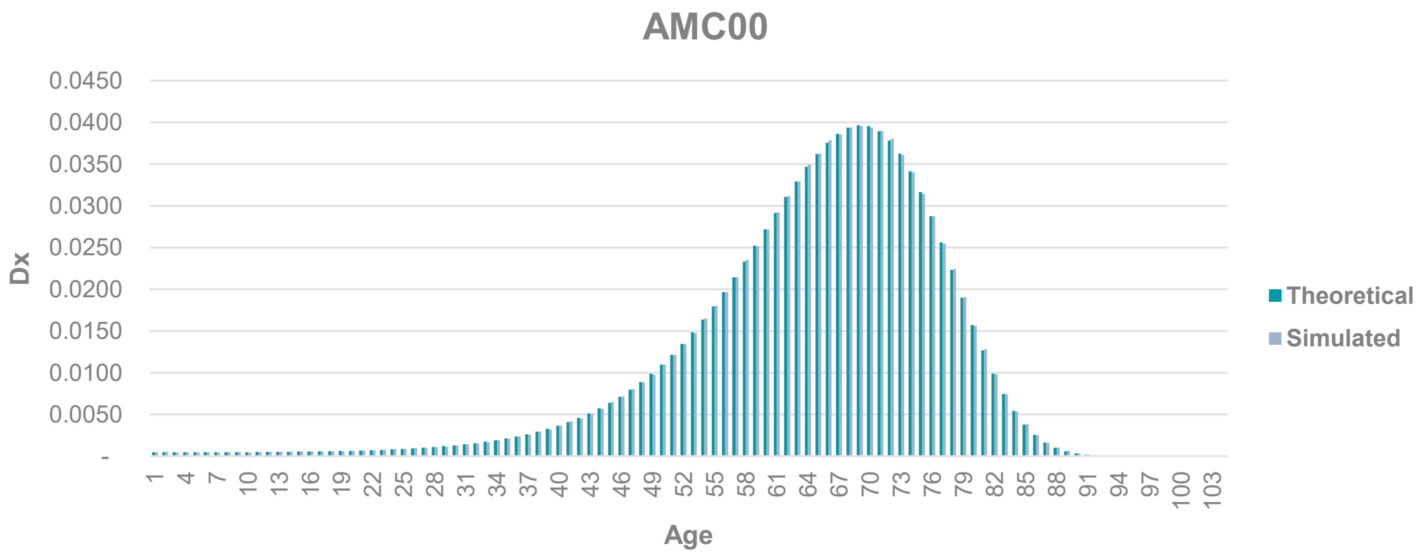




Dense Encoding

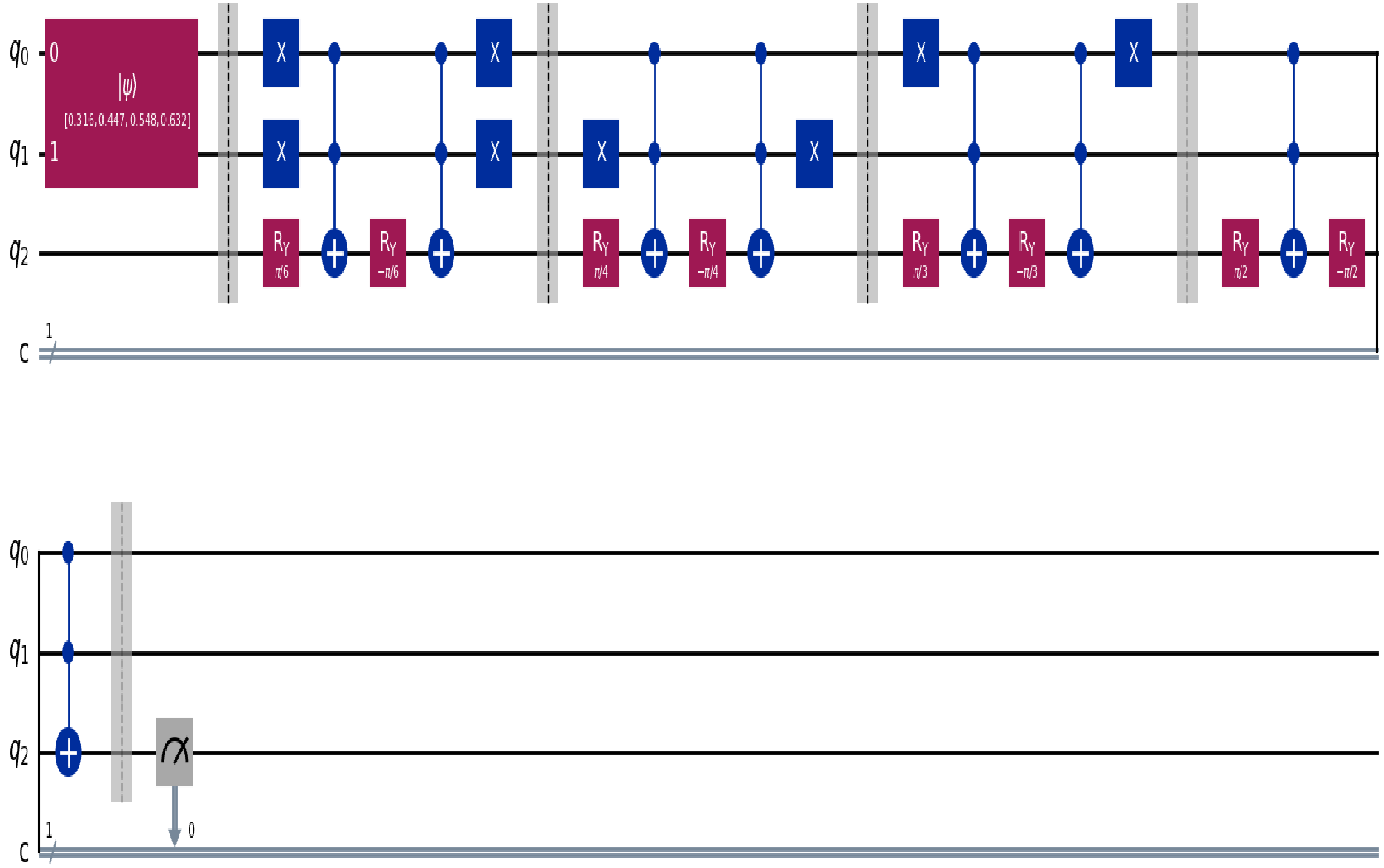
$$|\Psi> \ = a_{0000000}|0000000> \ + a_{0000001}|0000000> \ + a_{0000010}|0000010> \ + \ \ldots\ldots\ldots\ldots\ldots$$
$$+ a_{1111110}|1111110> \ + a_{1111111}|1111111>$$

