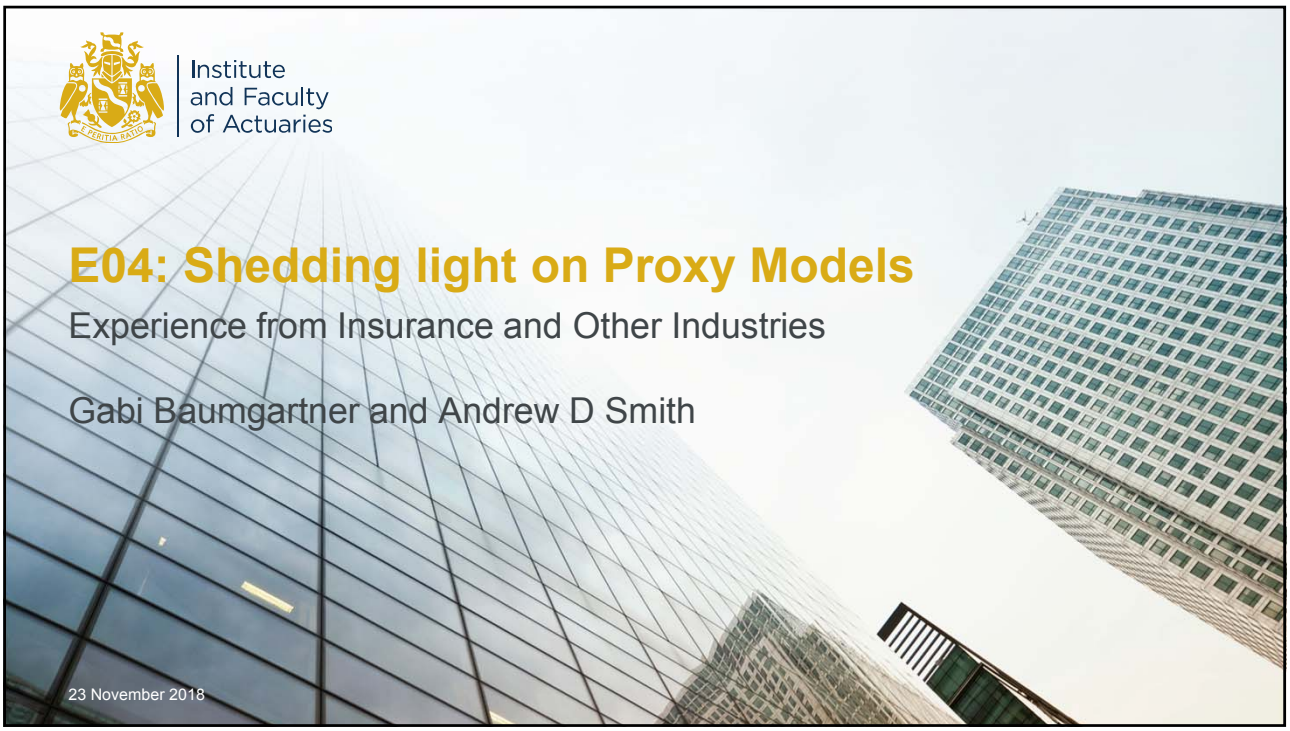

Purpose and content

Purpose

- Summary of industry standards to allow you to benchmark your approach against other life insurers
- Broader view of applications from an academic perspective, explaining areas of divergence between techniques popular in insurance compared to mainstream functional data analysis
- Basis for discussion on where proxy modelling might go next.

Content

- Introductions
- Proxy modelling in the life insurance industry
- Proxy modelling in other industries
- Conclusions
- Q&A: As we go along.

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Proxy modelling in the life insurance industry

Introduction

What are proxy models?

