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Pragmatic Stochastic Reserving: The One-Year View

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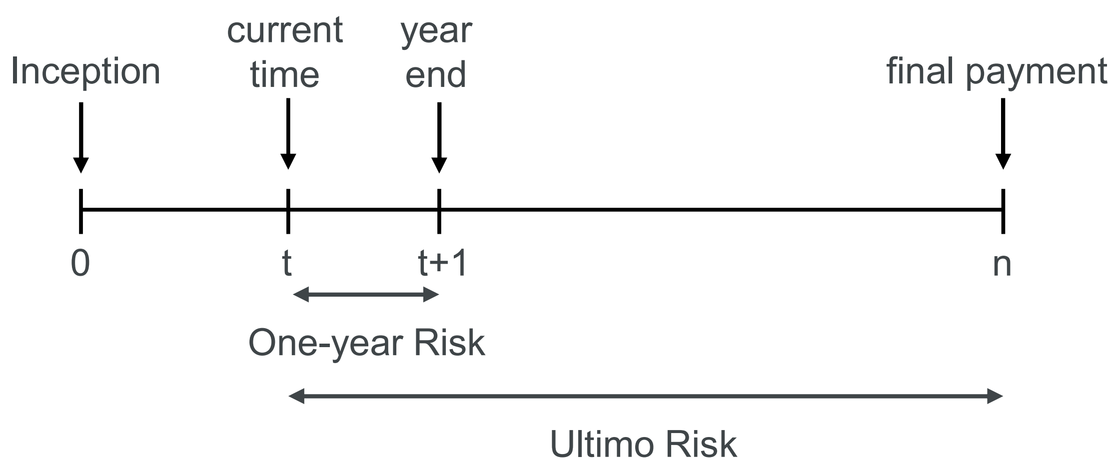
Ultimate vs one-year view

- Established practice is to consider risk to ultimate
- Solvency II requires a one-year view of capital
- We need to ensure reserve risk adequately included in this
- Need to recognise all sources of error
  - process, parameter, and model risk
- Need to consider ENIDs

Ultimate vs one-year view



Problem:
How do we estimate the closing reserve?

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Claims Development Result

Claims Development Result = Opening Estimate of Ultimate
– Closing Estimate of Ultimate

= Opening Reserve
– Closing Reserve
– Claims Paid During Year

Methods of estimating one-year risk

- Merz-Wüthrich
- Actuary-in-the-box
- Emergence Patterns
- Other methods
  - Bayesian methods
  - Hindsight re-estimation
  - Perfect foresight
  - Robbin's method

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Merz-Wüthrich: assumptions (Mack's model)

- Claims data is a triangle of cumulative paid or incurred claims
  $$\{C_{ij} : i = 1, \dots, n, j = 1, \dots, n - i + 1\}$$
- Origin periods are independent
- $(C_{ij} : j = 1, \dots, n)$ are all Markov processes for $i = 1, \dots, n$
- $E[C_{i,j+1} \mid C_{ij}] = f_j C_{ij}$
- $Var(C_{i,j+1} \mid C_{ij}) = \sigma_j^2 C_{ij}$

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Definition of MSEP

- Suppose we have
  - set of observations $\mathcal{D}$
  - from $\mathcal{D}$ we predict X with $\hat{X}$
- $MSEP_{X|\mathcal{D}}(\hat{X}) = E\left[\left(X - \hat{X}\right)^2 | \mathcal{D}\right] = Var(X|\mathcal{D}) + \left(E[X|\mathcal{D}] - \hat{X}\right)^2$

  Process error: $Var(X|\mathcal{D})$

  Parameter error: $\left(E[X|\mathcal{D}] - \hat{X}\right)^2$

- See Wüthrich and Merz's book "Stochastic claims reserving methods in insurance" section 3.1 for further discussion

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Merz-Wüthrich's formula

The MSEP for a single origin period is given by

$$\left(\hat{C}_{iJ}^{I}\right)^2 \left(\frac{\hat{\sigma}_{I-i}^2 / \left(\hat{f}_{I-i}^{I}\right)^2}{C_{i,I-i}} + \frac{\hat{\sigma}_{I-i}^2 / \left(\hat{f}_{I-i}^{I}\right)^2}{S_{I-i}^{I}} + \sum_{j=I-i+1}^{J-1} \frac{C_{I-j,j}}{S_j^{I+1}} \frac{\hat{\sigma}_{I-i}^2 / \left(\hat{f}_{I-i}^{I}\right)^2}{S_j^{I}} \right)$$

(See "Modelling the Claims Development Result for Solvency Purposes" by Merz and Wüthrich, equation 3.17)

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Merz-Wüthrich: strengths

- Based on a well-established model of the chain ladder (Mack's model)
- Can be implemented in a spreadsheet
- If ultimo result calculated using Mack's model then Merz-Wüthrich's one-year view is consistent with this
- Can be adapted to give MSEP for multi-year CDRs

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Merz-Wüthrich: limitations

- Formula only produces one statistic of the CDR – the MSEP
- Only applies to Mack's model of the chain ladder
- If Mack's model is not a good fit to the triangle of claims data then the formula is liable to give unreasonable results.
- Cannot give you one-year premium risk

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Actuary-in-the-box

- Term coined by Esbjörn Ohlsson
- First described in 2009 by Ohlsson and Lauzeningks
- Although not new at this point
- Very general method, but does make some assumptions, which we will highlight

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Actuary-in-the-box: Ohlsson and Lauzeningks

1. Obtain best estimate of the opening reserves using a well-defined algorithm
2. Extend the input data for the algorithm used in step 1 by simulating one further year of data
3. Apply exactly the same algorithm as in step 1 to the extended data set to get a distribution of the claims reserve at time 1

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Ohlsson and Lauzeningks' assumptions

