

# Assessing the Economic Impact of Longevity Hedges

Andrew J.G. Cairns

Heriot-Watt University, Edinburgh

and

Director, Actuarial Research Centre,

Institute and Faculty of Actuaries

International Congress of Actuaries, Berlin, June 2018



Actuarial  
Research Centre  
Institute and Faculty  
of Actuaries

- Introduction and motivation
- Hedging longevity risk with an index-based call-spread option contract
- Anatomy of a hedging calculation
- Numerical example
- Discussion

# Motivation

- Longevity risk
- Measurement
  - e.g. Capital Requirement
  - Best estimate + extra for risk
- Longevity risk management
  - customised hedges
  - index-based hedges

- Why use **General Population Longevity Index** based risk transfer instruments?
  - **Capacity and Price**
- Pros/cons
  - Transferred risk is efficiently priced
  - But hedger left with **basis risk**
- Thus we need
  - a clear and rigorous approach to quantify basis risk
  - hedger and regulator agreement on approach
  - to quantify properly the **Capital Relief**

- Life insurer
- Aim 1: measure mortality/longevity risk
- Aim 2: manage mortality/longevity risk
  - e.g. to *reduce* regulatory capital
  - e.g. to *reduce* economic capital
  - e.g. to *increase* economic value

Solvency II options:

- Solvency Capital Requirement,  $SCR =$  difference between Best estimate of annuity liabilities (BE) and Annuity liabilities following an immediate 20% reduction in mortality
- or  $SCR =$  extra capital required at time 0 to ensure solvency at time 1 with 99.5% probability
- or  $SCR =$  extra capital at time 0 to ensure solvency at time  $T$  with  $x\%$  probability

## Liability to be Hedged

- $L$  = random PV at time 0 of liabilities
- $L(0)$  = point estimate of  $L$  based on time 0 info
- $L(T)$  = point estimate of  $L$  based on info at  $T$   
= PV of actual cashflows up to  $T$   
+ PV of estimated cashflows after  $T$
- Risk  $\Rightarrow$  capital requirements

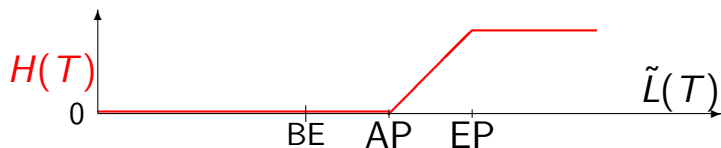
What type of hedge to modify capital requirements and manage risk?

# Hedging Options

Index-based hedge (derivative)

- Synthetic  $\tilde{L}(T) \approx$  true  $L(T)$
- Call spread derived from underlying  $\tilde{L}(T)$

Payoff at  $T$ , per unit



$$H(T) = \begin{cases} 0 & \text{if } \tilde{L}(T) < AP & \text{(Attachment Point)} \\ \tilde{L}(T) - AP & \text{if } AP \leq \tilde{L}(T) < EP & \text{(Exhaustion Point)} \\ EP - AP & \text{if } EP \leq \tilde{L}(T) \end{cases}$$