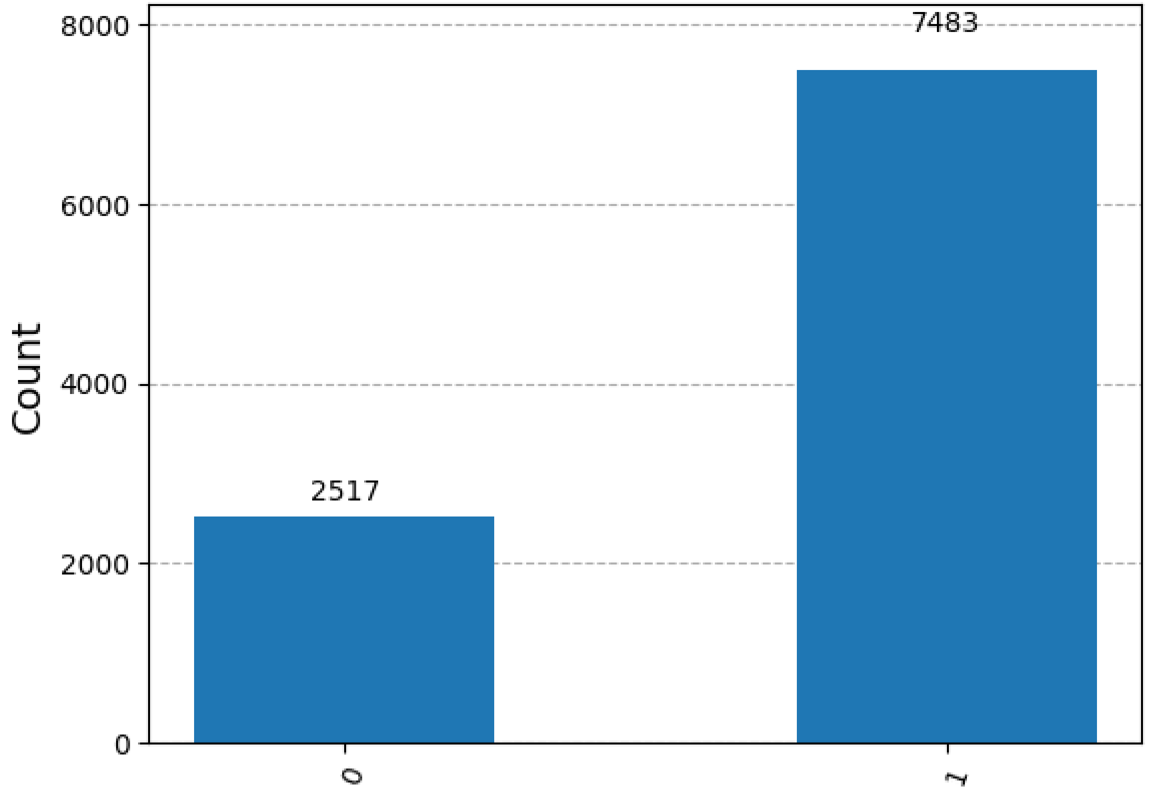




Annuity Valuation Using Quantum

|Ψ>= √0.1|00> + √0.2|01> + √0.3|10> + √0.4|11>

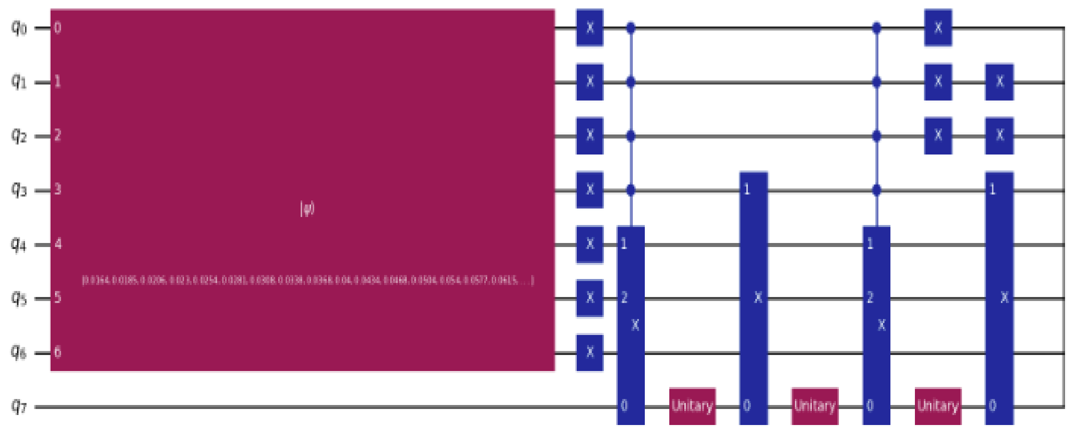


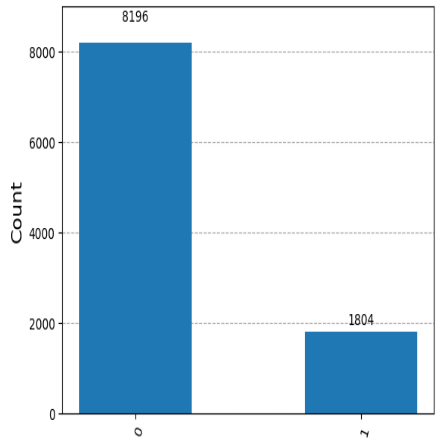


Option Valuation

- Partition Price Probability space
- Encode distribution
- Use controlled rotations
- Measure target qubit
- Scale