- The reserves are set using a well-defined algorithm that can be carried out automatically
- Exactly the same algorithm will be applied to estimate both the opening and closing reserves
- The model used for setting the reserves has a notion of claims development
- Parameters are calculated within the model
- If you want a distribution of the closing reserve then the claims data must include the paid claims

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Actuary-in-the-box: applied to bootstrap

1. Carry out bootstrap procedure
2. Extend the claims data by one year using the bootstrap output
3. Re-fit the underlying deterministic model to the extended claims data
4. Calculate the reserves from the extended claims data using the underlying deterministic model

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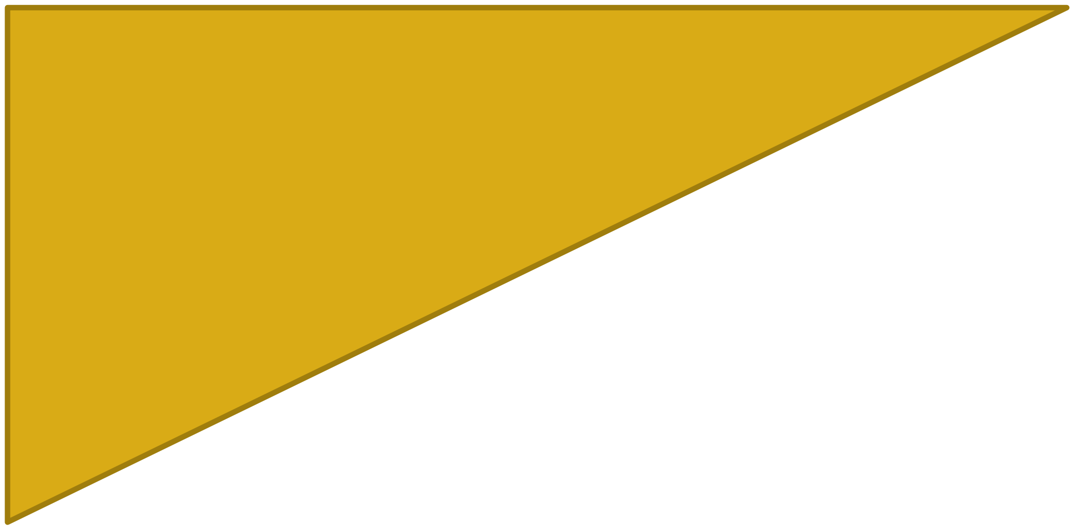
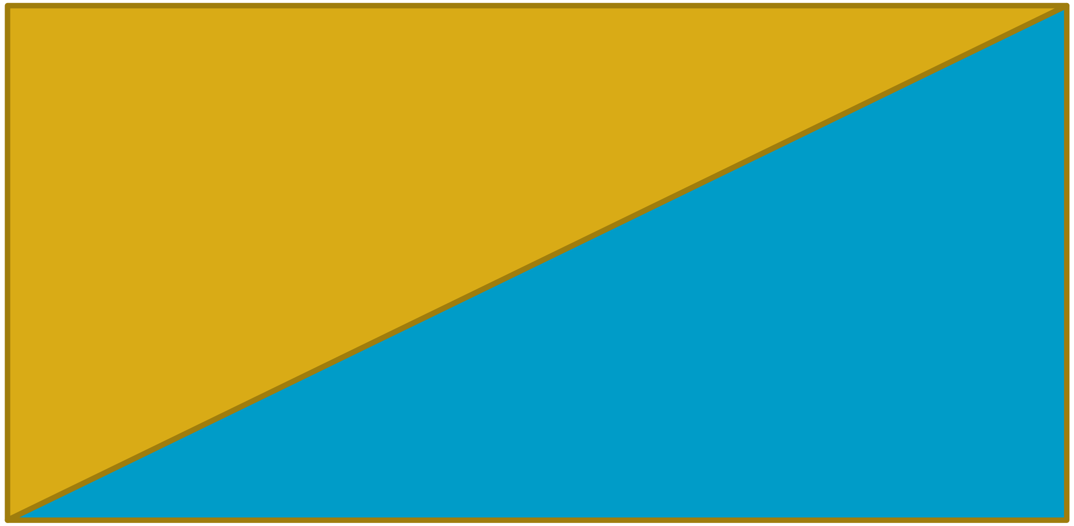
Actuary-in-the-box: bootstrapped Mack's model



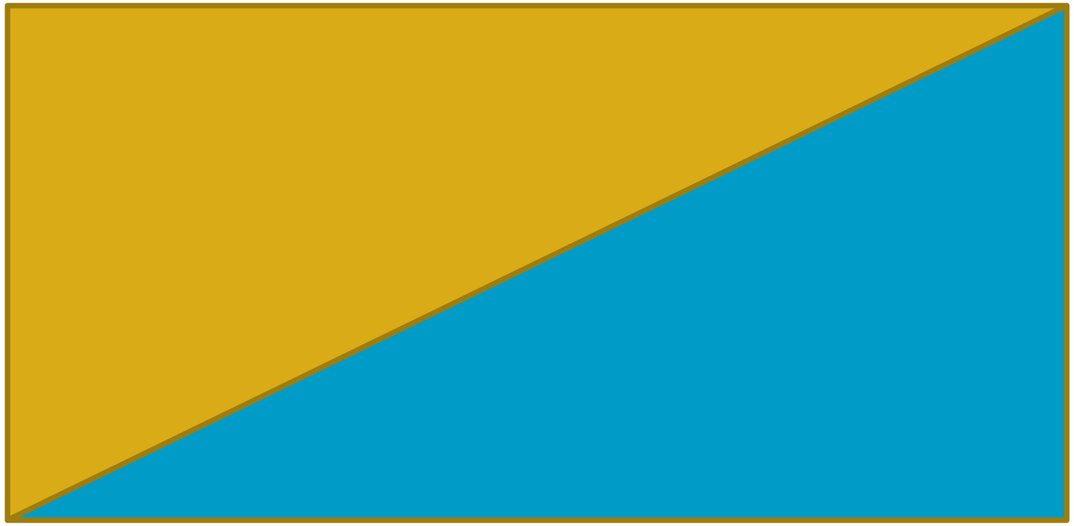
Fit model to triangle of observed claims

