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Setting Long Term Interest Rate Assumptions

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Introduction

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Long Term Interest Rate Forecasting

Setting long term interest rate assumptions is very challenging

- What do we even mean by long term? (should 30 years be different from 50 or 100 years?)
- Many forecasting methodologies at our disposal (different results)
- To what extent can we use history as a guide?
- Not enough data to adequately backtest long term targets (can we extrapolate the efficacy of shorter term back tests?)
- So much uncertainty (economic, social, geo-political etc.)

Once targets are set can they be incorporated into a parsimonious stochastic modelling framework?

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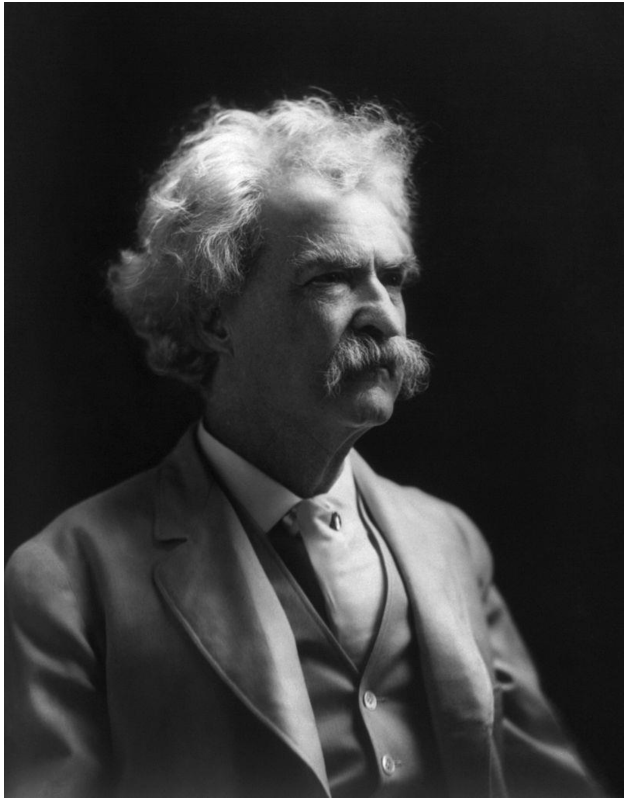
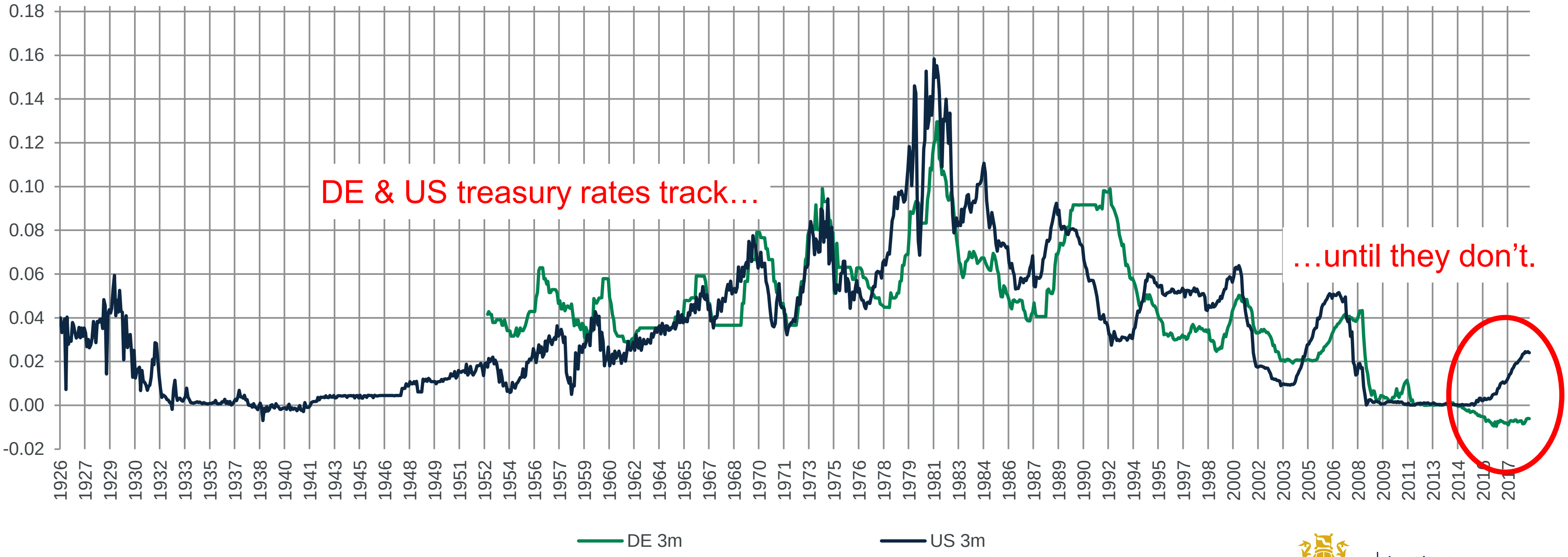
Long Term Interest Rate Forecasting