	CDF	Price	PDF
0	0.001000	64.096217	0.000270
1	0.008858	64.866932	0.000341
2	0.016717	65.637648	0.000425
3	0.024575	66.408364	0.000527
4	0.032433	67.179079	0.000647
...	...	...	...
123	0.967567	158.894248	0.000161
124	0.975425	159.664963	0.000145
125	0.983283	160.435679	0.000131
126	0.991142	161.206395	0.000119
127	0.999000	161.977110	0.000107





European Put Option

Current Price = 100

Strike = 105

Risk free rate = 3%

Volatility= 15%

Term = 1 Year

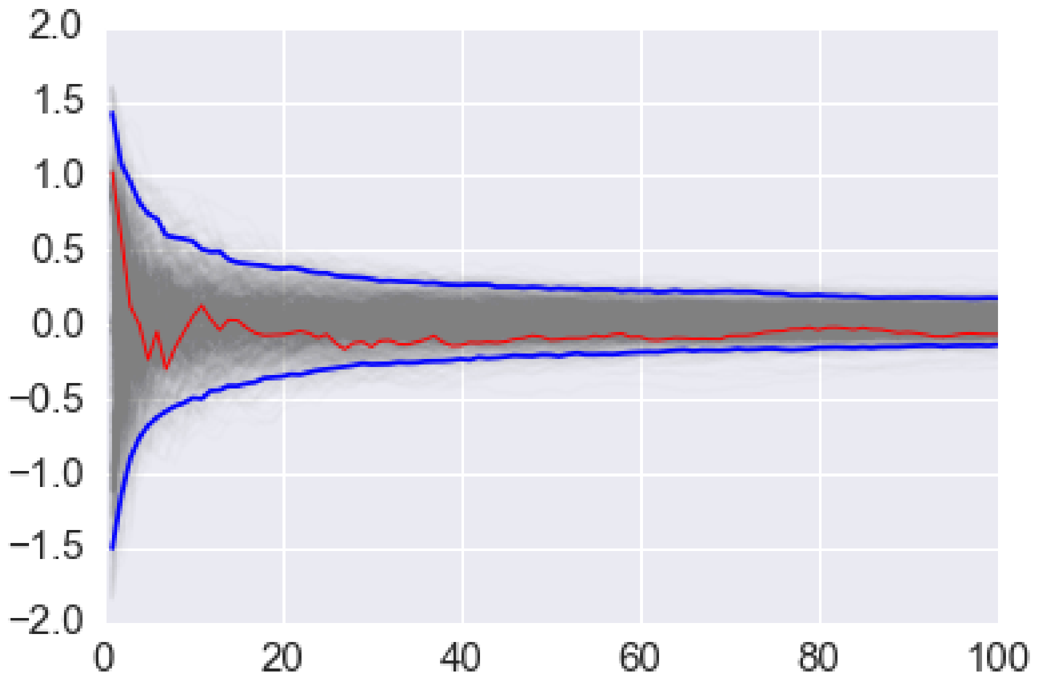
Put Price = 0.1804* max_payoff * DiscountFactor