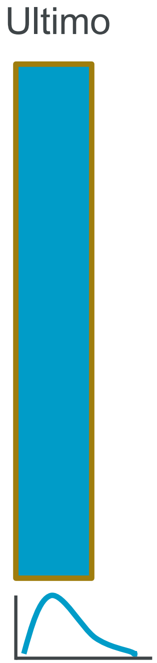
Actuary-in-the-box: bootstrapped Mack's model



Bootstrap to project to ultimate incorporating process and parameter error…

Actuary-in-the-box: bootstrapped Mack's model





…to get a distribution of the ultimate claims

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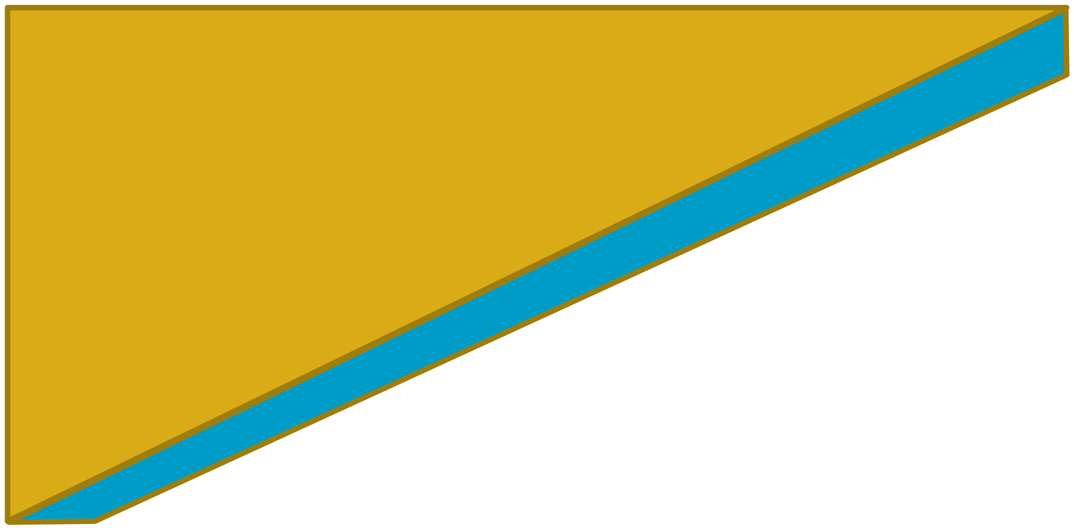
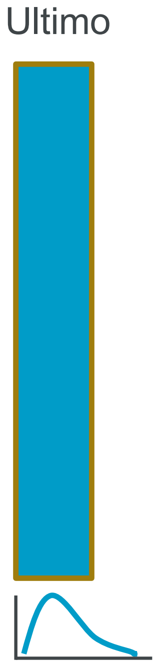
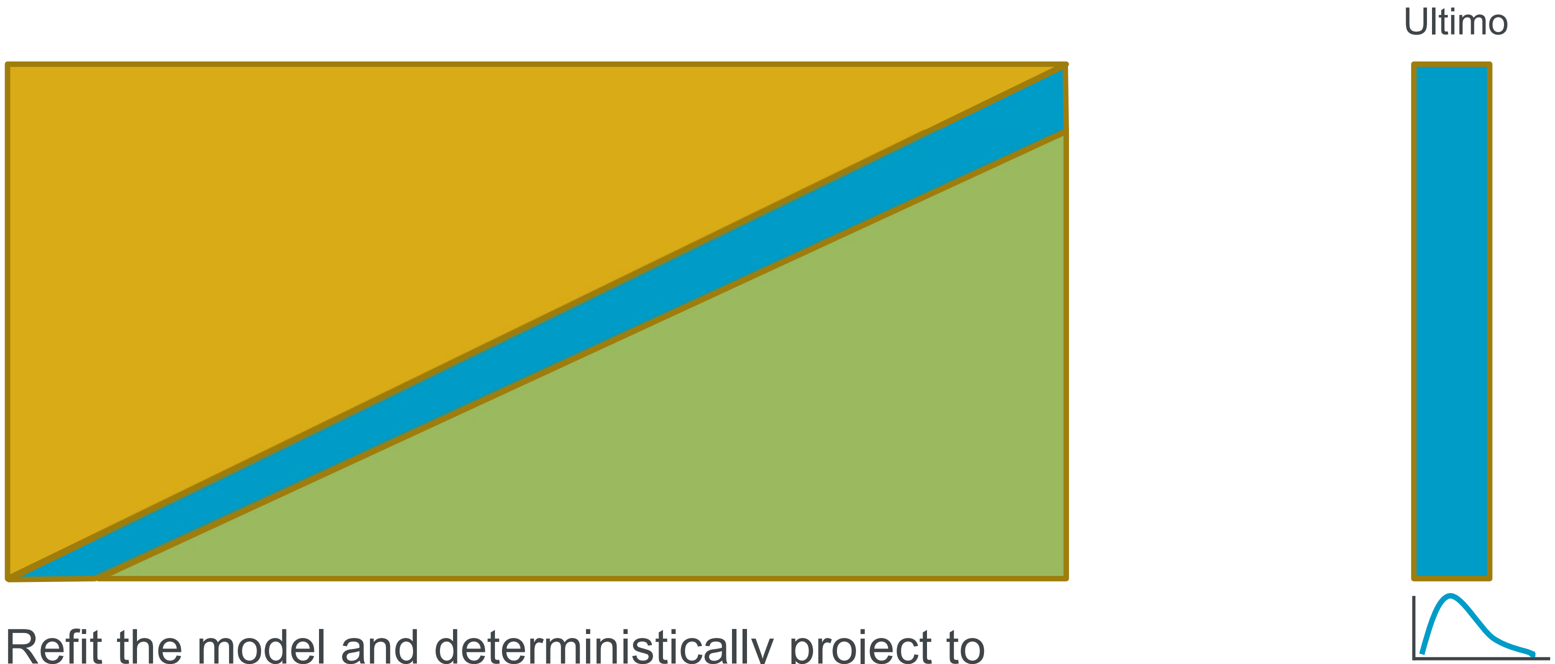
Actuary-in-the-box: bootstrapped Mack’s model