German and US 3-month Treasury Rates



Source: Conning Inc./Bloomberg

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Long Term Interest Rate Forecasting

We will compare two possible methodologies

- Use forward curve metric
- Econometric forecasts
- Quasi-Econometric

We will consider specifically the forecasts of UK Gilt Yields

- 2018 forecast start date
- Consider a 5 and 30 year forecasting time horizon

Finally we consider whether it is possible to implement such forecasts in a parsimonious stochastic interest rate model

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Forecasting Methods

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Forward Rate

Forward rates are interest rates that can be locked in today for an investment in a future time period.

Let's denote F(t, T, S) the simply compounded forward interest rate prevailing at time t for the expiry T > t, and maturity S > T .

$$F(t, T, S) = \frac{1}{S-T}\left(\frac{P(t,T)}{P(t,S)} - 1\right)$$

where P(t, T) is the T–maturity zero coupon.

Equivalently the continuously compounded forward rate F(t, T, S) is targeted

$$F(t, T, S) = \frac{1}{S-T} Log\left(\frac{P(t,T)}{P(t,S)}\right)$$

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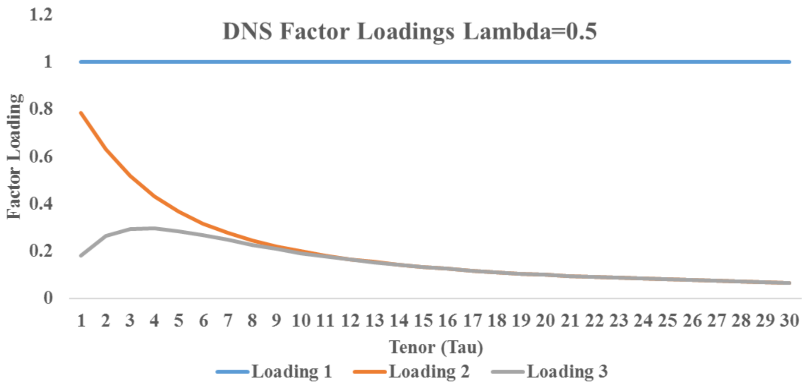
Dynamic Nelson Siegel (Basic Idea)

- Dynamic Nelson Siegel Model (DNS) is a popular framework for analysing and forecasting interest rates
  - Backed by a large body of research (e.g. Diebold and Li 2005/2006)
  - Outperforms other methods on data from multiple economies
  - Parsimonious, intuitive, relatively simple to estimate
- Three factor model
- Fix λ and fit β's to historical yield curves (OLS)
- For example with Gilt yields.....