Max_payoff = 105 – 64.1 = 40.9
Discount Factor = Exp (-0.03) = 0.97

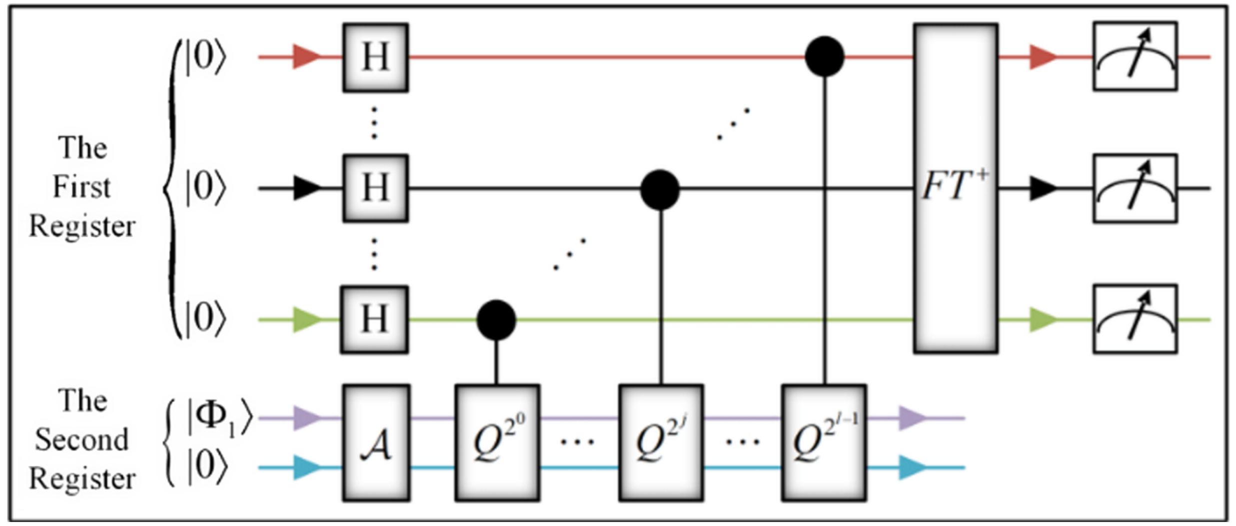
Quantum - Put = 7.16

Black Scholes- Put = 7.03

Quadratic Speedup via Quantum Amplitude Estimation