Extend the observed triangle with the first diagonal from the bootstrap projection

Actuary-in-the-box: bootstrapped Mack's model



Refit the model and deterministically project to ultimate…

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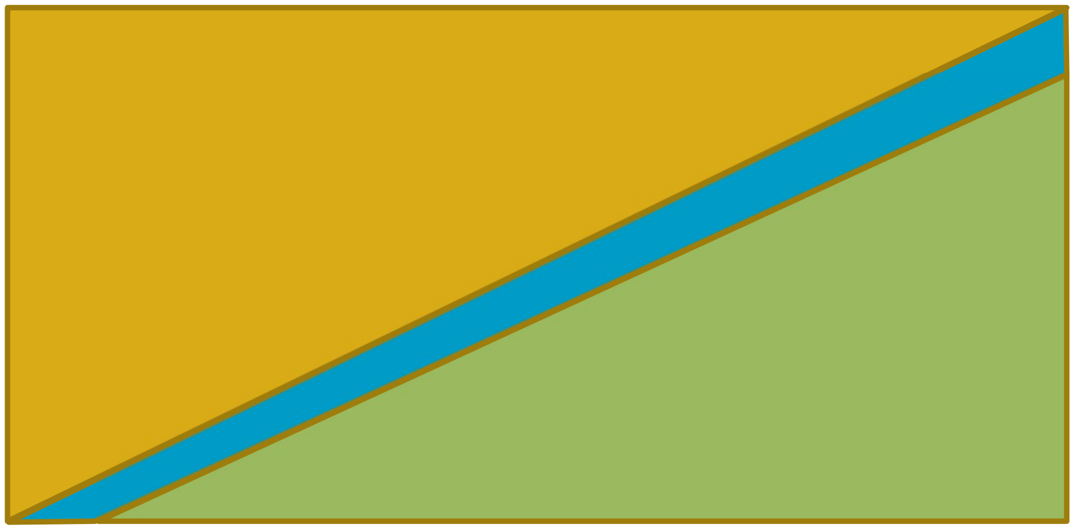
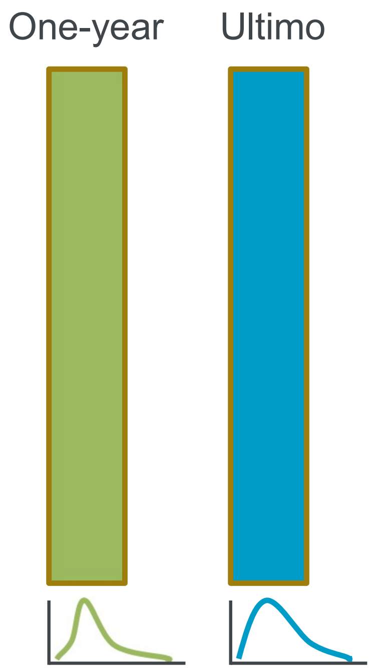
Actuary-in-the-box: bootstrapped Mack's model