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left[\frac{1-\exp(-\lambda_t\tau)}{\lambda_t\tau}\right] + \beta_{3t}\left[\frac{1-\exp(-\lambda_t\tau)}{\lambda_t\tau} - \exp(-\lambda_t\tau)\right] + \varepsilon_t.$$

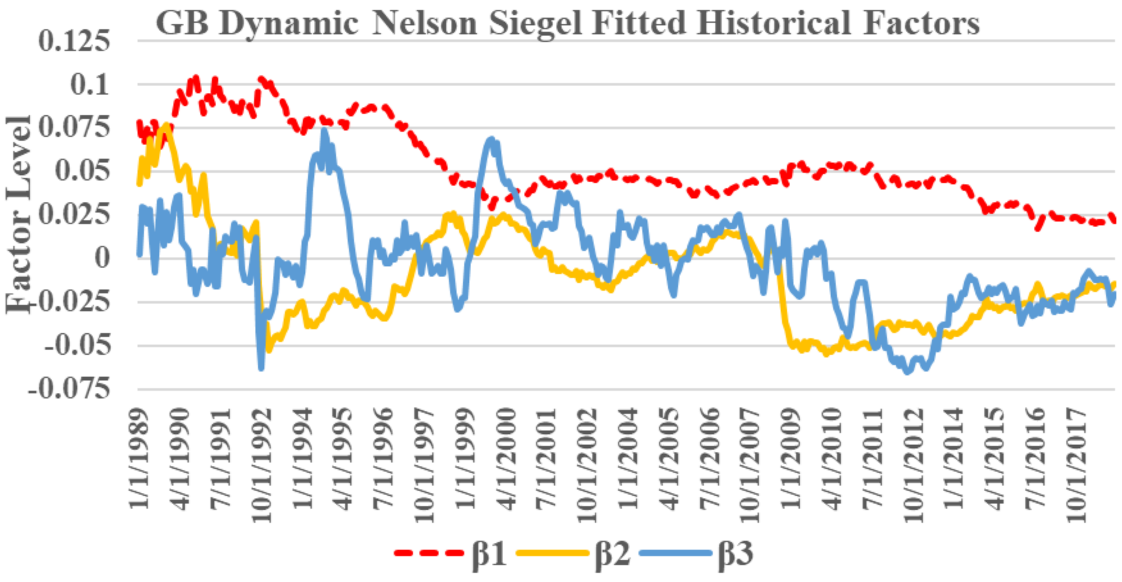
Level | Slope | Curvature



Source: Conning Inc.

Dynamic Nelson Siegel (Basic Idea)

- Factors β are dynamic
- B1,t closely follows the yield levels as expected
- “Shape” factor movements track term structure movements
- Build ARIMA model to forecast future yields curves



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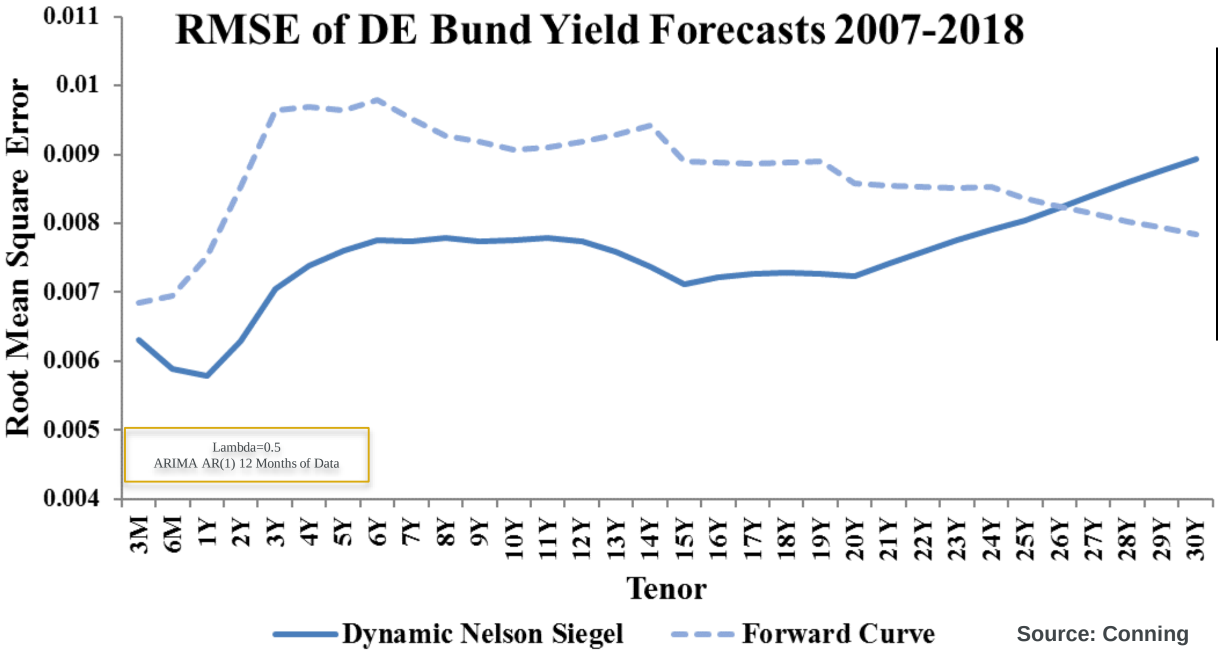
Dynamic Nelson Siegel - Performance



