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How to optimise your MA portfolio

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Agenda

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- Regulatory Constraints
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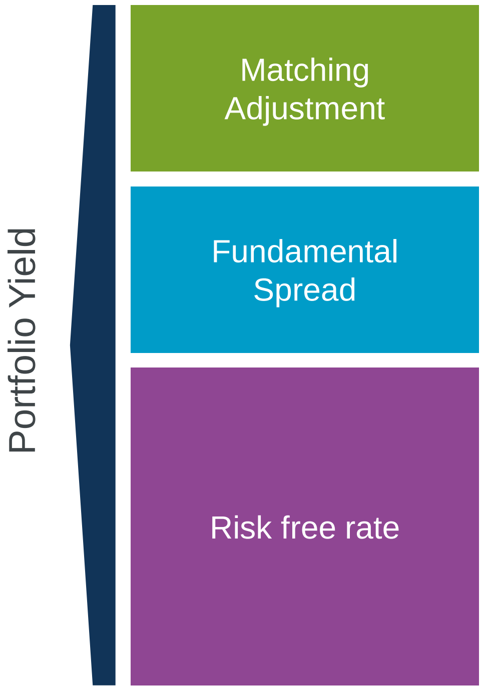
Introduction

- Matching Adjustment (MA) is a spread added to the risk-free rate used for valuation of certain long-term liabilities, mainly annuities. The basic idea behind the MA is that assets often trade at a spread over a reference risk-free rate after allowance for credit risk (fundamental spread). As insurers hold on to these assets until maturity or use a buy and maintain strategy, they expect to earn this excess spread arising from illiquidity.
- MA optimisation in the context of investing in private credit and other illiquid assets, and restructuring assets that are not outright eligible has been discussed a lot.
- We provide an insight into use of mathematical programming for optimising matching adjustment portfolios in terms of:
  - ✓ Asset allocation
  - ✓ Decision making and,
  - ✓ Operational aspects,

  while allowing for specific constraints.



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Optimisation Approaches

We evaluate the approaches that are typically used to optimise the MA portfolio and/or comply with the regulatory requirements.

	DESCRIPTION	CONCLUSION
Brute Force trail and error	A guided brute force may involve scoring each asset, and then performing a trial and error approach, taking out the worst scoring assets one at a time until pre-defined constraints are met.	• Intuitive • Inefficient • Given typical MA portfolios consisting of thousands of lines of assets, it is not possible to enumerate all possible combinations, resulting in non-optimal allocation
Metaheuristics / Machine Learning	Metaheuristics may include evolutionary algorithms in guiding the search towards a more optimal MA. This approach uses a form of adaptive memory to remember which asset allocation worked well before and recombines them into new, better allocation.	• Can be complex to develop • Inefficient • No guarantee as to finding an optimal or even feasible solution • Results are generally not reproducible
Mathematical programming / Operations Research methods	These are 'exact' methods that provide provably optimal solutions or approximate with some guarantee. The approaches include: • Reformulating the Matching Adjustment as a convex optimisation problem that can be solved efficiently using linear or mixed integer programming • Non-linear programming methods specialised for specific use cases may need to be developed	• Efficient • "Globally" optimal • Can scale up to large portfolios of 100k+ assets • Results are reproducible • Most real world constraints essential in practical portfolio construction can be incorporated • Runtime ranges from few seconds to few minutes

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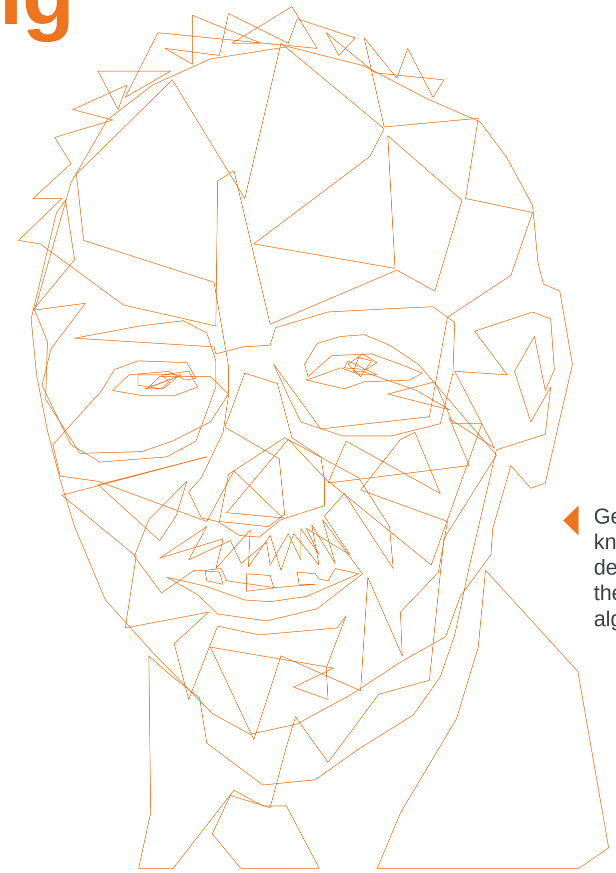
The Case for Mathematical Programming