…to get the one-year ultimate distribution

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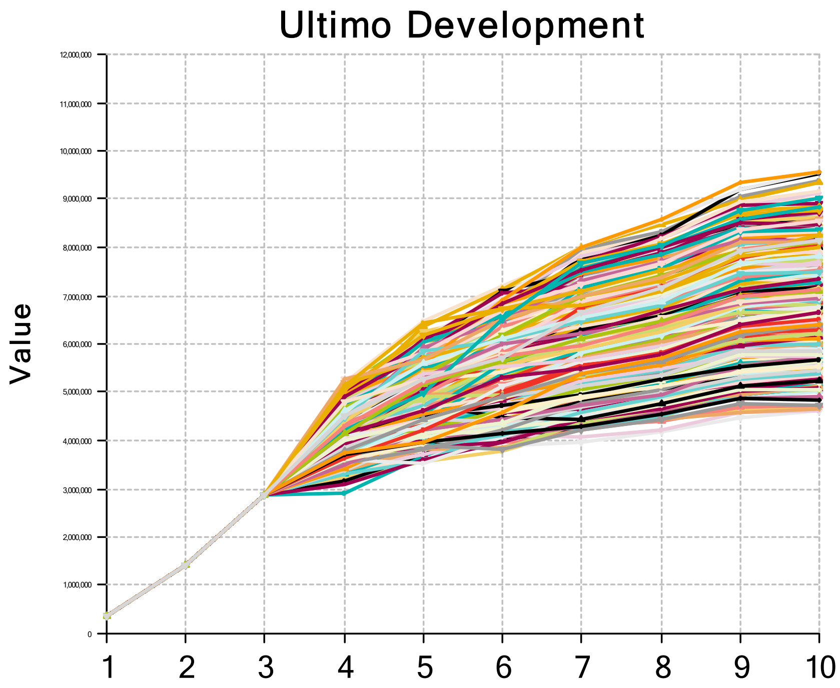
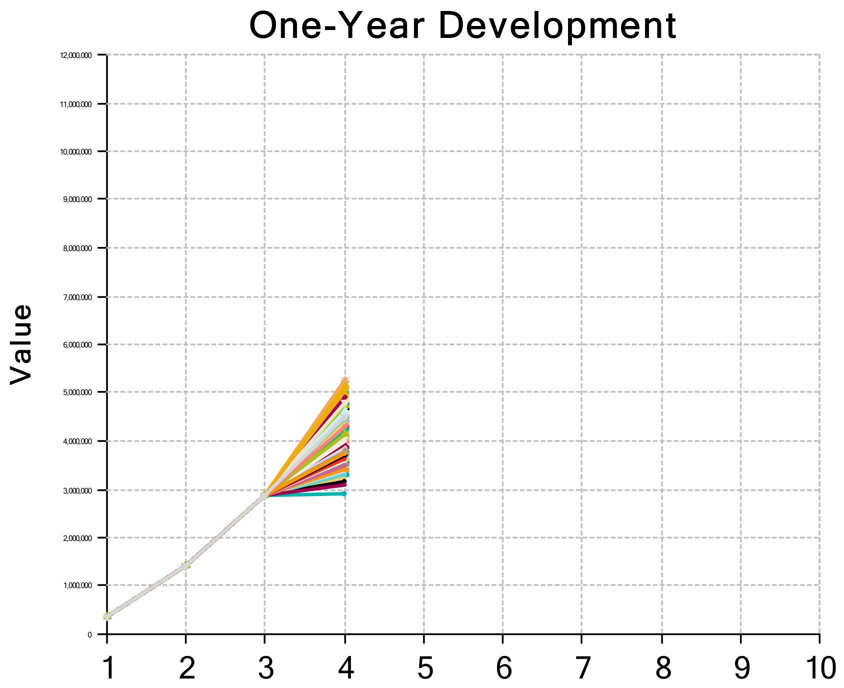
Actuary-in-the-box: bootstrapped Mack's model



Actuary-in-the-box: bootstrapped Mack’s model



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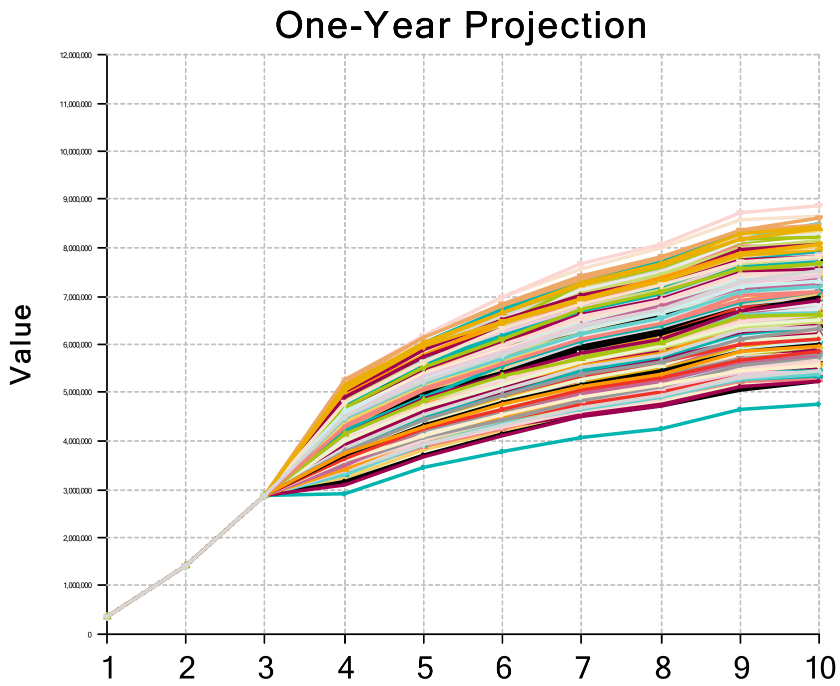
Actuary-in-the-box: bootstrapped Mack's model