- A proxy model estimates the outputs of a more complex model, e.g. a detailed actuarial cash flow model.
- It produces outputs in a fraction of the time needed for the complex model.
- Proxy models used by life insurers typically take the following form:

Example with two risks, $x = (x_1, x_2)$, up to quadratic terms:

$$f(x_1, x_2) = c + \underbrace{a_1 x_1 + a_2 x_2}_{\text{Linear relationship}} + \underbrace{a_{12} x_1 x_2}_{\text{Interaction between risks}} + \underbrace{a_{11} x_1^2 + a_{22} x_2^2}_{\text{Non-linear relationship}}$$

What are they used for?

Situations where a large number of runs of a detailed model are required. For example:

- Capital quantification (e.g. internal model SCR, economic capital)
- Regular solvency monitoring
- Hedging
- Asset allocation

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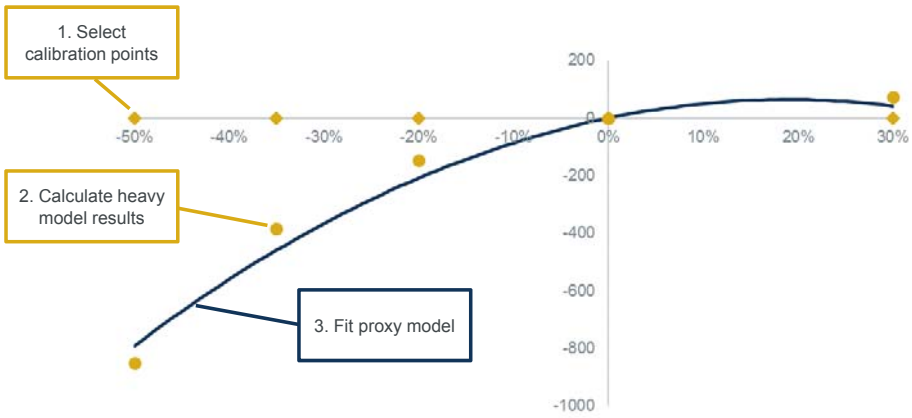
How are proxy models calibrated? – Illustrative example



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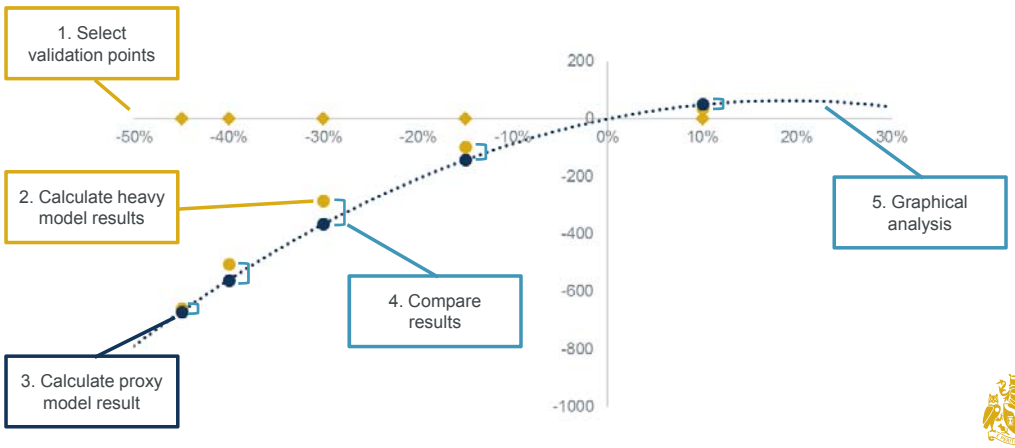
How are proxy models validated? – Illustrative example



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Deloitte's proxy modelling survey 2018

Refresh of Deloitte's 2016 proxy modelling survey

Presents an overview of the approaches taken in 2018

Nine participants covering mutuals and FTSE100 companies

Seven use the proxy models in their Internal Models.

Others are preparing for IMAP or use them for internal purposes

The following slides summarise the results of this survey.

