How much to put in a tontine

Thomas Bernhardt and Catherine Donnelly
Risk Insight Lab https://risk-insight-lab.com/

Actuarial and Financial Mathematics Conference, Thursday 7th February, 2019

The UK pension freedom since 2015

building a personal pension pot, and at retirement...

buy annuity

- savings for guaranteed income, $\mathbb{E}[\text{give}] = \mathbb{E}[\text{take}]$
- mortality pooling (law of large numbers)

go into drawdown

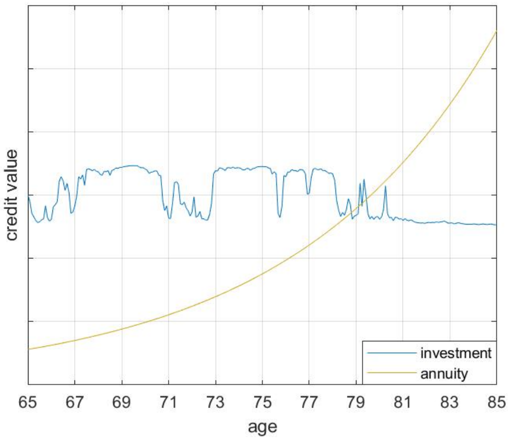
- savings spent over time
- investments (fluctuating)
- bequest

The UK pension freedom since 2015

building

buy ann

- savi
  inco
- mor
  larg



time

ıating)

The UK pension freedom since 2015

building a personal pension pot, and at retirement...

buy annuity

- savings for guaranteed income, $\mathbb{E}[\text{give}] = \mathbb{E}[\text{take}]$
- mortality pooling (law of large numbers)

$\Downarrow$

mortality credits at high ages, unpopular choice

go into drawdown

- savings spent over time
- investments (fluctuating)
- bequest

$\Downarrow$

investment returns at low ages, risk of outliving

Tontines

Tontine $=$ mortality credits $+$ investment return

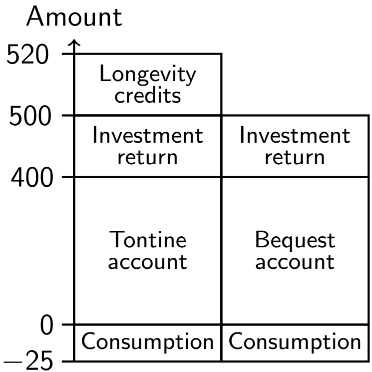
- surrender savings to a group of people, to get mortality credits
- no guarantees, to be able to invest

add bequest

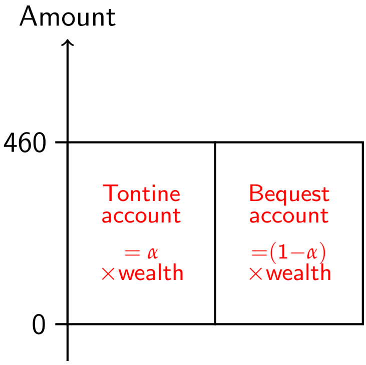
- allow to choose α, how much to surrender, to have a bequest (comes with reduction in mortality credits)

Tontines

in the background mortality credits boost wealth and bequest



(a) Before re-balancing.



(b) After re-balancing.

Tontines

Tontine = mortality credits + investment return

- surrender savings to a group of people, to get mortality credits
- no guarantees, to be able to invest

add bequest

- allow to choose α, how much to surrender, to have a bequest (comes with reduction in mortality credits)

mathematical description

- mortality credits = additional α-weighted stream of income
- in a Black-Scholes market and force of mortality λ...

$$\frac{\mathrm{d}X_t}{X_t} = r(1-\pi_t)\mathrm{d}t + \mu\pi_t\mathrm{d}t + \sigma\pi_t\mathrm{d}W_t - c_t dt + \alpha\lambda_t\mathrm{d}t$$

Numerical results

optimization problem including lifespan τ, bequest motive b, and constant relative risk aversion $1-\gamma$