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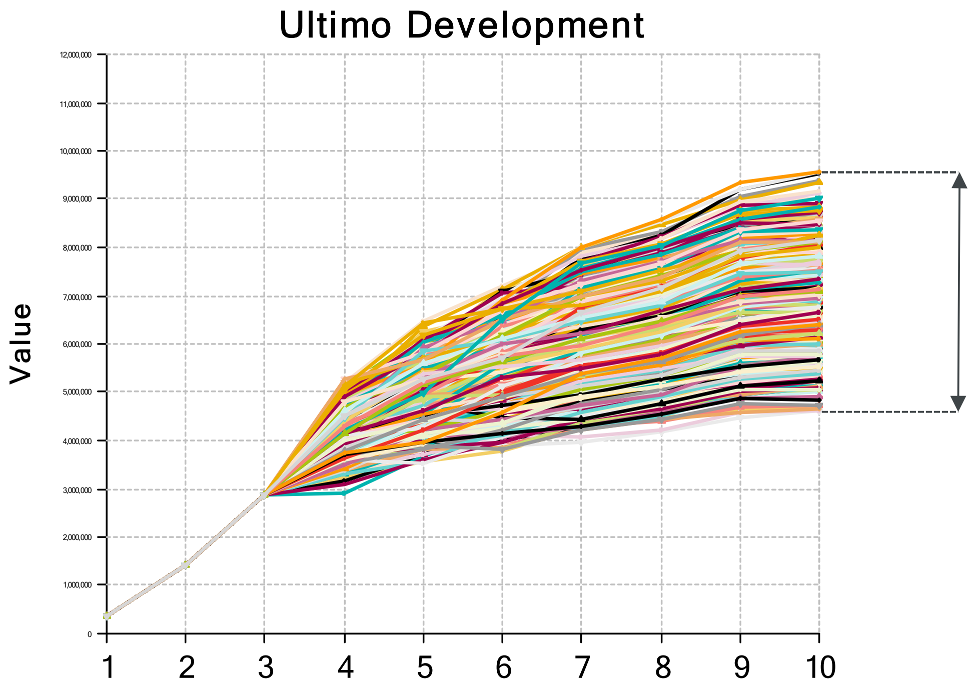
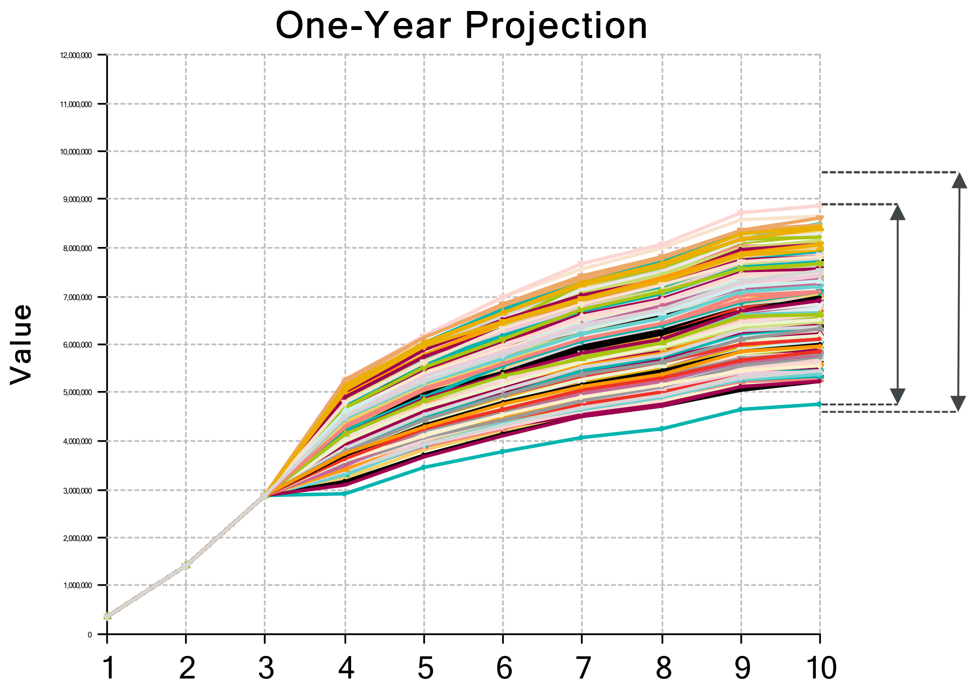
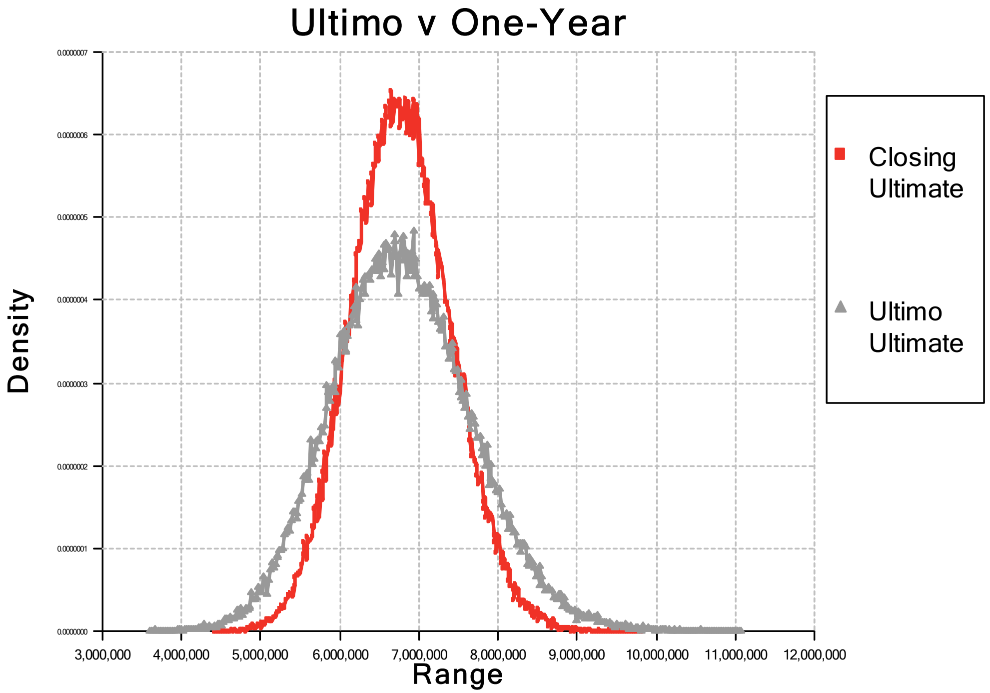
Actuary-in-the-box: bootstrapped Mack's model



Actuary-in-the-box: bootstrapped Mack's model



Actuary-in-the-box: ultimo v one-year



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Actuary-in-the-box: strengths

- Very general procedure
- Flexible
- Automatically consistent with ultimo view – if the same model is used
- Can be adapted to estimate one-year premium risk
- Outputs full distribution of closing reserve, and the CDR
- Can be iterated

