Institute and Faculty of Actuaries

Parameterising emergence factors: how hard can it be?

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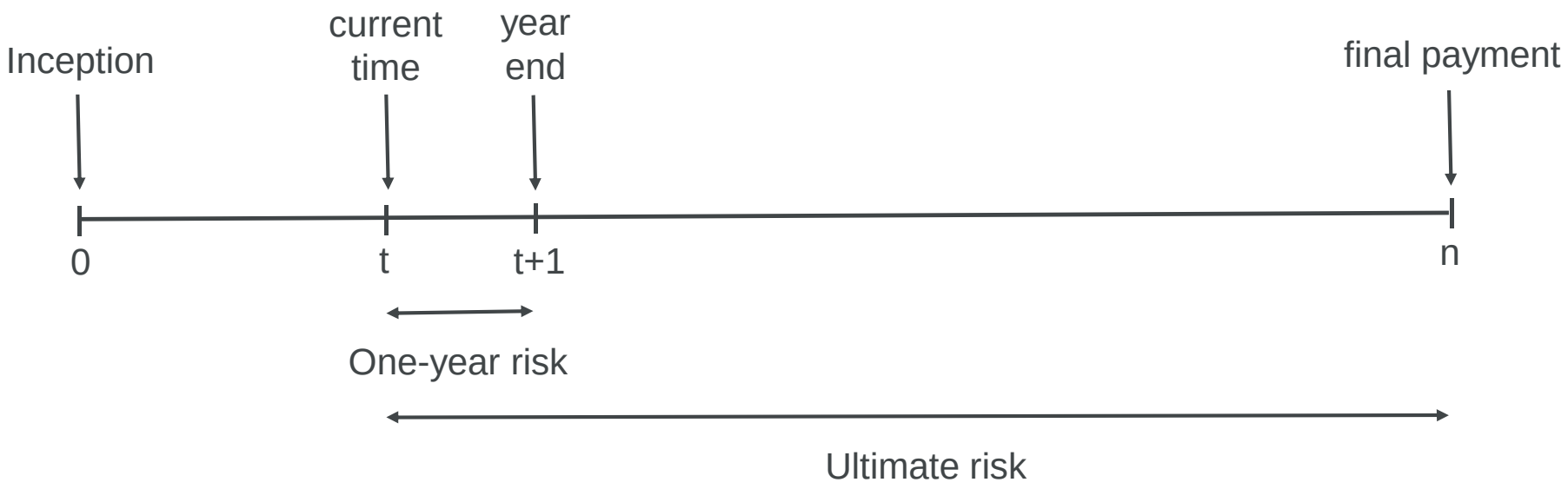
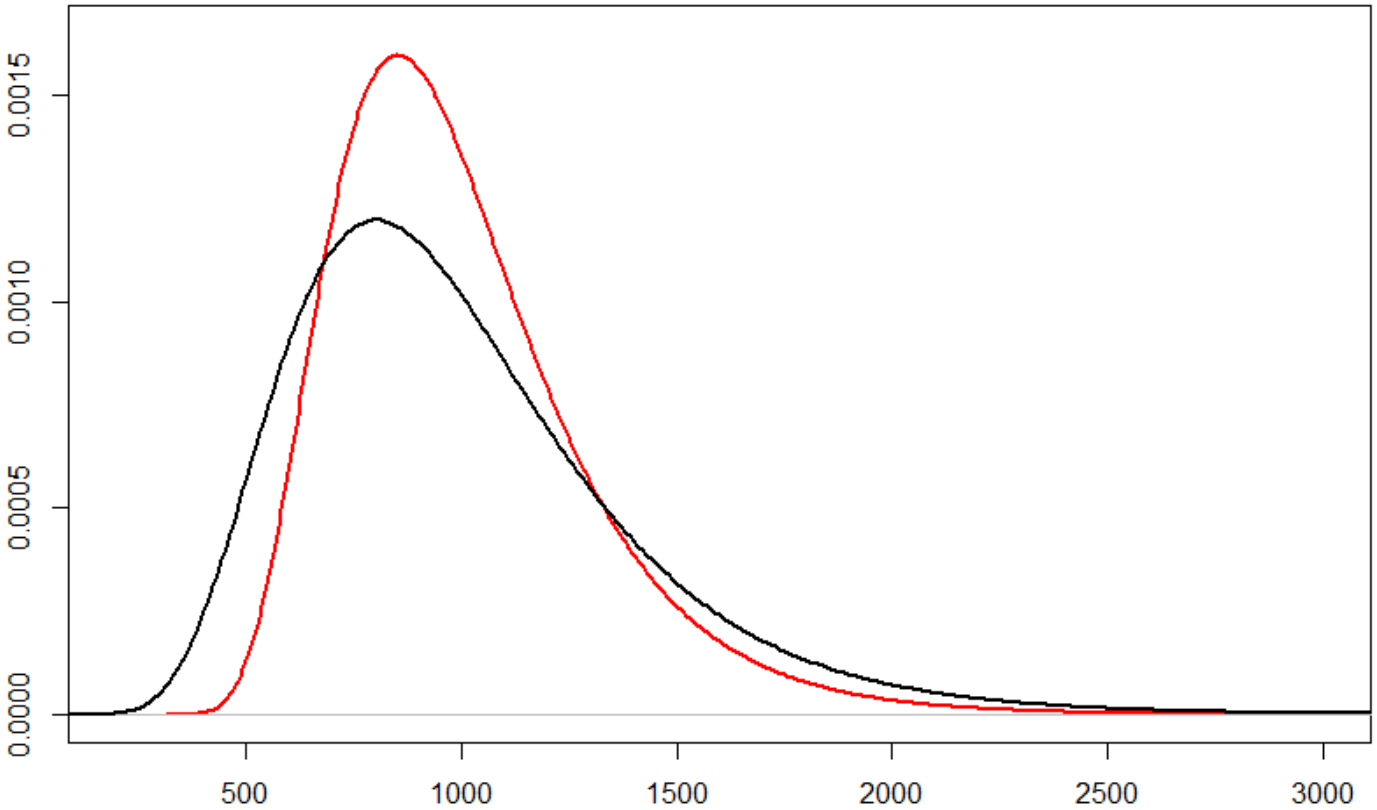
Ultimate view and one-year view