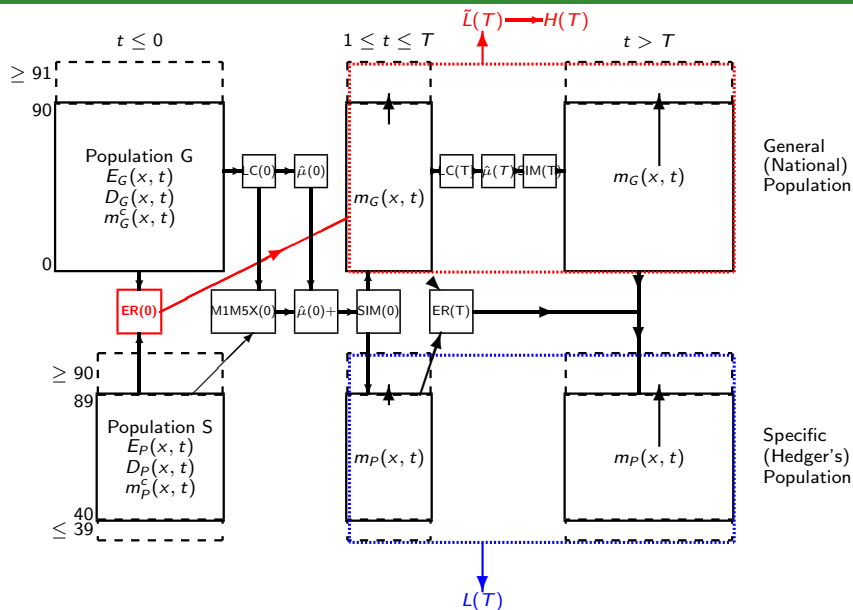
## The Synthetic $\tilde{L}(T)$

- $\tilde{L}$  = random PV at time 0 of a portfolio of synthetic liabilities
- Synthetic mortality experience
  - based on general population mortality
  - adjusted using **experience ratios**
- $\tilde{L}(T)$  = point estimate of  $\tilde{L}$  based on info at  $T$   
= **PV of actual *synthetic* cashflows up to  $T$**   
+ **PV of estimated *synthetic* cashflows after  $T$**

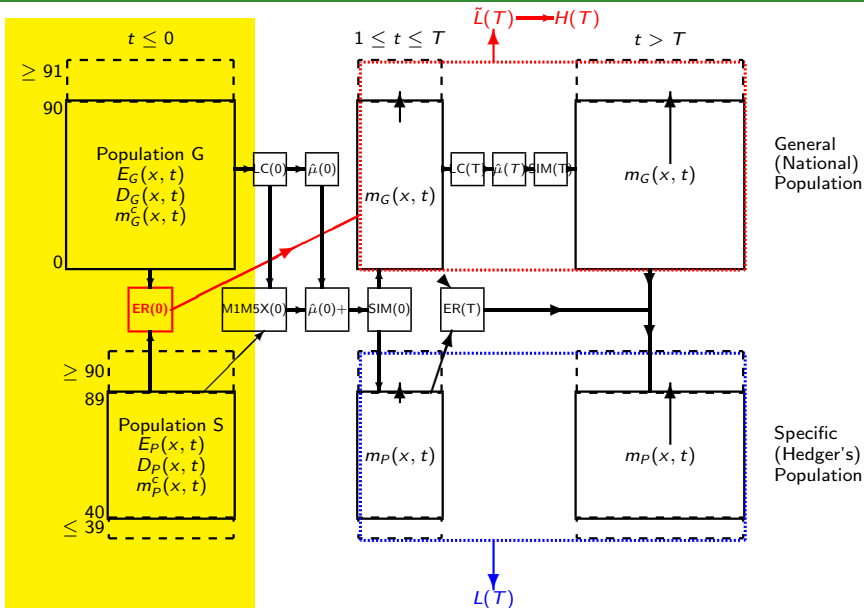
## Questions and Observations

- What is the impact of the hedge:  
 $L(T) \longrightarrow L(T) - H(T)$ ?
- Need a two population mortality model
- Practical reality: calculation is more complex than academic 'ideal world'
- What are good choices of  $AP$ ,  $EP$ ,  $T$ ?