Actuary-in-the-box: limitations

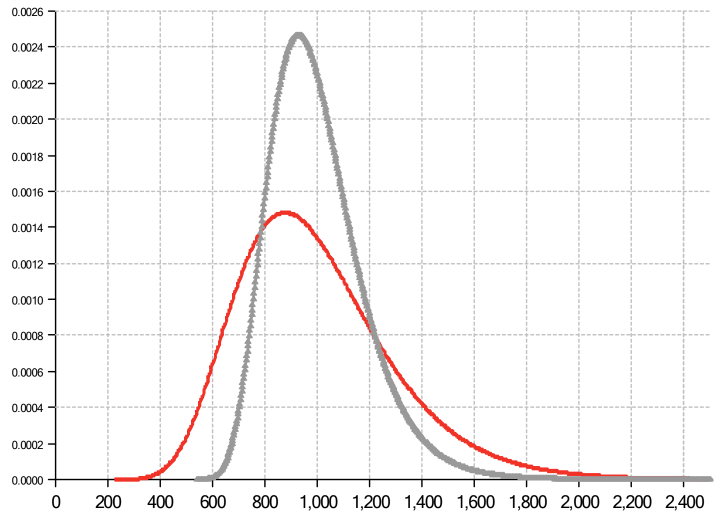
- Cannot incorporate judgement
- Cannot make use of information not in the claims data used by the model
- Often applied to paid claims data, therefore inconsistent with ultimo view if incurred is used for that
- Relatively computationally expensive
- Cannot be applied to model with no notion of claims development
- Cannot be applied to a model with parameters from outwith the model

Emergence Patterns

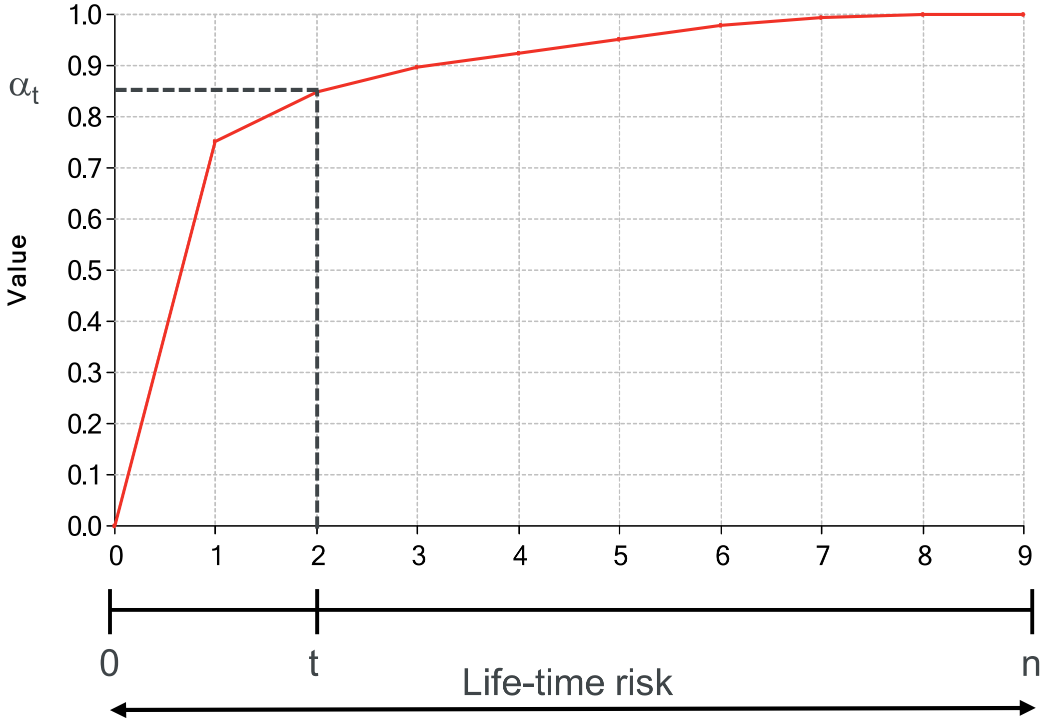
- Simple idea – rescale the ultimo distribution

$$\alpha(X - E[X]) + E[X]$$

- Many different interpretations: Which distribution?
- Some hidden subtleties that need to be understood