	DNS		*Forward Curve*	
	RMSE	**STDER**	**RMSE**	**STDER**
Tenor				
3 Month	*63.03*	*3.77*	*68.53*	*4.36*
1 Year	*57.91*	*3.58*	*75.24*	*4.91*
5 Year	*75.98*	*4.18*	*96.48*	*5.54*
10 Year	*77.52*	*4.05*	*90.65*	*5.14*
30 Year	*89.31*	*4.53*	*78.46*	*4.36*

Source: Conning Inc.

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Quasi-Economic

Central Banks globally have adopted inflation targeting over the last 30 years

- Inflation and interest rates are related
- Use this relationship to extrapolate a short maturity target
- Apply a term spread estimate to the short term target to get whole curve targets

Economy	*Inflation Targetter*	*Target Inflation or Represenative Value*	*Central Bank Statement*	*Year of Adoption (If Known)*
US	Yes	2%	https://www.federalreserve.gov/faqs/money_12848.htm	2012
GB	Yes	2%	https://www.bankofengland.co.uk/monetary-policy	1992
EU/DE	Yes	<2%	https://www.ecb.europa.eu/mopo/html/index.en.html	1999
CH	Yes	<2%	https://www.snb.ch/en/iabout/monpol/id/monpol_strat#t3	2000
NO	Yes	2%	https://www.norges-bank.no/en/about/Mandate-and-core-responsibilities/Monetary-policy-in-Norway/	2001
SE	Yes	2%	https://www.riksbank.se/en-gb/monetary-policy/	1993
DK	No	<2%	http://www.nationalbanken.dk/en/monetarypolicy/Pages/Default.aspx	N/A
PL	Yes	2.5% +/- 1%	https://www.nbp.pl/homen.aspx?f=/en/onbp/informacje/polityka_pieniezna.html	1998
AU	Yes	2.00% - 3.00%	https://www.rba.gov.au/inflation/inflation-target.html	1993
BR	Yes	4.5% +/-1%	https://www.bcb.gov.br/pec/metas/InflationTargetingTable.pdf	1999
CA	Yes	2% +/-1%	https://www.bankofcanada.ca/core-functions/monetary-policy/	1991
JP	Yes	2.00%	Bank of Japan	2013
KR	Yes	2.00%	https://www.bok.or.kr/eng/bbs/B0000179/view.do?nttId=10047248&menuNo=400063	2001
HK	No	2.50%	https://www.hkma.gov.hk/eng/key-functions/monetary-stability.shtml	N/A