We provide some background to how operations research methods are useful.

- The modern day linear programming started with the seminal work of George Dantzig in 1947 when he proposed the Simplex method, partly in connection with the logistic problems that arose during the World War II. Since then linear programming and its extensions such as integer programming revolutionised the way planning, scheduling and other decisions making problems are solved in various industries. Now days, these methods are used as a matter of routine for portfolio optimisation.
- The MA optimisation problem can be defined as a constrained optimisation problem. The constraints include regulatory requirements, such as cashflow matching tests and other related to risk management and portfolio management.
- We provide a mathematical formulation of the optimisation problem and several examples of constraints that may be relevant.
- The benefits of mathematical programming are not limited to optimisation of the balance sheet, but extends to operational aspects. The need for manual intervention is greatly reduced, and a fast runtime enables running many scenarios.

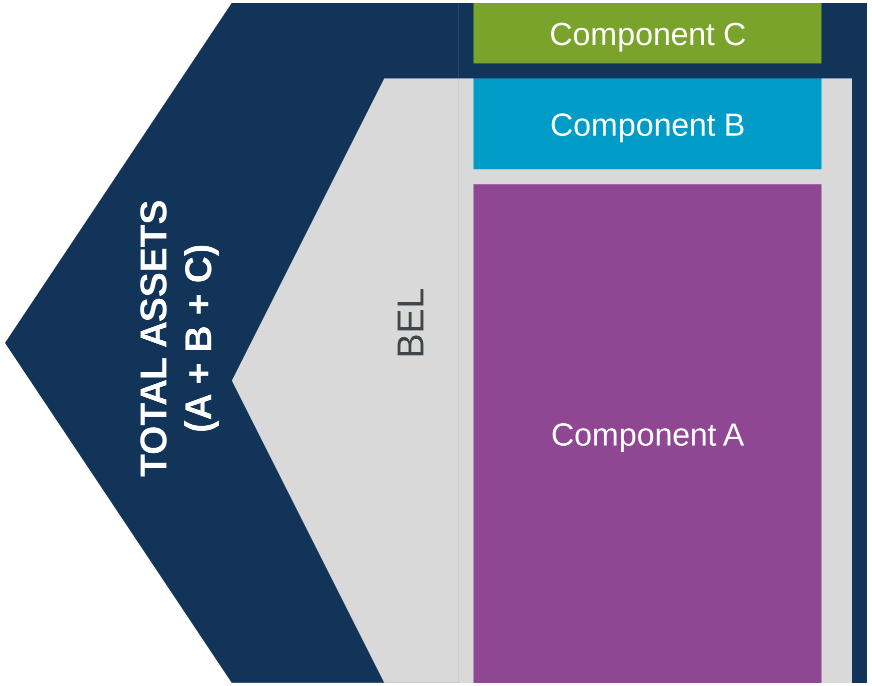


George Dantzig, known for the development of the simplex algorithm

Assets Categorisation

The MA portfolio assets are split into three components, namely A, B and C. This split impacts allocation decisions for reasons that will become apparent later.

- Component A assets are used to calculate the MA, subject to meeting the PRA tests 1 and 3 (see next slide). The cashflows after adjustment for expected default from these assets are used to match liability cashflows.
- Component B assets are additional assets required so that market value of assets is equal to the BEL.
- Component C assets are any surplus assets.



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Regulatory Constraints

The PRA has set out qualitative requirements and three quantitative tests that must be met to demonstrate compliance with the requirements. We describe the quantitative tests on this slide.