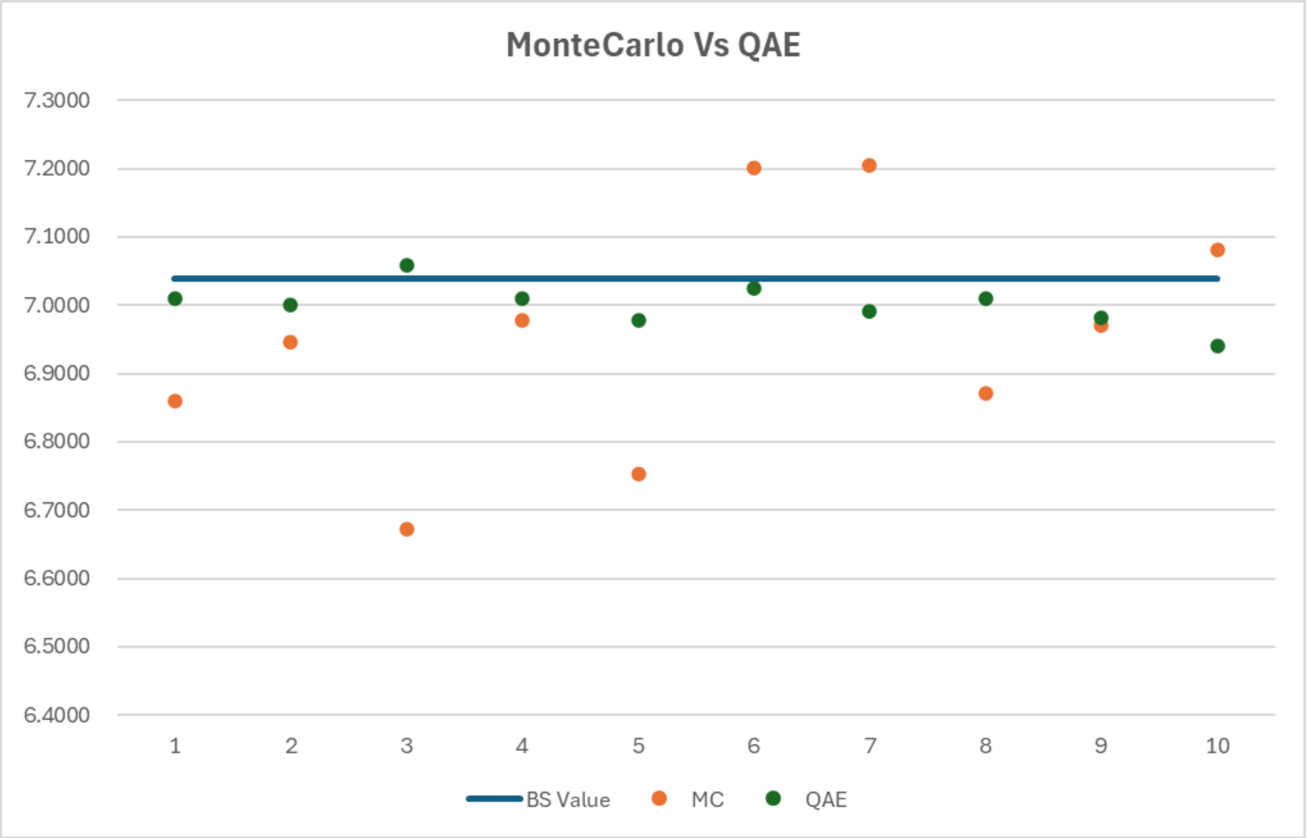


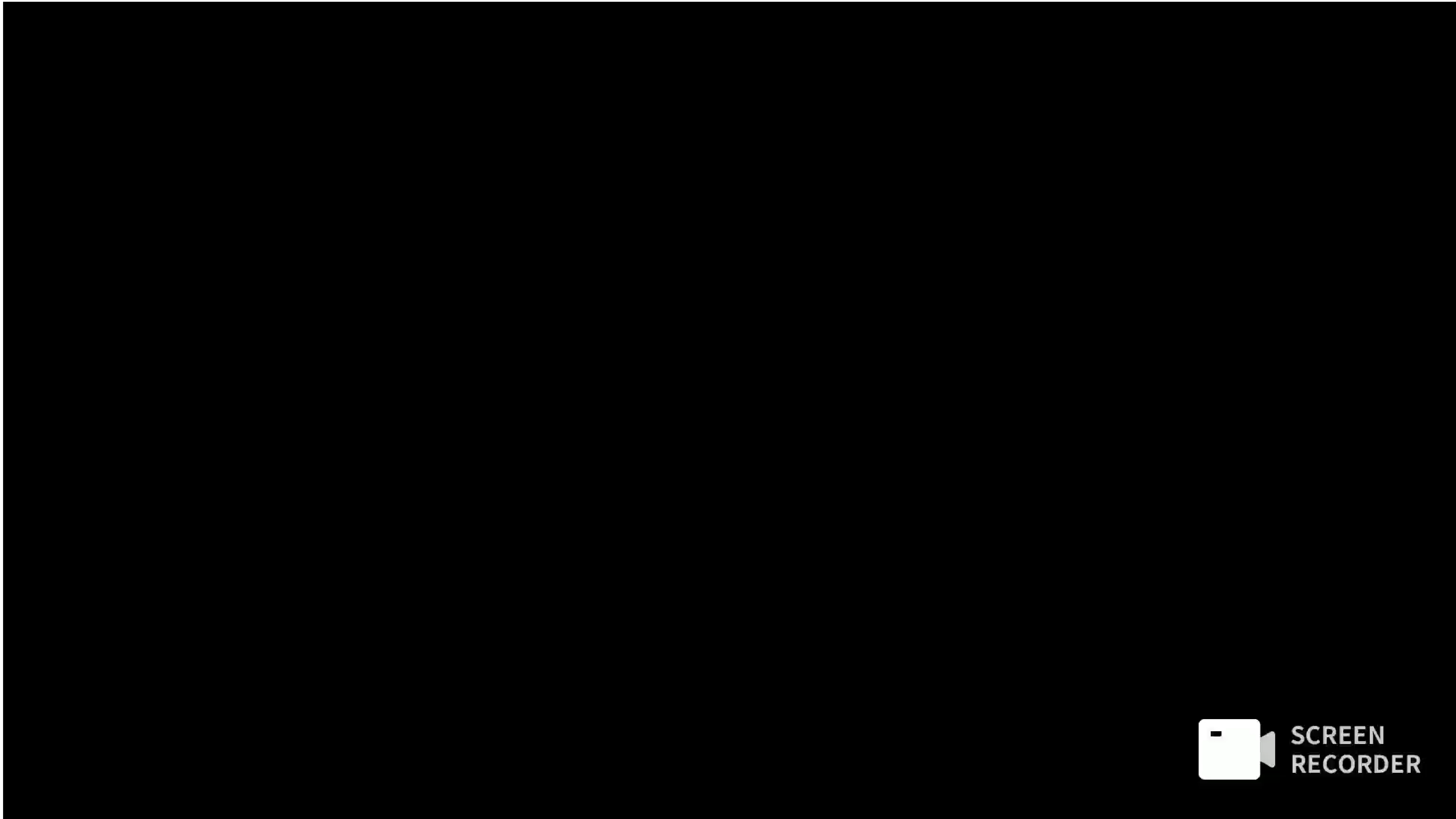
$$A|0\rangle_{n+1} = \sqrt{1-a}|\Psi_0\rangle_n|0\rangle + \sqrt{a}|\Psi_1\rangle_n|1\rangle$$