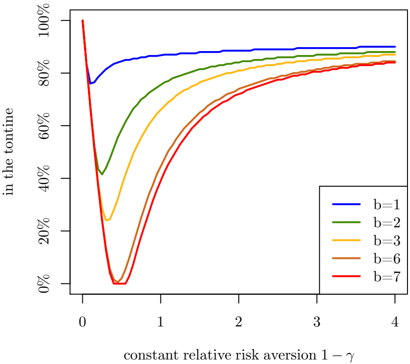
- $\sup\limits_{\alpha,c,\pi} \mathbb{E}\Big[\int_0^\tau U\big(s, cX_s\big)\, \mathrm{d}s + b\, B\big(\tau, (1-\alpha)X_\tau\big)\Big]$
- $U(s,x) = B(s,x) = \mathrm{e}^{-\rho s} x^\gamma / \gamma$
- $\mathbb{P}[\tau > x] = \exp\Big(-\int_0^x \lambda_s \mathrm{d}s\Big)$

Numerical results

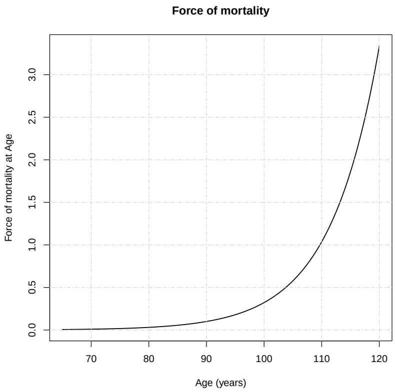
solution for optimal α,
given bequest motive b
and risk aversion $1-\gamma$