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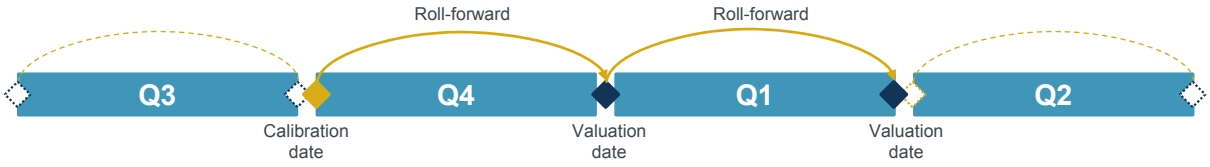
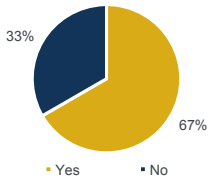
Frequency and timing of calibration

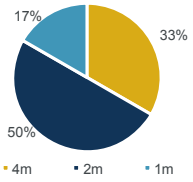
Example for half-yearly calibrations



Roll-forward applied



Roll-forward period



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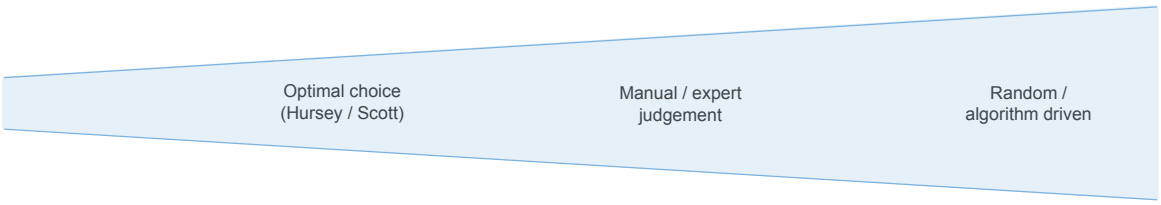
Calibration of proxy models

Choice of calibration points



	Optimal choice	Manual / Expert Judgment	Random / algorithm driven
Popularity	Low	High	Medium
Main use	Roll-forward	All business	Assets, annuities, WP (LSMC)
Typical fitting method	Exact fit	Least squares	Least squares

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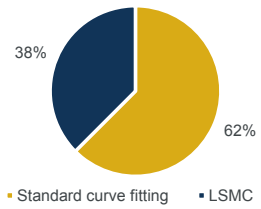
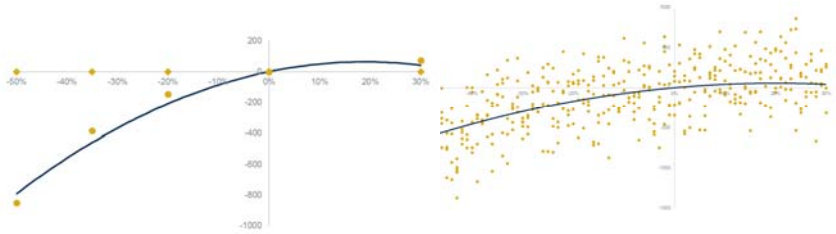
Calibration of proxy models for WP business

Standard curve fitting vs LSMC for WP



Standard curve fitting vs .LSMC illustration



LSMC approaches are being considered by some companies but its uptake has been slow due to significant implementation and validation effort and fears of “black-box” approach.

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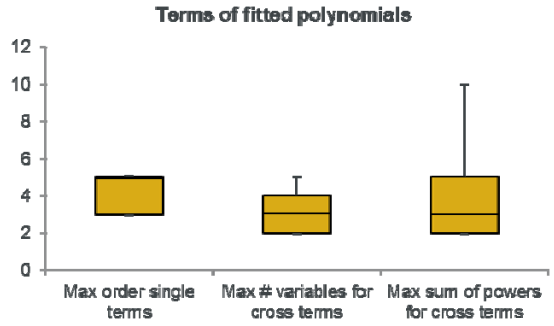
Calibration of proxy models

Complexity of proxy models



Legend:

- *Max order single term: Highest n^{th} order used for a single term x^n.*
- *Max # variables for cross terms: Maximum number of risks that can be included for cross terms.*
- *Max sum of powers for cross terms: Highest n^{th} order used for cross terms.*

Most companies aim to keep the polynomials to the simplest structure as possible.