What are emergence factors?



$$\hat{X} = \alpha(X - E[X]) + E[X]$$

Question:
Where does the factor α come from?

Why use emergence factors?

Alternatives

- Merz-Wüthrich
- Actuary-in-the-Box
- Direct modelling

No well established model

Inflexible

Need enough data to fit model

Computationally expensive

Depends on bootstrap

Difficult to explain

Need consistent data

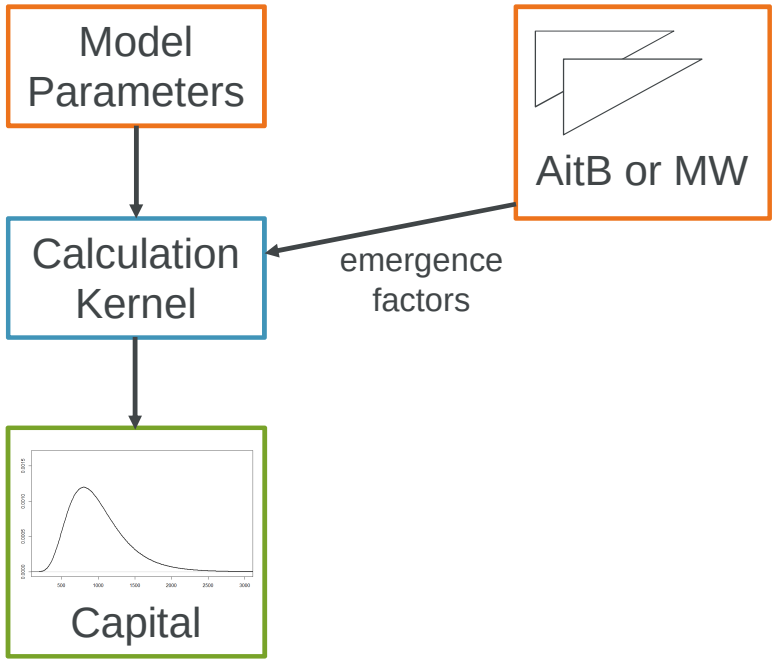
Emergence factors

- Computationally easy
- Flexible
- Simple to explain

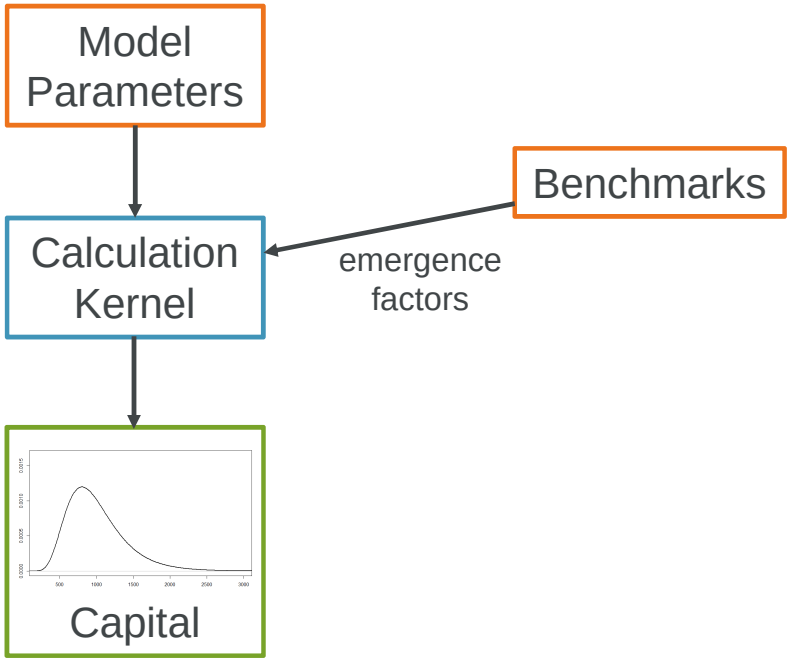
Parameterising emergence factors

	1	2	3	4	5	6	7	8	9	10	Ultimate	One-Year	Ratio
2007	357,848	1,124,788	1,735,330	2,218,270	2,745,596	3,319,994	3,466,336	3,606,286	3,833,515	3,901,463	0	0	
2008	352,118	1,236,139	2,170,033	3,353,322	3,799,067	4,120,063	4,647,867	4,914,039	5,339,085		75,535	75,535	100%
2009	290,507	1,292,306	2,218,525	3,235,179	3,985,995	4,132,918	4,628,910	4,909,315			121,699	105,309	87%
2010	310,608	1,418,858	2,195,047	3,757,447	4,029,929	4,381,982	4,588,268				133,549	79,846	60%
2011	443,160	1,136,350	2,128,333	2,897,821	3,402,672	3,873,311					261,406	235,115	90%
2012	396,132	1,333,217	2,180,715	2,985,752	3,691,712						411,010	318,427	77%
2013	440,832	1,288,463	2,419,861	3,483,130							558,317	361,089	65%
2014	359,480	1,421,128	2,864,498								875,328	629,681	72%
2015	376,686	1,363,294									971,258	588,662	61%
2016	344,014										1,363,155	1,029,925	76%
											2,447,095	1,778,968	73%