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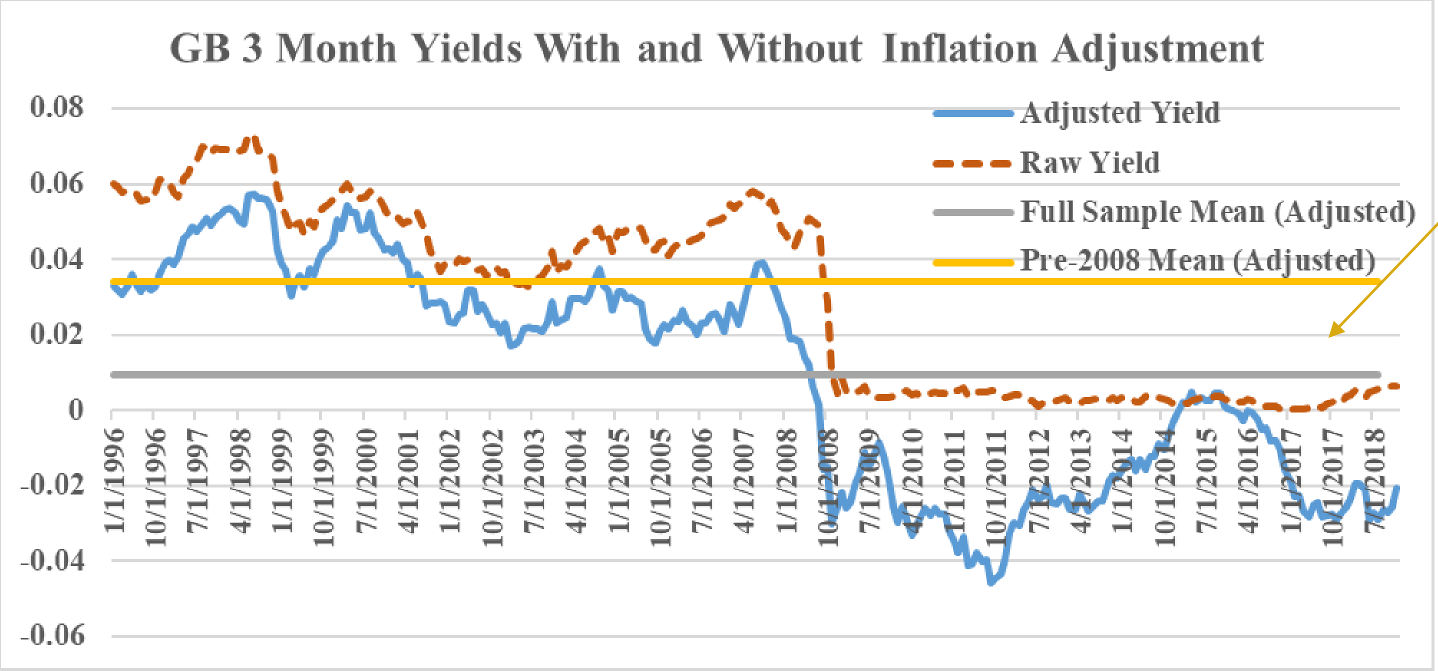
Quasi-Economic