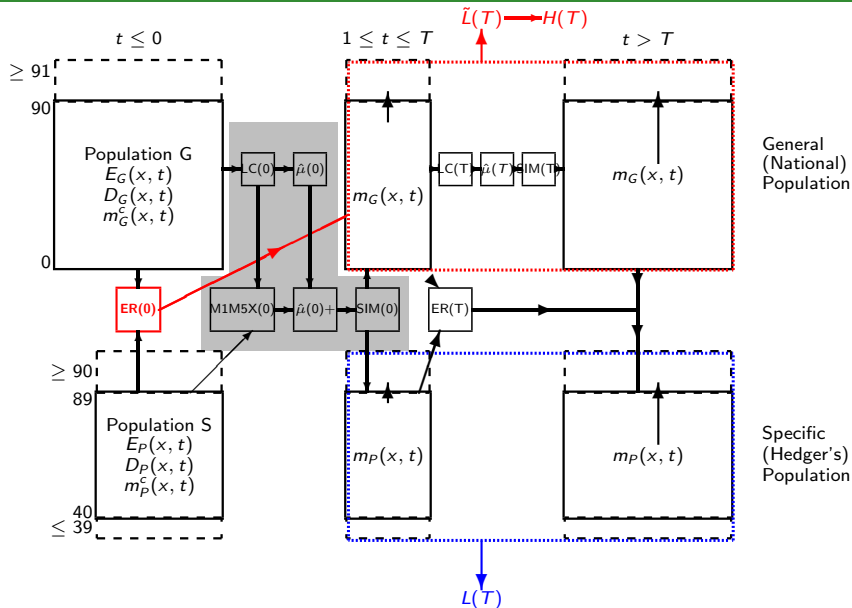
# Anatomy of a Hedging Calculation: Looks Complex!