Two ways of using emergence factors

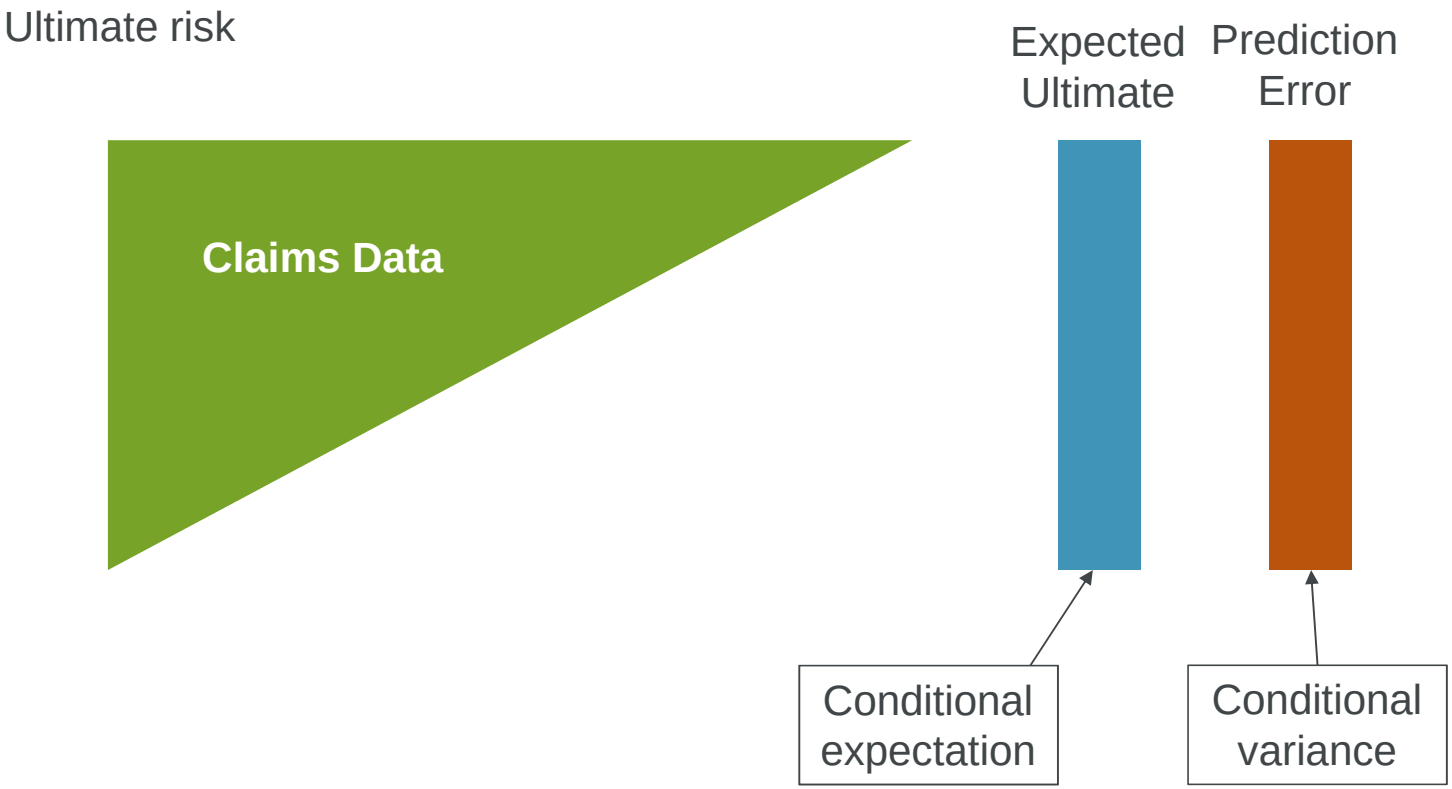


Class X ⟷ Class X

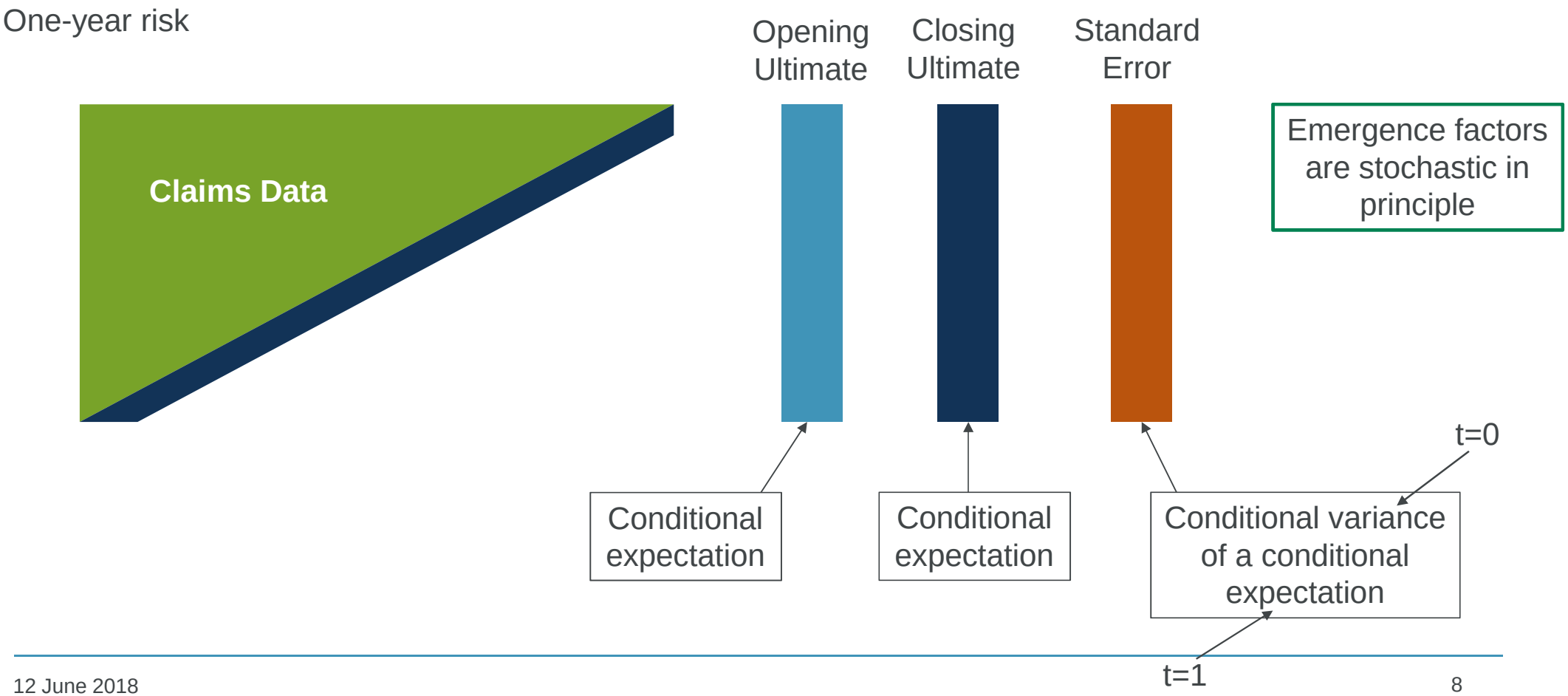


Class Y
Class Z
Class X

What are emergence factors, exactly? (1/2)



What are emergence factors, exactly? (2/2)



The impact on the two uses

	1	2	3	4	5	6	7	8	9	10
2007	357,848	1,124,788	1,735,330	2,218,270	2,745,596	3,319,994	3,466,336	3,606,286	3,833,515	3,901,463
2008	352,118	1,236,139	2,170,033	3,353,322	3,799,067	4,120,063	4,647,867	4,914,039	5,339,085	
2009	290,507	1,292,306	2,218,525	3,235,179	3,985,995	4,132,918	4,628,910	4,909,315		
2010	310,608	1,418,858	2,195,047	3,757,447	4,029,929	4,381,982	4,588,268			
2011	443,160	1,136,350	2,128,333	2,897,821	3,402,672	3,873,311				
2012	396,132	1,333,217	2,180,715	2,985,752	3,691,712					
2013	440,832	1,288,463	2,419,861	3,483,130						
2014	359,480	1,421,128	2,864,498							
2015	376,686	1,363,294								
2016	344,014									
		3.619	2.016	1.439	1.236	1.138	1.047	1.061	1.086	1.018

Is there a better method?