Assumed Real Reference Rate

+

Central Bank Inflation Target

+

Inflation Correction

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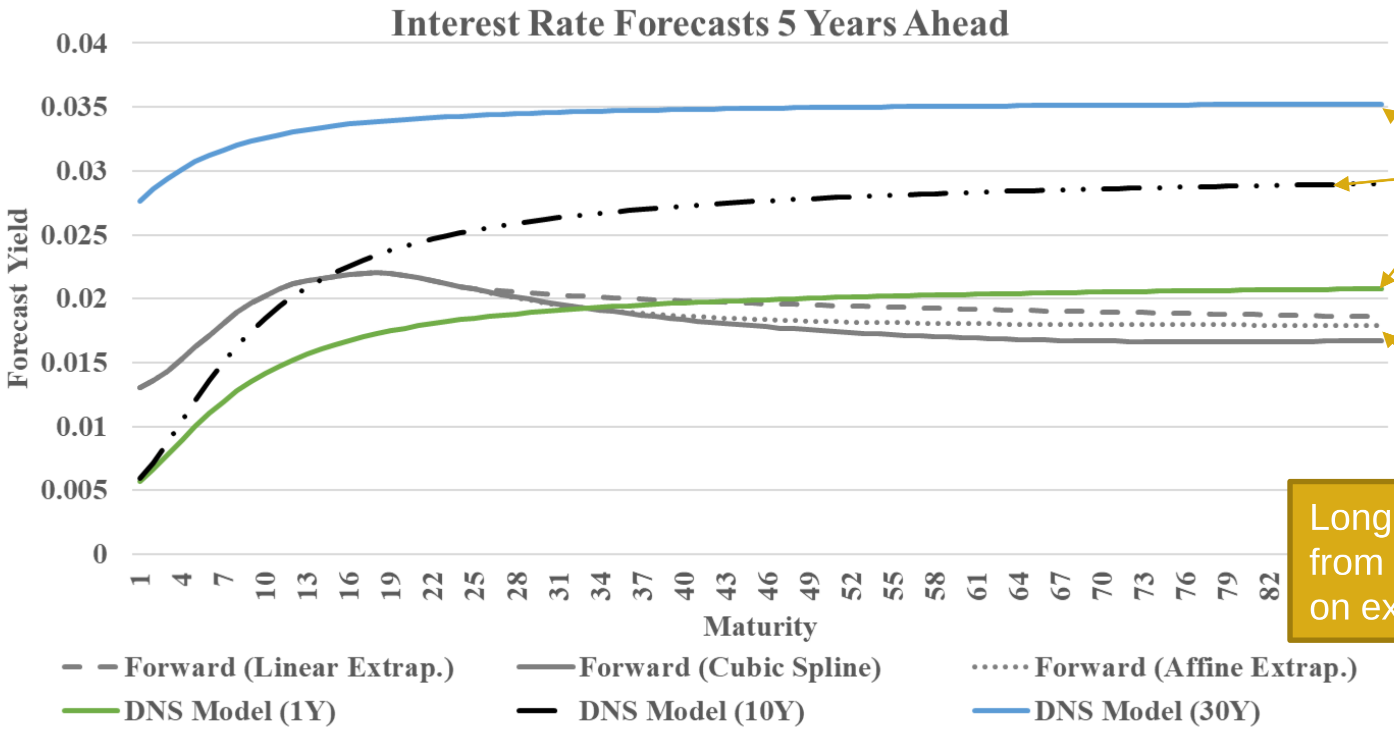
Results and Conclusion

2023 Forecasts – Medium Term



DNS Forecasts dependent on estimation time window used (1,10 or 30 years)

Long maturity forecasts from forwards dependent on extrapolation technique

Source: Conning Inc.

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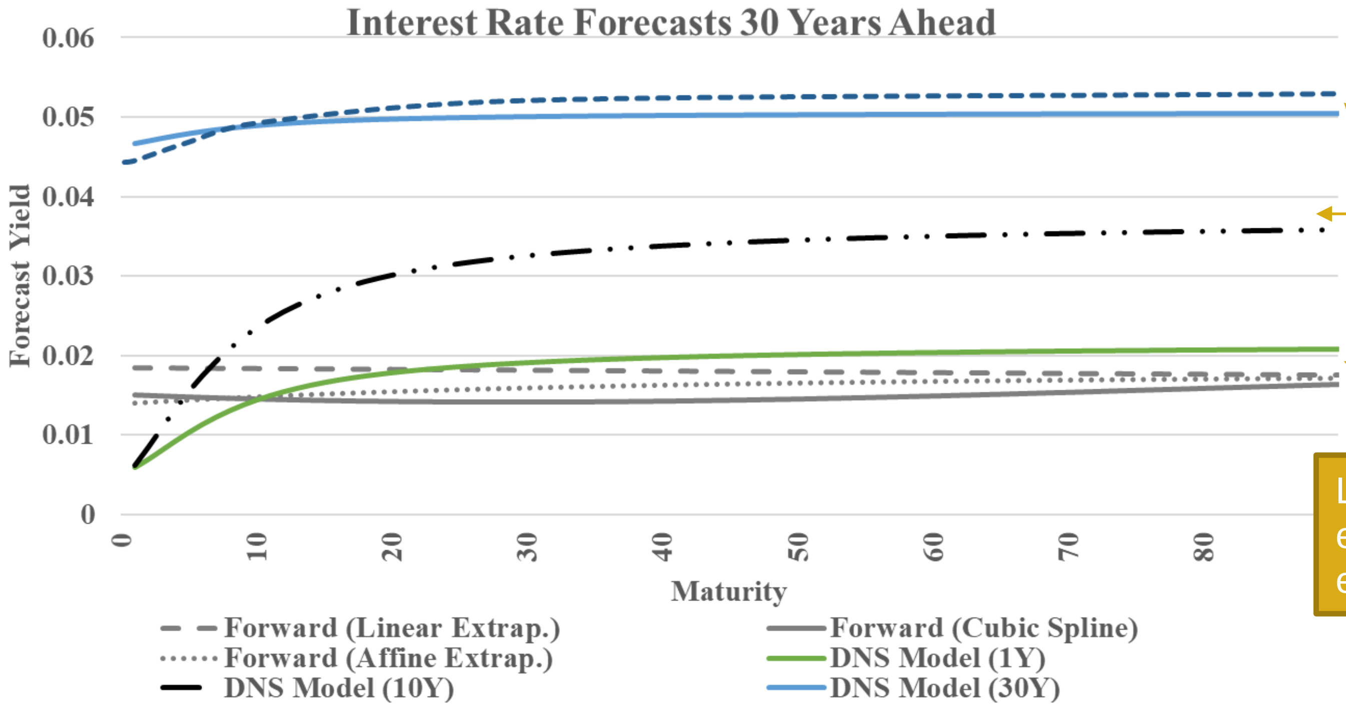
2048 Forecasts – Long Term