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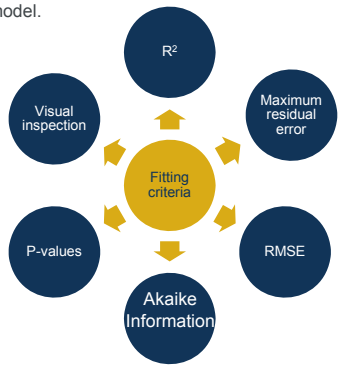
Calibration of proxy models

Fitting criteria

- A range of fitting criteria are used across the firms in our survey
- Over half of firms opt for a range of different fitting criteria, although some rely on only one goodness-of-fit test for fitting the model.



Fitting tolerance

- Firms applying standard curve fitting tend to use tolerances based on absolute and average fitting errors (e.g. Root Mean Square Error, aka RMSE) as opposed to statistical tests.
- Statistical tests are less meaningful in this context due to the relatively small number of fitting points and the residuals not being independent.

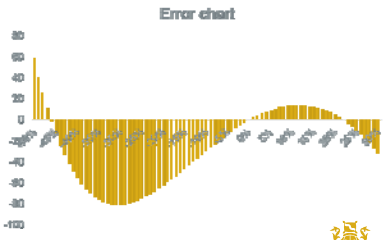


Illustration – continuation of example

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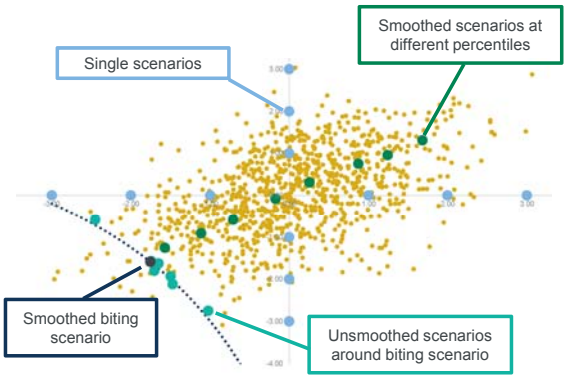
Validation of proxy models

Validation approach

- Out-of-sample testing plus
- Graphical analysis (e.g. bias in residuals)



Test criteria

- Variety of goodness-of-fit criteria
- A number of firms use a range of criteria, e.g.:
  - Individual errors
  - Absolute (average) errors
  - Statistical indicators
  - Graphical analysis
  - Ranking tests

Date	Validation intensity
Calibration date	Extensive validation including one-off investigations
Valuation date	Lighter touch validation due to Working Day Timetable constraints

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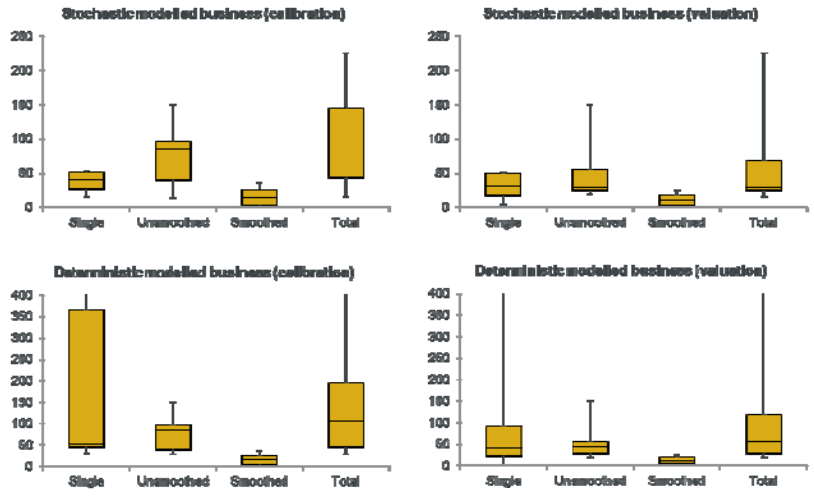
Validation of proxy models

Number of out-of-sample tests on calibration (L) and valuation (R) dates.



