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The Effect of Model Uncertainty on the Pricing of Critical Illness Insurance

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Outline:

1. Critical Illness Insurance (CII)
2. Data & Problems
3. Claim delay distribution modelling
4. CI diagnosis rates under uncertainty
5. CI pricing under uncertainty

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Critical Illness: Policy description

- Fixed term policy, usually ceasing at age 65
- A fixed sum insured payable on the diagnosis of one of a specified list of illnesses
- Benefit type:

  Full acceleration (FA): Death is included as a critical illness (88%)

  Stand alone (SA): Death is not included as a critical illness (12%)
- Covers:

  Cancer; Death; Heart attack; Stroke; Multiple Sclerosis; Total & permanent disability; Coronary artery bypass graft; Kidney failure; Major organ transplant; Other.

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Data

CI data for 1999 – 2005 supplied by CMI:

- Details of policies inforce at the start and end of each year
  - 18,000,000 policy-years of exposure
- Details of claims settled in 1999 – 2005
  - 19,000 claims

Covariates in the data:

Covariate	Number of levels
Age	Numerical
Sex	2 (Female = 0)
Smoker status	2 (NS = 0)
Policy duration	Numerical
Office	13
Benefit type	2 (FA = 0 & SA)
Benefit amount	Numerical
Policy type	2 (Single/Joint life = 0)
Settlement year	Numerical
Cause	10

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Diseases covered

Critical illnesses and percentage of claims in 1999 – 2005

Critical Illness	% claims	Critical Illness	% claims
Cancer	49.0	Total & permanent disability (TPD)	2.6
Death	17.6	Coronary artery bypass graft (CABG)	2.1
Heart attack (HA)	11.6	Kidney failure (KF)	0.6
Stroke	5.4	Major organ transplant (MOT)	0.2
Multiple sclerosis (MS)	4.3	Other causes	6.6
Males	57.3	Non-smokers	73.9
Females	42.7	Smokers	26.1

Source: Continuous Mortality Investigation, UK

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Modelling & estimation

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- Estimation & smoothing of CI diagnosis rates
  – how do these depend on **risk factors?**
- Diagnosis is the **insured event and there is a delay between diagnosis** and **settlement**
  – diagnosis date often not recorded (18%); need to model it
  – does delay also depend on risk factors?
- The **exposure corresponds to claims settled, not to claims diagnosed;** need to adjust it
- Premium **pricing**
- Also take into account uncertainty

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Diagnosis to settlement delay

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Mean Delay = 185 days; SD Delay = 263 days

Fit a **claim delay distribution** (CDD):

$F^{(j)}(u : x; \theta)$ = Pr[claim diagnosed age x, cause j, covariates θ, will be settled within u years]

- Estimate missing dates of diagnosis as:

  Date of settlement – median of appropriate CDD

  (posterior distribution available)
- Use the CDD to adjust the exposure
- Office-specific growth weights introduced

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Observed delay & null model fit

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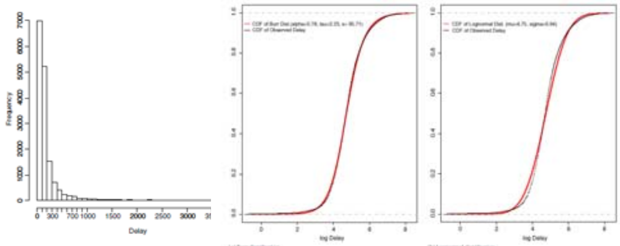


Fig. 1. Histogram of claim settlement delay (in days).

(a) Burr distribution.

(b) Lognormal distribution.

Claim Delay (D) Distribution

Include **risk factors** in GLM setting:

M_1: $D_i \sim \text{LN}(\mu_i, \sigma^2)$

$$E(D_i) = \exp(\eta_i + \sigma^2/2)$$

where

$$\eta_i = \mu_i = \beta_0 + \sum_{j=1}^{8} \beta_j z_{ij} + \beta_{9,k} + \beta_{10,l}$$

M_2: $D_i \sim \text{Transformed Gamma}(\alpha, \tau, s_i)$

$$f_D(u) = \frac{\tau (u/s)^{\alpha\tau} \exp(-u/s)^{\tau}}{u\Gamma(\alpha)} \quad , \quad E(D_i) = \exp(\eta_i)$$

where η_i as above and s_i given as function of η_i, α, τ.