DNS Forecast window dependent but shapes are reasonable. But which is most appropriate?

Long term forward forecasts entirely dependent on extrapolation technique!!!!

Source: Conning Inc.

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Practical Considerations - Calibrating to Forecasts

Is it possible to incorporate these forecasts into a stochastic simulation?

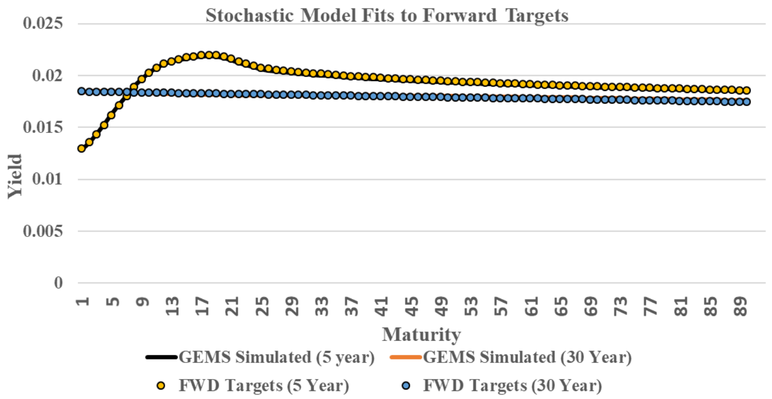
Use an extended 3 Factor CIR model

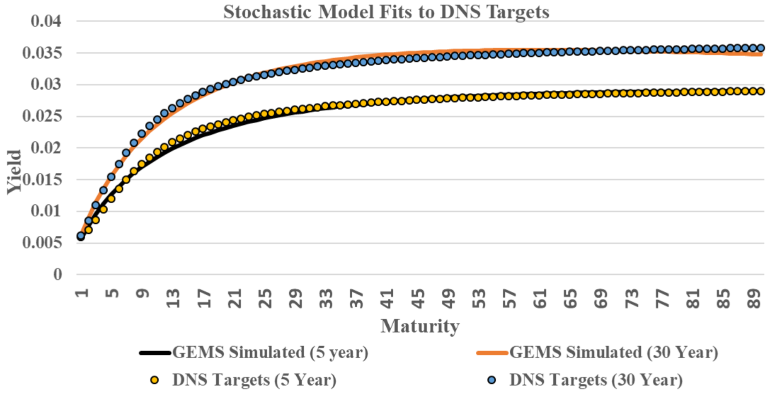
$$dx_i(t) = [\vartheta_i - \kappa_i x_i(t)]\,dt + \sigma_i \sqrt{x_i(t)}\,dW_i(t)$$

$$\mathbb{E}_0^Q\left[\exp\left(-\int_t^T r(\tau)d\tau\right)\right] = e^{\left(-\int_t^T l(s)ds + \vec{A}(\tau) + \vec{B}(\tau)\cdot\vec{x}(t)\right)}$$