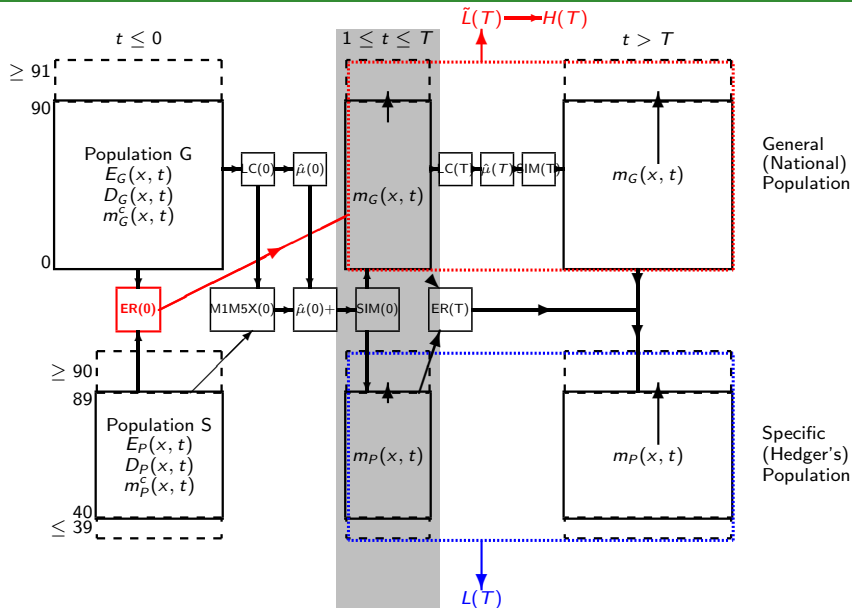
# Historical Data



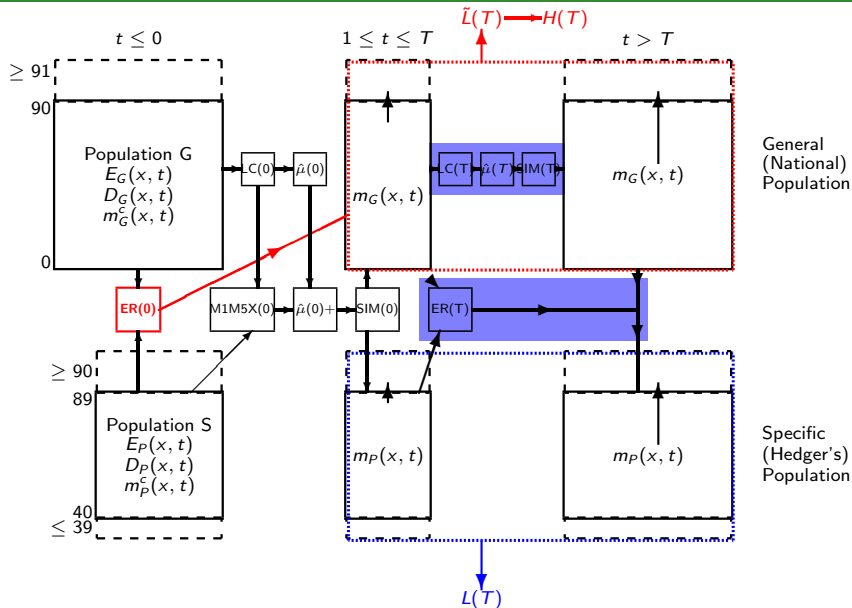
# Modelling Based on Data Up To Time 0



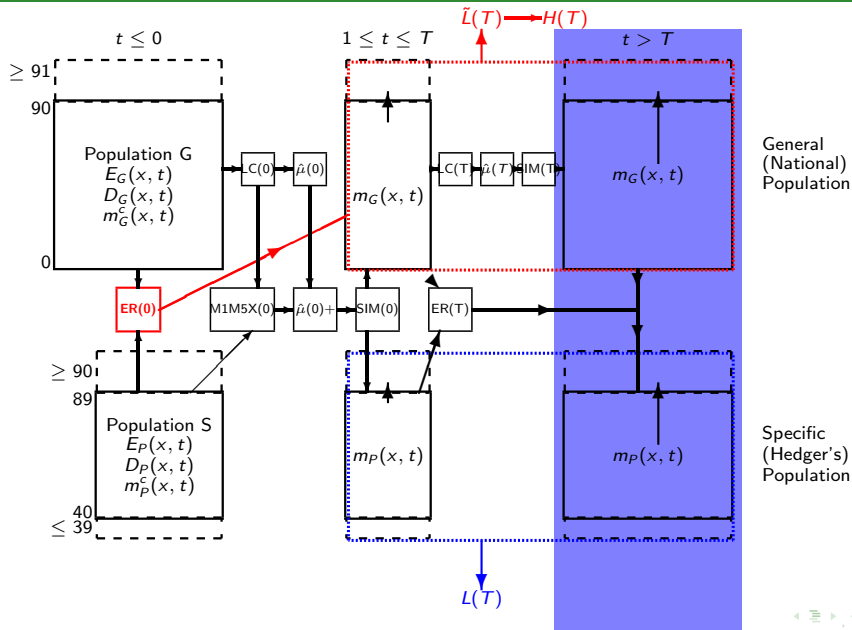
# Generate Stochastic Scenarios Up To Time $T$