Claim Delay (D) Distn (cont)

M_3: $D_i \sim \text{Burr}(\alpha, \tau, s_i)$

$$f_D(u) = \frac{\alpha\tau(u/s)^{\tau}}{u[1+(u/s)^{\tau}]^{\alpha+1}} \quad , \quad E(D_i) = \exp(\eta_i)$$

with s_i given as function of η_i, α, τ.

M_4: $D_i \sim \text{Generalised beta}(\alpha, \tau, \gamma, s_i)$

$$f_D(u) = \frac{\Gamma(\alpha+\gamma)}{\Gamma(\alpha)\Gamma(\gamma)} \frac{\tau(u/s)^{\tau\gamma}}{u\,[1+(u/s)^{\tau}]^{\alpha+\gamma}} \quad , \quad E(D_i) = \exp(\eta_i)$$

again, with s_i given as function of $\eta_i, \alpha, \tau, \gamma$.

Claim Delay (D) Distn (cont)

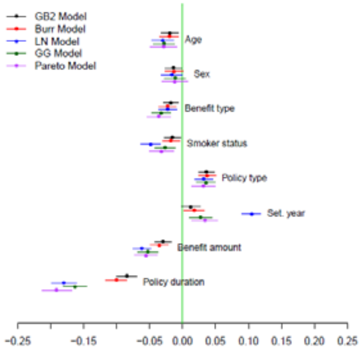
Fit the 4 models under a Bayesian framework using *MCMC*:

- Compare fit using *Deviance Information Criterion*:

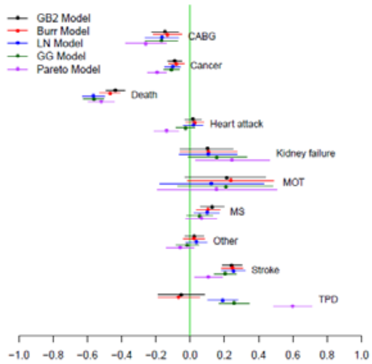
	LN	Tr. Gamma	Burr	Gen. Beta
DIC	194,356	193,025	191,262	190,992

- Model fit also compared using *posterior predictive checking*.

Risk factor estimates



Illness coefficient estimates



Prediction of Claim Delay

Find 'best' predictive model using Bayesian variable selection – Ozkok et al. (2012a))

	Scenario				
Covariate	1	2	3	4	5
Benefit type	FA	FA	FA	FA	FA
Policy type	JL	JL	JL	JL	JL
Amount (£)	50k	250k	50k	50k	50k
Duration (yrs)	4	4	1	4	4
Office	11	11	11	11	11
Cause	Cancer	Cancer	Cancer	Death	TPD
$E[D]$ (days)	174	156	195	112	217
CI	(167, 182)	(146, 166)	(187, 204)	(106, 119)	(193, 243)

Scenario 1: reference; Red: changes from reference

Estimation of CI diagnosis rates

- Non-recorded **diagnosis dates estimated through CDD model**
- Suppose Office 1 contributes data for 2000 to 2003. For this office, let:

θ be a set of **covariates**, including office

$\lambda^{(j)}_{x;\theta}$ be the **diagnosis inception rate** for cause j at age x with covariates θ

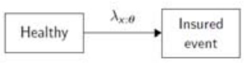
$E(u:x;\theta)$ be the **number of policies** (age x, covariates θ) inforce at time u, $0 \le u \le 4$

$N^{(j)}(x;\theta)$ be the **number of claims** (cause j, age x, covariates θ) diagnosed and settled in 2000 – 2003