Cumulative ultimate life-time emergence



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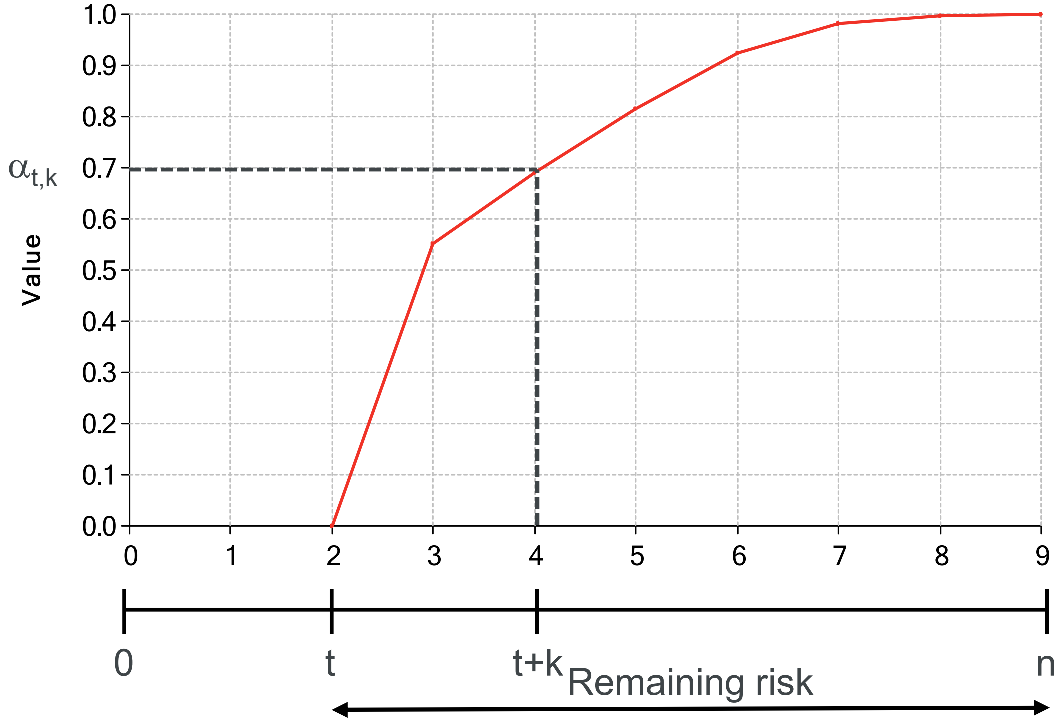
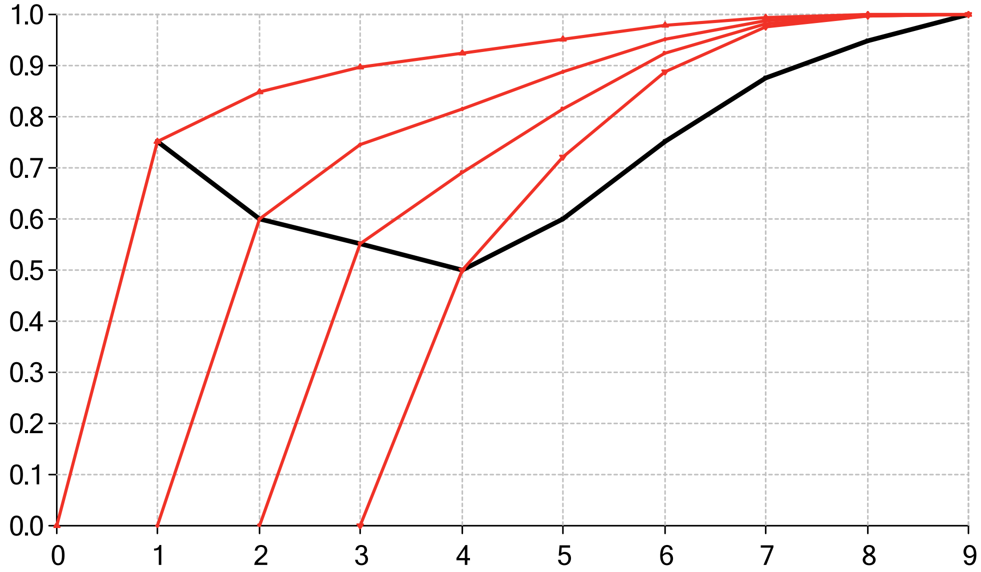
Conditional cumulative ultimate emergence



Conditional cumulative ultimate emergence



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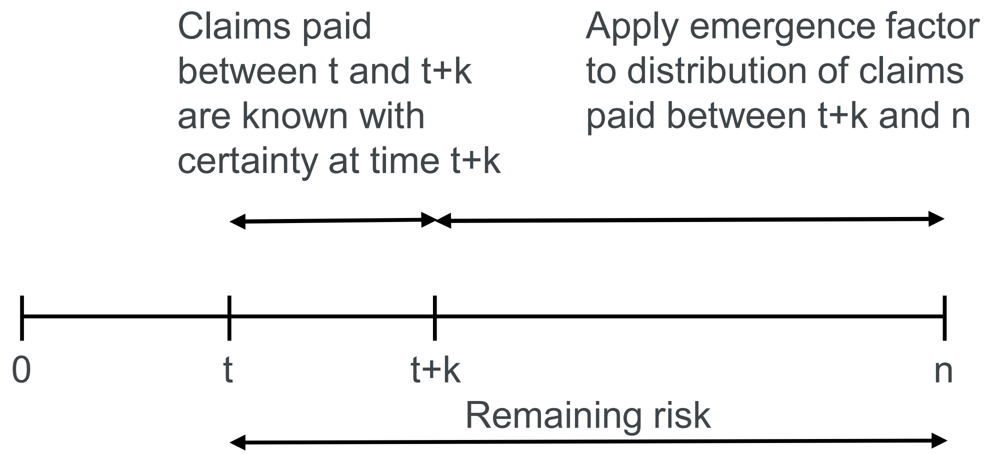
Conditional outstanding emergence



Calibration of emergence factors

- No generally accepted method
- Could use actuary-in-the-box
- Dependence issues mean totals and origin years can't all be matched

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Emergence factors: strengths

- Can be applied to any ultimo distribution
- Easy to understand
- Simple to apply
- Can be parameterised using actuary-in-the-box but is computationally much quicker to apply
- Automatically consistent with ultimo view

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Emergence factors: limitations

- Calibration is difficult
- Different interpretations could lead to confusion
- Inherits ultimo dependencies
- Can only target one risk measure – others might not be correct

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Conclusions

- All these methods have critical assumptions at their heart
- It is unlikely that these assumptions will truly apply
- Purely data derived variability estimates subject to sampling error
- Therefore essential to consider ENIDs
- Other working parties considering wider uncertainties
  - Managing Uncertainty Qualitatively
  - Managing Uncertainty with Professionalism

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Questions

Comments

Expressions of individual views by members of the Institute and Faculty of Actuaries and its staff are encouraged.

The views expressed in this presentation are those of the presenter.

Further reading

- Stochastic claims reserving methods in insurance
  by Mario Wüthrich and Michael Merz
  Book published by Wiley & Sons
- Practitioner's Introduction to Stochastic Reserving
  by Alessandro Carrato, Grainne McGuire, Robert Scarth
  Available from the Institute's website
- Modelling the Claims Development Result for Solvency Purposes
  by Michael Merz and Mario Wüthrich
  CAS E-Forum, Fall 2008 pp. 542-568
- The one-year non-life insurance risk
  by Esbjorn Ohlsson and Jan Lauzeningks, 2009
  Insurance: Mathematics and Economics 45, pp203-208

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