	1	2	3	4	5	6	7	8	9	10
2007	357,848	1,124,788	1,735,330	2,218,270	2,745,596	3,319,994	3,466,336	3,606,286	3,833,515	3,901,463
2008	352,118	1,236,139	2,170,033	3,353,322	3,799,067	4,120,063	4,647,867	4,914,039	5,339,085	
2009	290,507	1,292,306	2,218,525	3,235,179	3,985,995	4,132,918	4,628,910	4,909,315		
2010	310,608	1,418,858	2,195,047	3,757,447	4,029,929	4,381,982	4,588,268			
2011	443,160	1,136,350	2,128,333	2,897,821	3,402,672	3,873,311				
2012	396,132	1,333,217	2,180,715	2,985,752	3,691,712					
2013	440,832	1,288,463	2,419,861	3,483,130						
2014	359,480	1,421,128	2,864,498							
2015	376,686	1,363,294								
2016	344,014									

CDR standard errors

0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	Ultimate
0	0	0	0	0	0	0	0	0	0
75,535	0	0	0	0	0	0	0	0	75,535
105,309	60,996	0	0	0	0	0	0	0	121,699
79,846	91,093	56,232	0	0	0	0	0	0	133,549
235,115	60,577	82,068	51,474	0	0	0	0	0	261,406
318,427	233,859	57,825	82,433	51,999	0	0	0	0	411,010
361,089	328,989	243,412	59,162	85,998	54,343	0	0	0	558,317
629,681	391,249	359,352	266,320	64,443	94,166	59,533	0	0	875,328
588,662	554,574	344,763	318,493	236,576	56,543	83,645	52,965	0	971,258
1,029,925	538,726	511,118	317,142	293,978	218,914	51,661	77,317	49,055	1,363,155

Emergence factors from multi-year CDRs (1/2)

CDR standard errors

0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	Ultimate
0	0	0	0	0	0	0	0	0	0
75,535	0	0	0	0	0	0	0	0	75,535
105,309	60,996	0	0	0	0	0	0	0	121,699
79,846	91,093	56,232	0	0	0	0	0	0	133,549
235,115	60,577	82,068	51,474	0	0	0	0	0	261,406
318,427	233,859	57,825	82,433	51,999	0	0	0	0	411,010
361,089	328,989	243,412	59,162	85,998	54,343	0	0	0	558,317
629,681	391,249	359,352	266,320	64,443	94,166	59,533	0	0	875,328
588,662	554,574	344,763	318,493	236,576	56,543	83,645	52,965	0	971,258
1,029,925	538,726	511,118	317,142	293,978	218,914	51,661	77,317	49,055	1,363,155

Cumulative CDR standard errors

0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	Ultimate
									0
75,535									75,535
105,309	121,699								121,699
79,846	121,133	133,549							133,549
235,115	242,793	256,289	261,406						261,406
318,427	395,077	399,287	407,707	411,010					411,010
361,089	488,487	545,773	548,971	555,666	558,317				558,317
629,681	741,332	823,837	865,814	868,209	873,301	875,328			875,328
588,662	808,749	879,168	935,080	964,543	966,199	969,813	971,258		971,258
1,029,925	1,162,313	1,269,729	1,308,737	1,341,348	1,359,095	1,360,076	1,362,272	1,363,155	1,363,155