- Test 1: the maximum (discounted accumulated) cashflow shortfall must be less than or equal to 3% of the BEL (calculated at the basic risk free rate)

$$max\left\{(1+r_s)^{s-0.5}\sum_{j=1}^{s}\left(CF_j^{Liab}-\sum_i x_i CF_{ij}^{PD}\right)(1+r_j)^{-j+0.5}, \forall s \in S\right\} \leq 3\% \sum_j CF_j^{Liab}(1+r_j)^{-j}$$

where x_i is allocation of asset i to component A

CF_{ij}^{PD} is asset i cashflow adjusted for expected loss at time j

S is the set of annual time intervals over the projection period

- Test 2: the 99.5% value at risk for interest rate, inflation and currency risk must not exceed 1% of the BEL. This calculation is based on considering component A and B assets.

- Test 3: notional swap test requires that the ratio of the present value of liabilities to the presented value of asset cashflows (adjusted for expected loss) discounted at the basic risk free rate should lie in the range 99-100%

$$0.99 \leq \sum_j CF_j^{Liab}(1+r_j)^{-j} / \sum_j \sum_i x_i CF_{ij}^{PD}(1+r_j)^{-j} \leq 1$$

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MA Optimisation Problem Definition

We define the MA optimisation problem, subject to tests 1 and 3 set as constraints. The regulation does not specify a particular method for calculation of the (residual) fundamental spread (FS) and we set out a possible approach that can be used.

A flat rate that when used to discount liability cashflows results in a value equal to market value of component A assets defined by (2)

A flat rate that when used to discount liability cashflows results in a value equal to the BEL

Discounted accumulated cashflow shortfall

$$Maximise\ MA$$

$$s.t.$$

$$MA = IRR^{AL} - IRR^{BEL} - FS \quad (1)$$

$$\sum_i x_i P_i = \sum_j CF_j^{Liab} (1 + IRR^{AL})^{-j} \quad (2)$$

$$max\left\{(1+r_s)^{s-0.5} \sum_{j=1}^{s} \left(CF_j^{Liab} - \sum_i x_i CF_{ij}^{PD}\right)(1+r_j)^{-j+0.5}, \forall s \in S\right\} \leq 3\% \sum_j CF_j^{Liab} (1+r_j)^{-j} \quad (3)$$

$$0.99 \leq \sum_j CF_j^{Liab} (1+r_j)^{-j} \Big/ \sum_j \sum_i x_i CF_{ij}^{PD} (1+r_j)^{-j} \leq 1 \quad (4)$$

$$\sum_i x_i P_i = \sum_j \left(CF_j^{Liab} - \sum_i x_i \left(CF_{ij}^{PD} - CF_{ij}^{FS}\right)\right)(1 + r_{FS})^{-j} \quad (5)$$

$$0 \leq x_i \leq 1, \qquad \forall i \in \{1,2, \dots n\}$$

The fundamental spread is defined as $FS = IRR^{AL} - r_{FS}$ where r_{FS} is a variable (see constraint (5)).