Requirements;

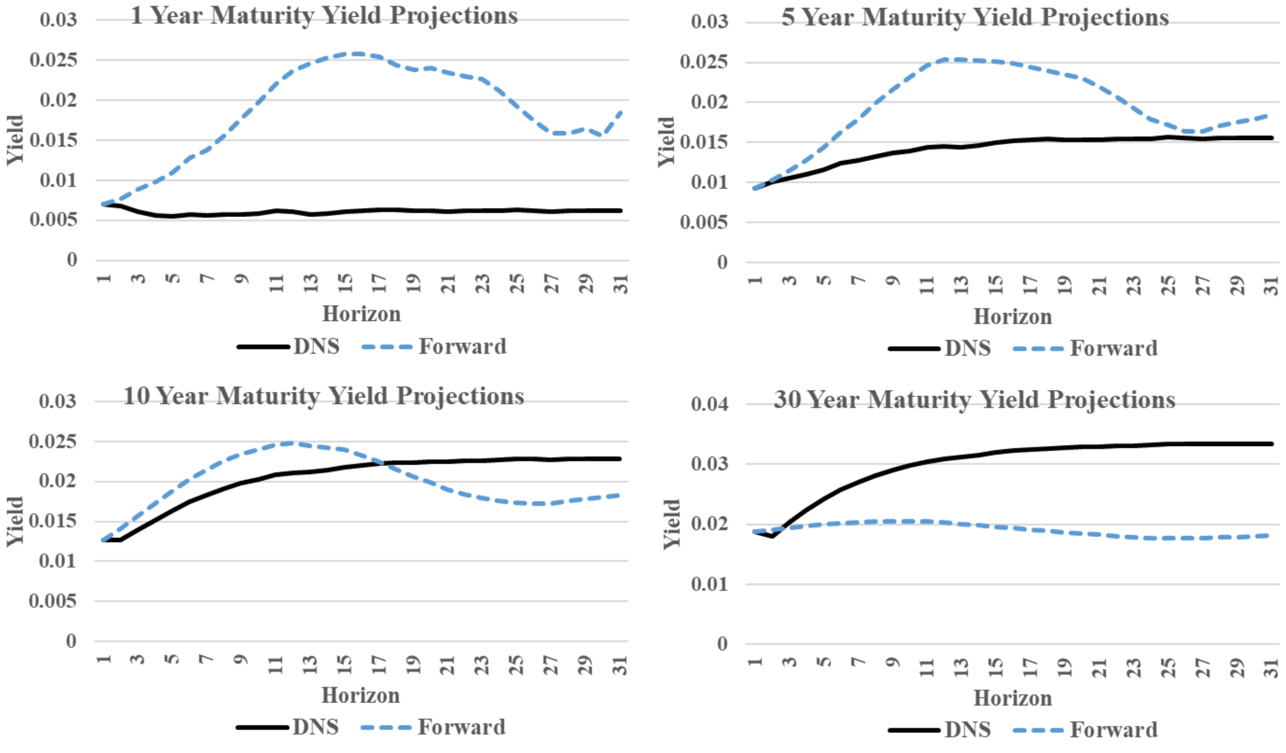
- Must fit the initial curve to 120 years
- Fit DNS or FWD targets at the 5 and 30 year horizon simultaneously
- Must remain arbitrage free





Source: Conning Inc. GEMS ESG

Practical Considerations - Calibrating to Forecasts



Source: Conning Inc. GEMS ESG

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Summary and Conclusions

There is no right or wrong view of future interest rates in the long term

More important that we have a robust, automatable, repeatable, explainable, justifiable approach

Forward Curve

- Fulfils many of the requirements
- Mostly suitable for short and medium term forecasting
- Unclear how to apply it to longer term forecasting

Econometric forecasting or quasi-econometric with DNS model

- Is a valuable tool for setting long term interest rate assumptions
- Data window to use is the only judgment required

A combined approach is possible using the forward curve for short term forecasts and econometric modelling for the longer term.

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