Cumulate use square-root of sum-of-squares along each origin period

$$121{,}133^2 = 79{,}846^2 + 91{,}093^2$$

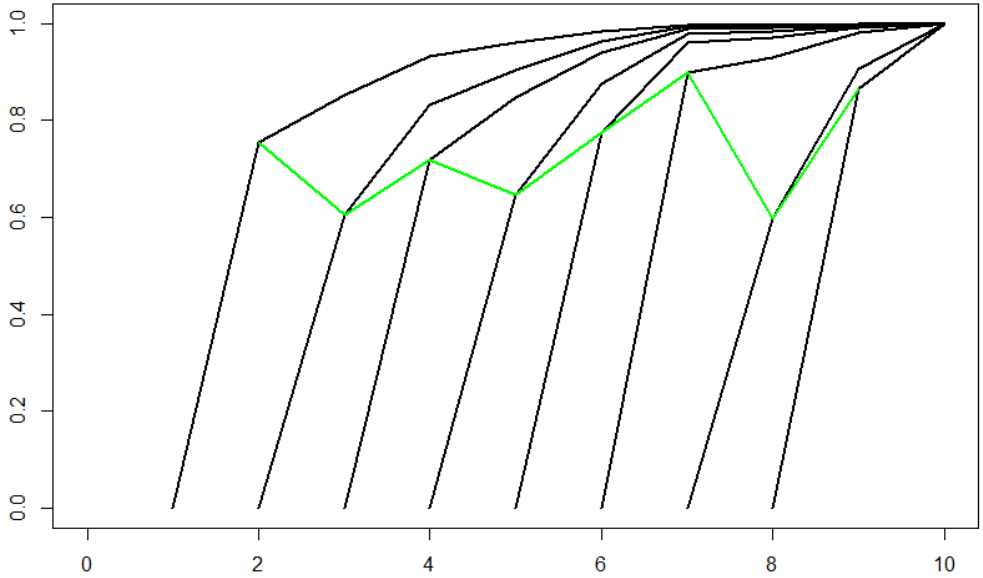
$$133{,}549^2 = 121{,}133^2 + 56{,}232^2$$

CDRs over non-overlapping periods are uncorrelated

Emergence factors from multi-year CDRs (2/2)

Cumulative % CDR standard errors

0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9
100.0%								
86.5%	100.0%							
59.8%	90.7%	100.0%						
89.9%	92.9%	98.0%	100.0%					
77.5%	96.1%	97.1%	99.2%	100.0%				
64.7%	87.5%	97.8%	98.3%	99.5%	100.0%			
71.9%	84.7%	94.1%	98.9%	99.2%	99.8%	100.0%		
60.6%	83.3%	90.5%	96.3%	99.3%	99.5%	99.9%	100.0%	
75.6%	85.3%	93.1%	96.0%	98.4%	99.7%	99.8%	99.9%	100.0%



Emergence factor = $\alpha_{t,k}$

t – prior development
k – future development

Problems to overcome

- Do different origin periods have a common emergence pattern?
- Could we assume that emergence factors are deterministic?
- Different origin periods have seen different amounts of development – can the emergence factors from different origin periods be compared in a meaningful way?
- Emergence factors are ratios – with this difficulty can we find an unbiased estimator for emergence factors?

Formula linking emergence factors

$$1 - \alpha^2_{t+k_1,k_2} = \frac{1 - \alpha^2_{t,k_1+k_2}}{1 - \alpha^2_{t,k_1}}$$

If emergence factors are deterministic then this equation holds

Application of formula linking emergence factors

Cumulative % CDR standard errors

0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9
100.0%								
86.5%	100.0%							
59.8%	90.7%	100.0%						
89.9%	92.9%	98.0%	100.0%					
77.5%	96.1%	97.1%	99.2%	100.0%				
64.7%	87.5%	97.8%	98.3%	99.5%	100.0%			
71.9%	84.7%	94.1%	98.9%	99.2%	99.8%	100.0%		
60.6%	83.3%	90.5%	96.3%	99.3%	99.5%	99.9%	100.0%	
75.6%	85.3%	93.1%	96.0%	98.4%	99.7%	99.8%	99.9%	100.0%

Implied one-year emergence

100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
86.5%	84.4%	84.5%	84.5%	84.5%	84.6%	84.7%	85.1%	
59.8%	49.1%	49.6%	50.1%	50.3%	51.0%	53.0%		
89.9%	90.1%	90.1%	90.0%	90.0%	90.0%			
77.5%	77.1%	77.2%	77.2%	77.3%				
64.7%	63.9%	64.1%	64.3%					
71.9%	71.8%	71.8%						
60.6%	60.3%							
75.6%								

Conclusions

- Emergence factors are simple to explain and calculate with
- Emergence factors can be used in different ways
- No satisfactory way of parameterising emergence factors is known
- Parameterisation is a fundamentally difficult problem
- Simplicity has been gained by packaging-up complexity into a single parameter
- Great care and judgement should be exercised if using externally parameterised emergence factors

Questions

Comments

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