MA Optimisation Problem Definition

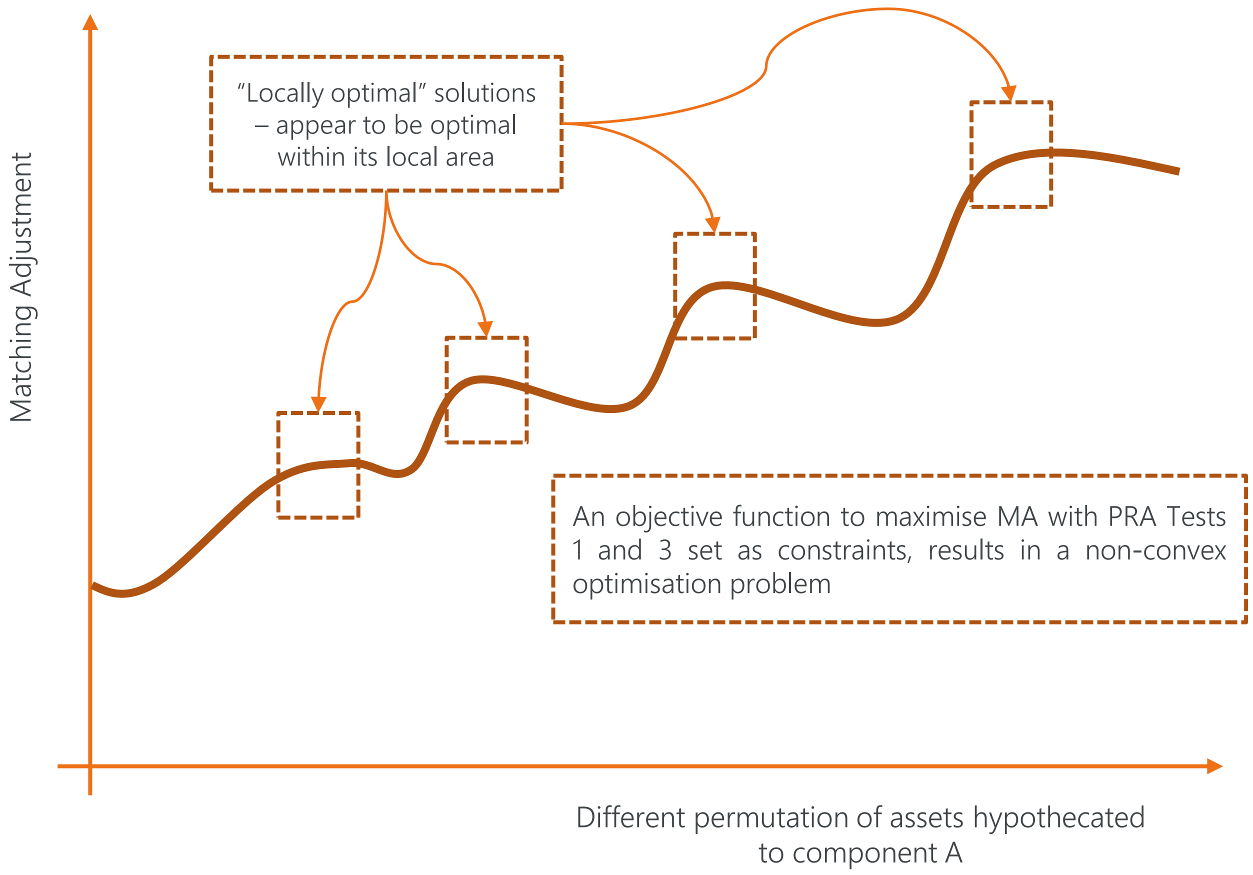
- The constraints (3) and (4) setting out PRA tests 1 and 3 can be convexified with little algebra. For example, constraint (3) can be reformulated as piecewise linear constraint.

- The constraint (5) cannot be convexified as it is resulting in a product of variables x_i and terms $(1 + r_{FS})^{-j}$

- This results in a non convex problem as illustrated on the next slide.

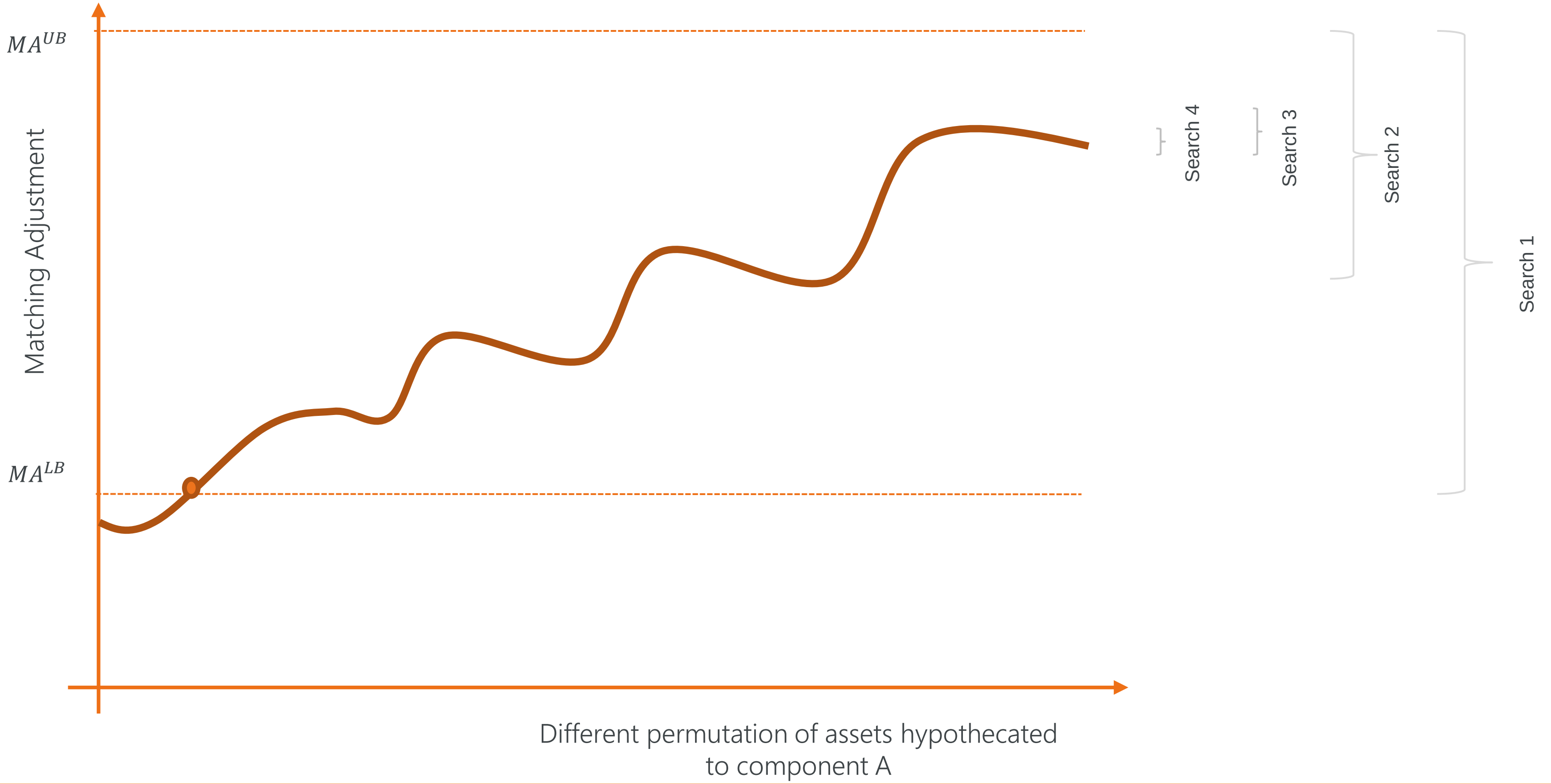
- A global optimisation approach is illustrated on slide 12. This uses a linear programming approach coupled with a line search strategy for global convergence.