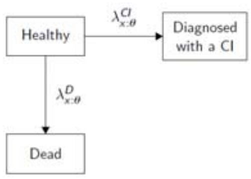
$$N^{(j)}(x;\theta) \sim \text{Poisson}\left(\lambda^{(j)}_{x;\theta} \int_{u=0}^{4} E(u:x;\theta)\, F^{(j)}(4-u:x;\theta)\, du\right)$$

Model

For all causes:



Have also considered:



Rate smoothing (graduation)

Gompertz-Makeham-Cox-type Model:

$$\lambda^{(j)}_{x;\theta} = \lambda^{(j)}_1(x) + \exp(\lambda^{(j)}_2(x))\, \exp(\beta\, \mathbf{z}^T)$$

where $\lambda^{(j)}_i(x)$ is a polynomial function of age only, $i = 1, 2$

$\lambda^{(j)}_1(x) \equiv 0$ for each cause except death

$\lambda^{(j)}_1(x) \equiv 0 \rightarrow$ log–linear (Cox-type) model for $\lambda^{(j)}_{x;\theta}$

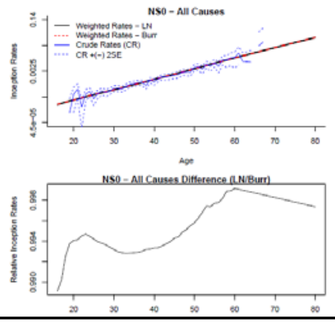
Rate smoothing (cont.)

Fit model – perform variable selection using *BIC*

Cause	Predictive model (covariates)							
CABG	Age	Sex			Smoker			
Cancer	Age	Sex	Year		Smoker	Age×Sm		
Death	Age	Sex			Smoker	Age×Sm		
Heart Attack	Age	Sex			Smoker	Age×Sm		
Kidney Fail		Sex						
MOT								
MS		Sex			Smoker		Pol Durn	
Other	Age	Sex		Office				Benefit type
Stroke	Age	Sex			Smoker	Age×Sm		
TPD	Age		Year				Pol Durn	
All causes	Age			Office	Smoker	Age×Sm	Pol Durn	

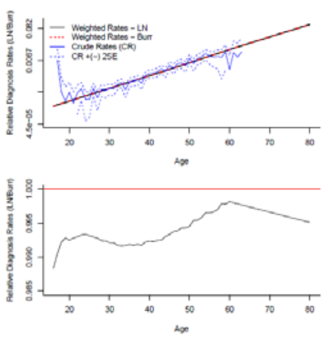
Comparison of diagnosis rates

Claim rates for **Non-smokers**, **Pol Durn 0**, **All causes**: Burr v LN



Comparison of diagnosis rates (2)

Claim rates for <u>**Smokers**</u>, **Pol Durn 0**, **All causes**: Burr v LN

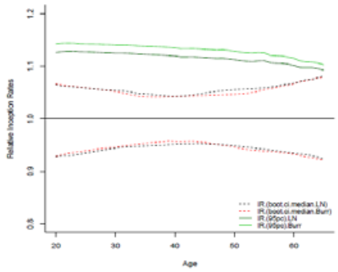


Sensitivity to diagnosis estimates

Relative rates (divided by rate obtained with Median of CDD)

– Rate using 97.5th percentile of Burr or LN CDD

– CIs derived using parametric bootstrap (Burr v LN)



Non-smokers, Pol durn 0, All causes

Premium pricing - derivation

Annual premium, paid at constant rate, n-year term

- Can be calculated using

$$\text{Net Premium } = \text{Benefit Amount} \times \frac{\int_{t=0}^{n} v^t \, {}_t p_x \, \lambda_{x+t} \, dt}{\int_{t=0}^{n} v^t \, {}_t p_x \, dt}$$

where ${}_t p_x$: survival probability
λ_{x+t}: total claim rate at age $x+t$
v: discount factor