- **QAE Setup**
- 128 shots
- Iterations 4
- QAE samples = 512
- **MonteCarlo samples = 10,000**

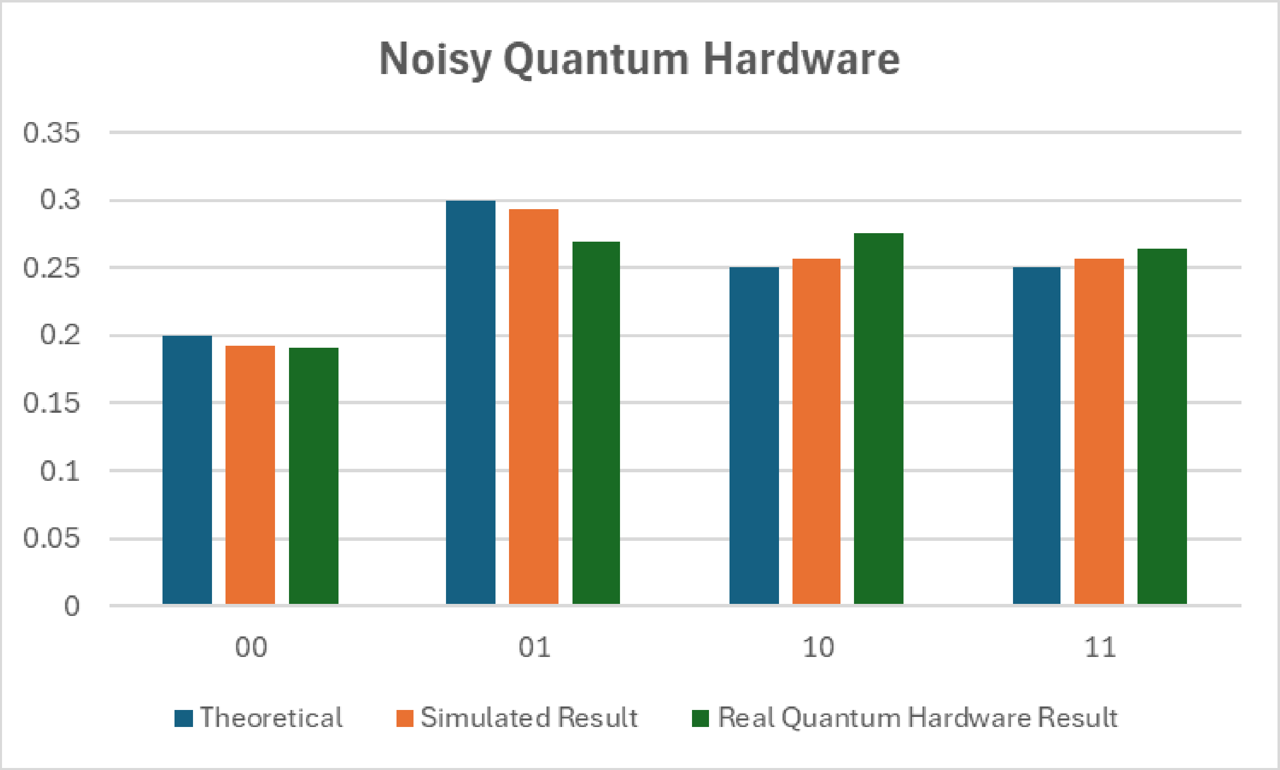
- **MonteCarlo Standard Error = 0.17**
- **QAE Standard Error = 0.03**