Global Optimisation of the MA Portfolio



Global Optimisation of the MA Portfolio

We set a lower bound of MA^{LB} that is feasible and an arbitrary upper bound MA^{UB}. We then search downwards from the upper bound to the lower bound until we find an optimal solution. At each iteration, a linear program with constraints (3) and (4) is solved, and a cut is added using constraint (5) by fixing the variable r_{FS} to the trial value. The search interval is successively reduced until difference between the upper bound and lower bound hits a desired tolerance.



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Risk Management Constraints

In practice several constraints in addition to regulatory requirements are important for risk management and efficient portfolio management reasons. We provide examples of several of these constraints that are important to consider in asset allocation.

Assets that are paired together for the purpose of eligibility, e.g., a USD bond with a cross currency swap, must be allocated by the same percentage
Allocation is binary for certain assets, i.e., these are allocated either 0% or 100%
Hedging, e.g., matching PV01 and IE01 measures or key rate durations of assets with liabilities.
Constraints related to ensuring that inflation linked liabilities are backed by inflation linked assets
Minimum and maximum allocation by asset type, and diversification by sector or geographical location
Liquidity constraints, ensuring sufficient liquid assets are allocated
Minimum investable amount, maximum investable amount and lot size limits when trading-in or out other assets
Transaction costs, e.g., minimum and maximum number of trades when rebalancing the portfolio or acquiring new assets
Credit rating constraints to ensure that the portfolio is of an appropriate credit quality

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Management Actions

Future management actions can be assumed under stress or when modelling capital as in certain scenarios the assets could be insufficient to meet the liabilities or compliance with the PRA tests is breached. Without delving into what management actions could be acceptable for regulatory purposes, we provide examples of management actions that can be automated or considered explicitly in the optimisation process.

The optimisation can determine if a problem is infeasible, i.e., no combination of available assets can satisfy the PRA tests and/or other constraints.

A management action could be to inject cash from elsewhere in the business to restore compliance with the requirements. This may not be always an option and some more realistic examples are:

- Trade out some assets allowing for transaction costs and haircuts and trade in 'safer' assets (government bonds). A haircut may need to be applied due to uncertainty with modelling the selling price of assets under stressed market conditions. The market value of assets assumed to be traded-in must be explicitly constrained to be less than or equal to assumed proceeds from asset traded out. A combination of cash and rebalancing of the portfolio could be assumed for this purpose until compliance with the MA requirements is restored.
- The threshold conditions at which trading can begin, such as minimum investable amount and any maximum limits by asset type or issuer must be set as constraints to ensure that the assumed management action is realistic.

Questions

Comments

The views expressed in this presentation are those of the presenters.

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