# Modelling Based on Data Up To Time $T$

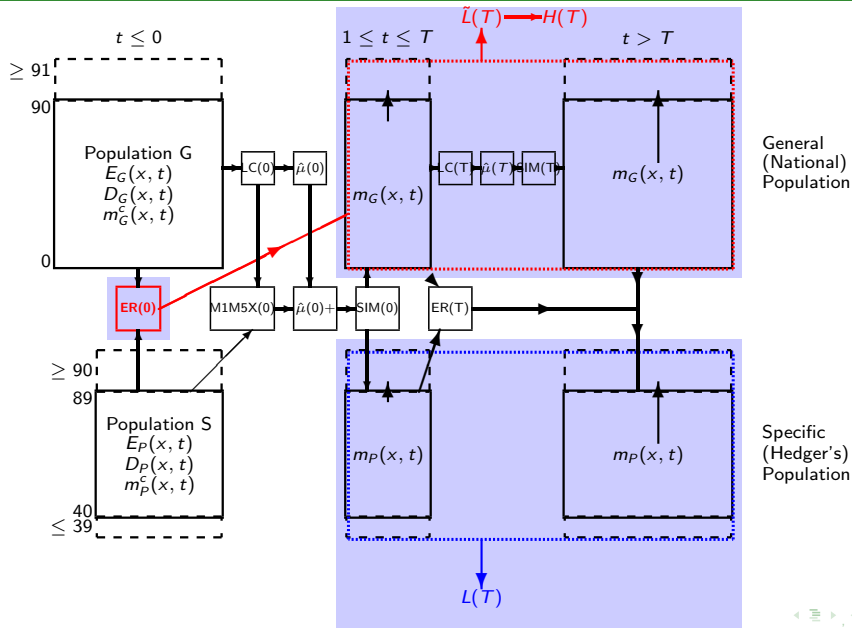


# Central Forecast After $T$ For Each Scenario Up To $T$





# Extract $m_{G/P}(x, t)$ : Calculate $L(T)$ , $\tilde{L}(T)$ , $H(T)$



# How many models do you need?

*Academic 'ideal':* One model

*In practice:*

- Time 0:
  - Liability valuation model (BE + SCR)
  - Simulation model ( $0 \rightarrow T$ )
- Time  $T$ :
  - Hedge instrument valuation model
  - Liability valuation model
- 'Models' for extrapolating to high (and low) ages

- **Unhedged Liabilities:**  
Deterministic BE + 20% stress

- **Simulation:** (by way of example)
  - General population: (Lee-Carter/M1)

$$\ln m_{gen}(x, t) = A(x) + B(x)K(t) \quad (\text{Lee-Carter/M1})$$

- Hedger's own population: (M1-M5X)

$$\ln m_{pop}(x, t) = \ln m_{gen}(x, t) + a(x) + k_1(t) + k_2(t)(x - \bar{x})$$

- Hedge instrument:
  - Lee-Carter (M1) for general population
  - Recalibration: *on basis specified at time 0*

$$q_{pop}^H(x, t) = q_{gen}^H(x, t) \times ER(x, 0) \rightarrow \tilde{L}(T) \rightarrow H(T)$$

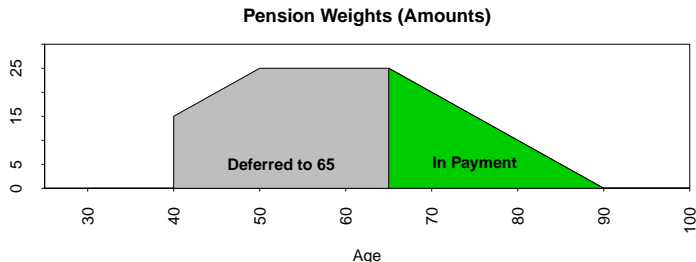
- Liability: specific (hedger's) population
  - Lee-Carter (M1) for general population
  - Possibly different calibration from the hedge instrument
  - $q_{pop}^L(x, t) = q_{gen}^L(x, t) \times ER(x, T) \rightarrow L(T)$
  - Approach must mimic local practice

## Hedging Example

- Data: Netherlands
  - CBS national data
  - CVS insurance data (Dutch aggregated industry experience data)
  
- Hedge instrument maturity:  $T = 10$
- Attachment and exhaustion points at 60% and 95% quantiles of  $\tilde{L}(T)$
- Key point:  $EP \ll 99.5\%$  quantile of  $\tilde{L}(T)$

# Hedging Example

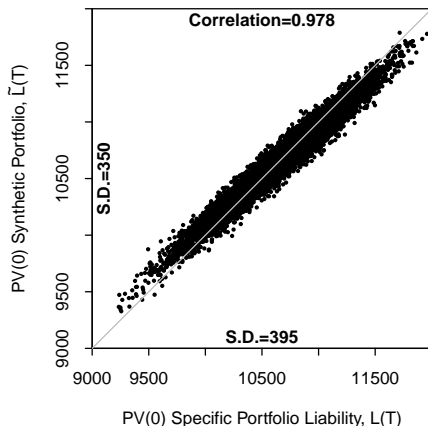
- Portfolio of deferred and immediate annuities
- Current ages 40 to 89
- Weights ( $\equiv$  pension amounts):