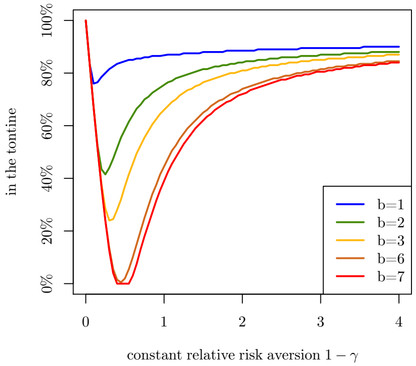
risk seeking, low $1-\gamma$

- down and up
- changes from 0% to 100%

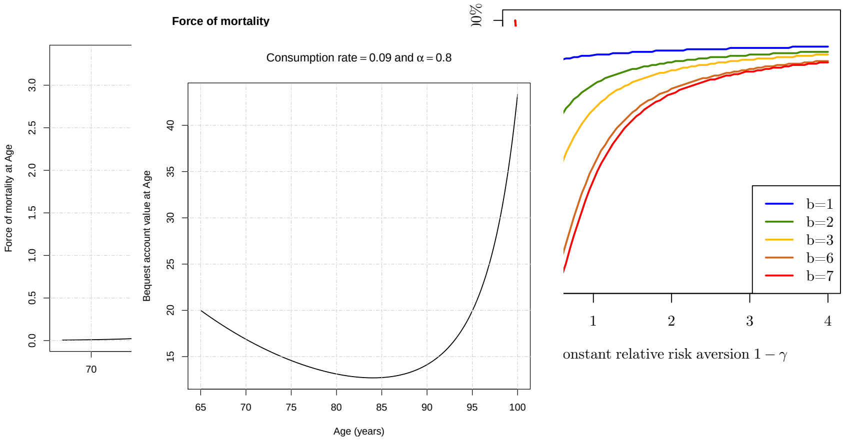


Numerical results

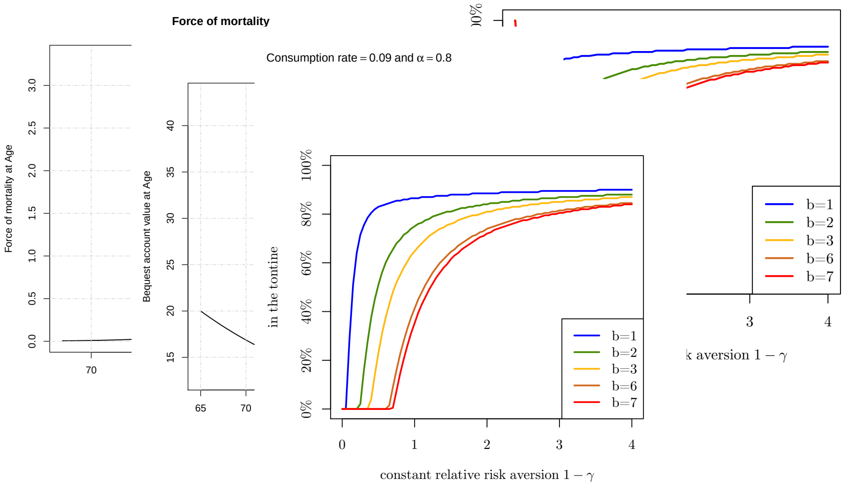




Numerical results



Numerical results

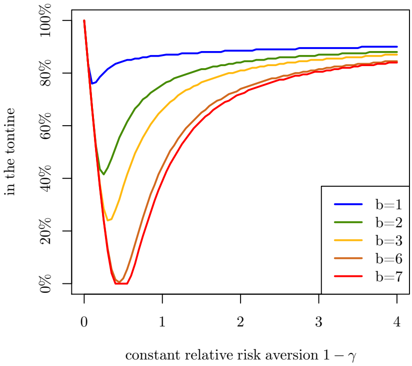


Numerical results

solution for optimal α,
given bequest motive b
and risk aversion $1-\gamma$

risk seeking, low $1-\gamma$

- down and up
- changes from 0% to 100%



risk averse, high $1-\gamma$

- around 80%
- stable even for changes in μ, σ, r and slight changes with ρ, λ

Numerical results

Age 70 with £100,000 pot

	Annuity	Drawdown	Tontine with bequest
Annual income	£6,000		
Age of default	Never		
Money left to heirs	Nothing		
Basis	S1PMA, UK yield curve		

Numerical results

Age 70 with £100,000 pot

	Annuity	Drawdown	Tontine with bequest
Annual income	£6,000	£6,600	
Age of default	Never	87 years	
Money left to heirs	Nothing	left pot	
Basis	S1PMA, UK yield curve	S1PMA, 2% p.a.	

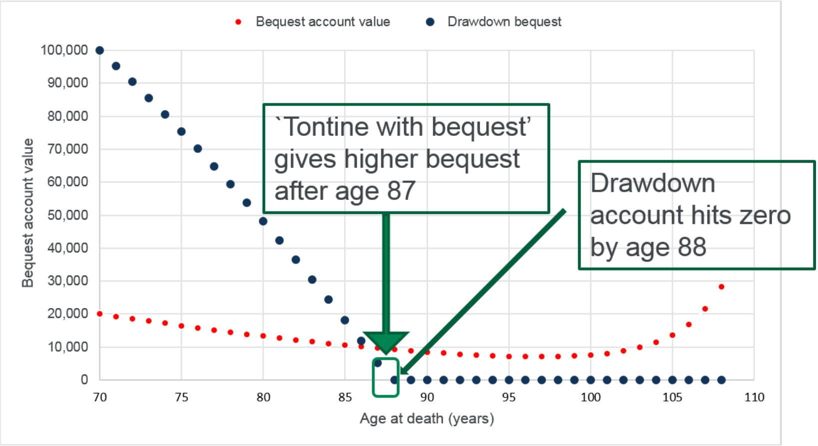
Numerical results

Age 70 with £100,000 pot

	Annuity	Drawdown	Tontine with bequest
Annual income	£6,000	£6,600	£6,600
Age of default	Never	87 years	120+ years (constant amount withdrawn)
Money left to heirs	Nothing	left pot	20% of left pot
Basis	S1PMA, UK yield curve	S1PMA, 2% p.a.	S1PMA, 2% p.a., 80% in the tontine

Numerical results

direct comparison drawdown vs. tontine with bequest



Future research

- how many members so that law of large numbers holds true?
- is risk sharing possible to achieve stability?

Thank you for your attention.
Do you have any questions or feedback?