- Then bootstrap distribution of λs used to derive CIs for premiums

Premium pricing - comparisons

Age 40, Pol Durn 0, All causes, Benefit amount £100k, *i* = 3%

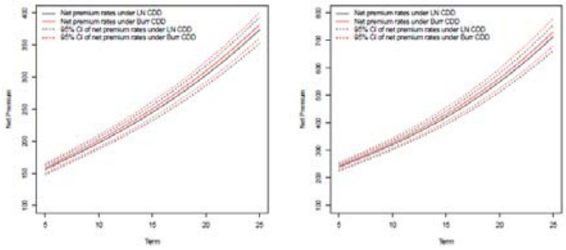
(LN v Burr CDD)

	LN CDD		Burr CDD	
	Non-smokers		Non-smokers	
Term	Net premium rate	95% CI	Net premium rate	95% CI
5-years	156.05	148.05, 162.73	158.84	150.67, 165.00
25-years	373.69	352.67, 393.23	381.71	360.49, 400.63
	Smokers		Smokers	
Term	Net premium rate	95% CI	Net premium rate	95% CI
5-years	239.70	225.96, 250.89	244.61	230.64, 256.17
25-years	714.03	662.88, 756.37	731.39	680.05, 779.60

Premium pricing - comparisons

Age 40, Pol Durn 0, All causes, Benefit amount £100k, i = 3%

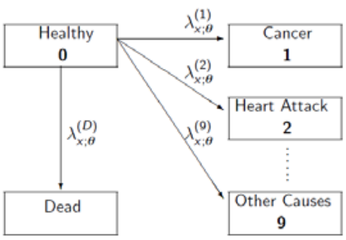
(Burr v LN CDD)



Non-smokers Smokers

Premiums with cause-specific model

Have also considered model for specific causes
(this can also distinguish between FA and SA policies)

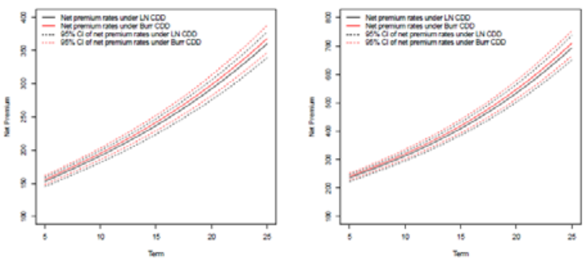


Useful for comparing rates or premiums for certain causes;
or when a specific illness is excluded – e.g. TPD

Premiums with TPD *not* covered

Age 40, Pol Durn 0, Benefit amount £100k, i = 3%

(Burr v LN CDD)



Non-smokers Smokers

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Summary

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- Delay between diagnosis and settlement in CII is important (e.g. IBNR, IBNS)
- Have developed **delay model: depends on risk factors**
- Bayesian analysis **accounts for non-recorded diagnosis dates**
- 4-parameter G.Beta distn fits data best – followed by 3-parameter Burr
- **CII rates and premiums estimated & smoothed** – including parameter and model uncertainty
- Estimates of **CDD are model-sensitive**
- **But claim rates and premiums are not**

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More details in:

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Ozkok, E., Streftaris, G., Waters, H.R., and Wilkie, A.D. (2012a) Bayesian modelling of the time delay between diagnosis and settlement for Critical Illness Insurance using a Burr generalised-linear-type model. *Insurance: Mathematics & Economics*, 50, 266–279.

Ozkok, E., Streftaris, G., Waters, H.R., and Wilkie, A.D. (2012b) Modelling critical illness claim diagnosis rates I: Methodology. *Scandinavian Actuarial Journal*, doi: 10.1080/03461238.2012.728537

Ozkok, E., Streftaris, G., Waters, H.R., and Wilkie, A.D. (2013) Modelling critical illness claim diagnosis rates II: Results. *Scandinavian Actuarial Journal*, DOI:10.1080/03461238.2012.728538.

Dodd, E., Streftaris, G., Waters, H.R. and Stott, A.D. (2014) The effect of model uncertainty on the pricing of critical illness insurance. To appear in *Annals of Actuarial Science*.