- Before and after: Compare  $L(T)$  with  $L(T) - H(T)$
- SCR = 99.5% quantile – mean

# Hedging Example ( $n = 10,000$ scenarios)

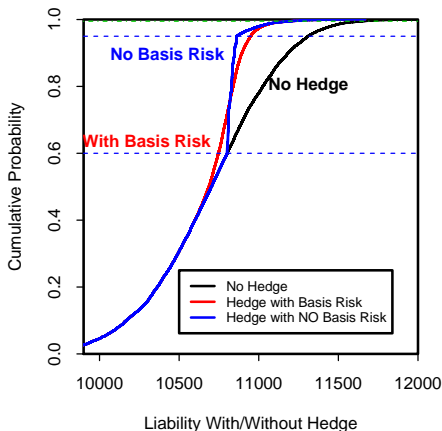
Simulated Annuity Portfolio Present Values



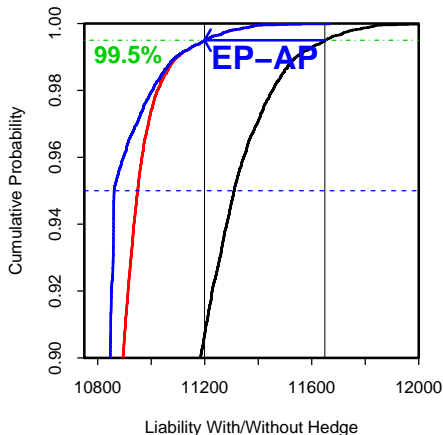
Note: Population basis risk typically increases SCR (without hedge) as a percentage of BE.

# What is the Impact of Population Basis Risk?

Liability Distribution Functions



Liability Distribution Functions



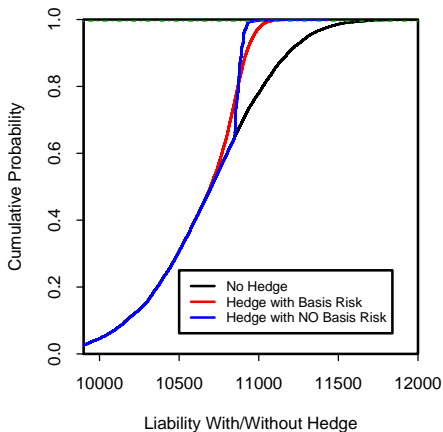
With  $EP = 95\%$  quantile

At the much higher 99.5% level:  $H(T)$  pays off in full  
with or without population basis risk.

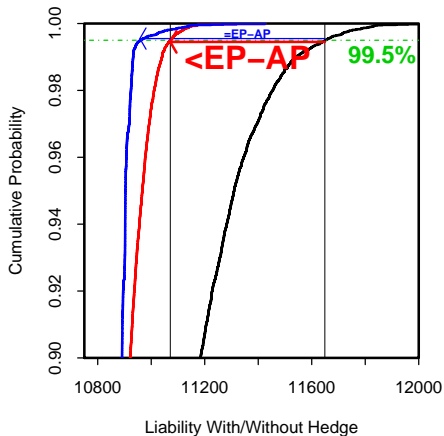


# Hedging Example: Higher AP (0.65) and EP (0.995)

### Liability Distribution Functions



### Liability Distribution Functions



## Numerical Example: AP, EP = 60% and 95% quantiles

$L(0):$	$SCR_{20\%stress}$	840	
$\tilde{L}(T):$	$SCR_{10}$	840	(Pop 1; no hedge)
$\tilde{L}(T) - H(T):$	$SCR_{11}$	478	(Pop 1; with $\tilde{L}(T)$ hedge)
$L(T):$	$SCR_{20}$	960	(Pop 2; no hedge)
$L(T) - H(T):$	$SCR_{21}$	598	(Pop 2; with $\tilde{L}(T)$ hedge)

**Table:** SCR values in excess of the mean liability. For the hedging instrument  $AP = 10779$  (60% quantile) and  $EP = 11228$  (95% quantile). Pop 1: synthetic  $\tilde{L}(T)$ . Pop 2: true  $L(T)$ .