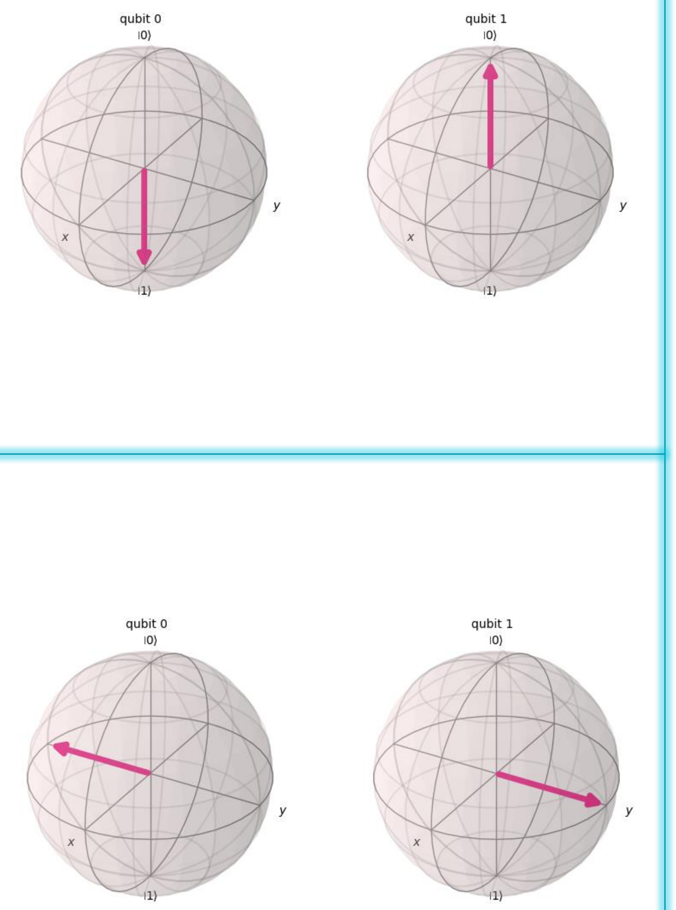



SCREEN
RECORDER

Hardware Challenges –
Noisy Intermediate State Quantum (NISQ)

$|\Psi\rangle = \sqrt{0.2}|00\rangle + \sqrt{0.3}|01\rangle + \sqrt{0.25}|10\rangle + \sqrt{0.25}|11\rangle$





- Amplitude dampening
- Dephasing

- Gate precision / Pulse imperfections
- Single and two-qubit gate errors

- Crosstalk and Leakage

- Measurement & readout errors

Wrap Up!

What we have covered

- **Bits vs Qubits**
- **Superposition** – multiple states at once
- **Entanglement** – like dependencies between risks
- **Encoding** – how to represent information on a quantum computer
- **Dense Encoding** – how a whole Life table can be encoded in just 7 qubits
- **Measurement** – turns quantum to classical
- **Examples** – Annuity Valuation & Option Pricing
- **IBM hardware** – Running on a real quantum computer

Your quantum journey

- Simply, **JUST DO IT!**
- Grasp the **fundamentals**
- Experiment with **toy problems**
- **Follow** industry developments
- **Collaborate**
- Stay **curious** and have **fun**

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