- “Good”  $\Rightarrow$  price and risk reduction
- “Good”  $\leftrightarrow$  Types of basis risk
  - Structural (e.g. non-linear payoff)
  - Population basis risk
    - Within population (e.g. linkage to different cohort)
    - Different population
- Hedge effectiveness  $\Rightarrow$  % reduction in required capital
- Haircut  $\Rightarrow$  impact on capital relief as a result of population basis risk
- EIOPA Solvency II guidelines  $\Rightarrow$  regulatory approval should focus on the haircut

## Numerical Example: AP, EP = 60% and 95% quantiles

$L(0):$	$SCR_{20\%stress}$	840	
$\tilde{L}(T):$	$SCR_{10}$	840	(Pop 1; no hedge)
$\tilde{L}(T) - H(T):$	$SCR_{11}$	478	(Pop 1; with $\tilde{L}(T)$ hedge)
$L(T):$	$SCR_{20}$	960	(Pop 2; no hedge)
$L(T) - H(T):$	$SCR_{21}$	598	(Pop 2; with $\tilde{L}(T)$ hedge)

**Table:** SCR values in excess of the mean liability. For the hedging instrument  $AP = 10779$  (60% quantile) and  $EP = 11228$  (95% quantile). Pop 1: synthetic  $\tilde{L}(T)$ . Pop 2: true  $L(T)$ .

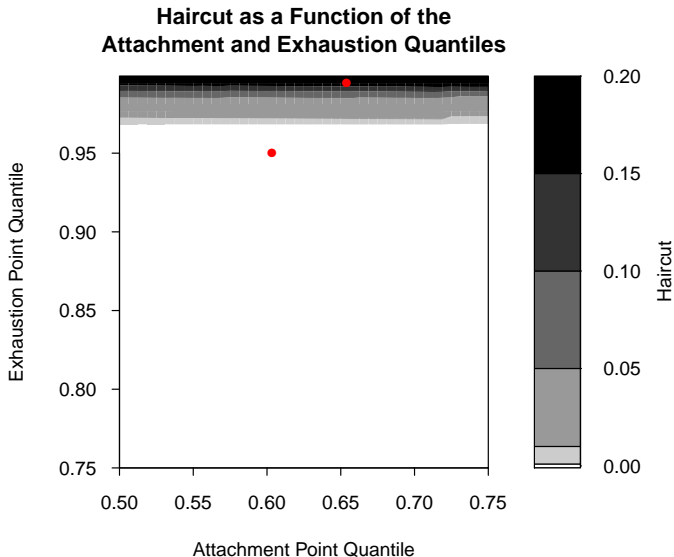
What is the impact of Population basis risk on hedge effectiveness?

$$\text{Haircut } HC = 1 - \frac{SCR_{20} - SCR_{21}}{SCR_{10} - SCR_{11}} = 0.000.$$

## Haircut $\approx 0$ : Interpretation

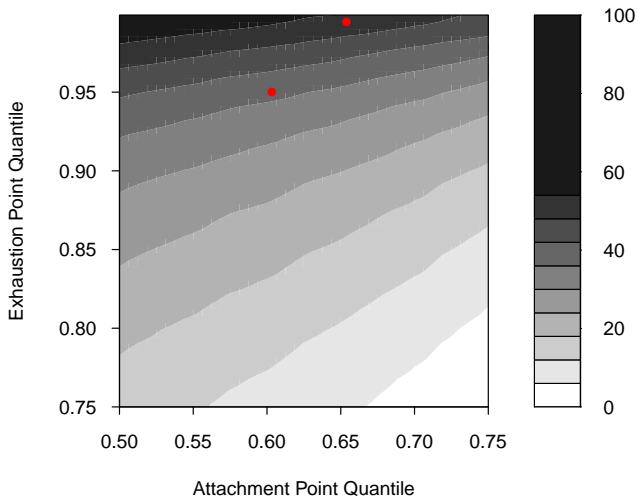
- Here  $EP \ll 99.5\%$  quantile
  - Above the 99.5% quantile the call spread (almost) always pays off in full
  - So **population basis risk**  $\Rightarrow$  little impact
  - **Structural basis risk** prevails
- 
- More detailed analysis  $\Rightarrow$   
Haircut is *worst* (highest) when EP is close to the 99.5% quantile.

# Haircut: Dependence on AP and EP



# Reduction in SCR: Dependence on AP and EP

Reduction in SCR with Hedge  
as a Percentage of SCR without Hedge



Purpose of hedge:

- To manage and reduce risk
- To reduce statutory or economic capital requirements ( $t = 0$ )
- To enhance *economic/shareholder value*



## Economic Value (work in progress)

Payments:

- Fixed  $P_t$  payable at  $t = 0, \dots, T - 1$
- Contracted at time 0
- Time 0 value,  $V_P = \sum_{t=0}^{T-1} P_t \exp(-rt)$

Benefits:

- $H(T)$  at time  $T$
- Capital reduction,  $CR_t$ , at  $t = 0, \dots, T - 1$
- Time 0 value

$$\begin{aligned} V_B &= \text{value of } H(T) \\ &+ \tilde{C}oC \times \text{'value' of } CR_0, \dots, CR_{T-1} \end{aligned}$$

Compare  $V_B$  with  $V_P$ .

# Discussion

- Rigorous approach: practical assessment of the impact of a longevity hedge
- Call spread: choice of EP  $\Rightarrow$  impact on haircut  $\Rightarrow$  impact on regulatory approval
- Choice of AP and EP  $\Rightarrow$  impact on SCR reduction
- Interaction: SCR reduction  $\leftrightarrow$  price  $\Rightarrow$  tradeoff
- Applies equally well to economic capital

# Thank You!

## Questions?

Paper online at:

[www.macs.hw.ac.uk/~andrewc/ARCresources](http://www.macs.hw.ac.uk/~andrewc/ARCresources)

# Bonus Slides



# Tradeoffs and Other Considerations

How to choose Maturity, AP and EP?

- Reduction in SCR ↗
- Cat Bond nominal ↘
- Bull spread price ↘
- Shareholder value added ↗
- Insurer risk appetite, hedging objectives etc.



## Sensitivity to Hedge Maturity, $T$

- e.g.  $T = 20$
- % reduction in SCR is *slightly* higher
- Haircut is *slightly* worse
- Haircut is still  $\approx 0$  for  $EP \leq 99.5\%$  quantile
- The longer the maturity:
  - less liquid market
  - less confidence in future reserving method
  - more future capital relief (everything else held constant)



# The Actuarial Research Centre (ARC)

A gateway to global actuarial research

The Actuarial Research Centre (ARC) is the Institute and Faculty of Actuaries' (IFoA) network of actuarial researchers around the world. The ARC seeks to deliver cutting-edge research programmes that address some of the significant, global challenges in actuarial science, through a partnership of the actuarial profession, the academic community and practitioners.

The 'Modelling, Measurement and Management of Longevity and Morbidity Risk' research programme is being funded by the ARC, the SoA and the CIA.

[www.actuaries.org.uk/arc](http://www.actuaries.org.uk/arc)

## Actuarial Research Centre (ARC):

funded research arm of the Institute and Faculty of Actuaries

Three major programmes started in 2016, including

### **Modelling, Measurement and Management of Longevity and Morbidity Risk**

- New/improved models for modelling longevity
- **Management of longevity risk**
- Underlying drivers of mortality
- Modelling morbidity risk for critical illness insurance