Typical range of number of tests and average of number of tests within this range

Date	Calibration	Valuation
Stochastic	50-120 (85)	20-50 (32)
Deterministic	50-130 (95)	30-65 (43)

Note: Number of tests excludes single risk stresses)

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Validation of proxy models

What if validation scenarios fail the tests?

- Respondents consider a range of actions:

Review inputs and correct these for any errors, update fitting parameters (e.g. polynomial terms allowed) and refit.

Investigate the cause of errors and explain why these errors are immaterial (if they are). No further action will be taken.

Adjust the SCR. This is typically based on the 99.5th smoothed scenario and errors of scenarios around this smoothed scenario.

- Many of the respondents do not have a prescribed mechanism of calculating SCR adjustments or have restrictions in place with the PRA (e.g. the adjustment cannot be smaller than £xm).

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Improvements of proxy models

1st line comments

- In general the respondents are satisfied with the quality of their proxy models, but several areas for development were identified, including:
  - Refining the interaction terms
  - Increasing the number of fitting points used without slowing down the loss-function fitting process
  - Reducing the fitting errors observed for specific risks
- Most respondents have no planned developments in the near future. Some highlighted that they intend to streamline the process using increased automation to accommodate an increased number of runs.

2nd line comments

- Quality of fit
- Implication of fitting errors on SCR
- Cycle of improvements

Note that the 2nd line and PRA comments are not first hand and filled in by the 1st line team

PRA comments

- Cycle of improvements
- Extensiveness and coverage of the out-of-sample testing
- Definition of goodness-of-fit criteria

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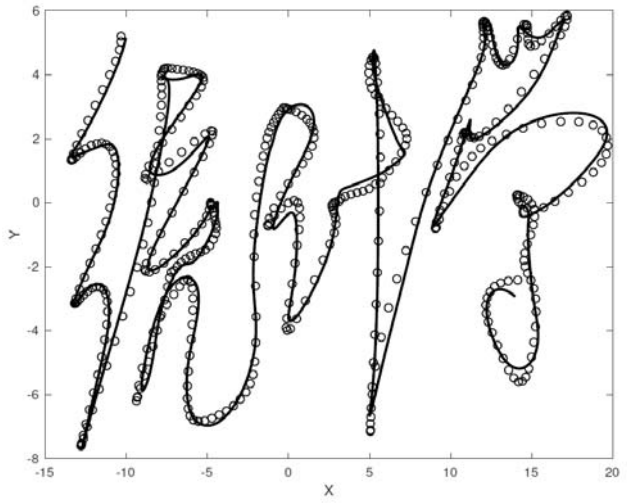
Proxy modelling in Other Industries

Proxy Model = Functional Data Analysis

- Applications to:
  - Handwriting
  - Athletics
  - Ballistics
  - Epidemic prediction
- Common thread:
  - Approximate ODEs



Source:
Ramsay & Silverman, 2002
See R package DATA2LD.

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Curves to Fit = Choice of Basis Functions

In UK life actuarial work:

- Polynomials the most popular choice
- Because?
  - Software easy
  - Peers do too
  - Weierstrass theorem
- But
  - Stone's generalisation applies to many other function families too
  - And speed of convergence matters

In other industries:

- FDA mostly uses splines
- Eg cubic spline basis

$$g(x) = \max\left\{3\frac{|x-k|}{h} - 1, \left(3 - \frac{|x-k|}{h}\right)\frac{(x-k)^2}{h^2}\right\}$$

- Knots *k*, bandwidth *h.*
- Cubic for $|x| \le k$, linear for $|x| \ge k$
- Exact fit #knots = #calibration points
- LSMC also possible

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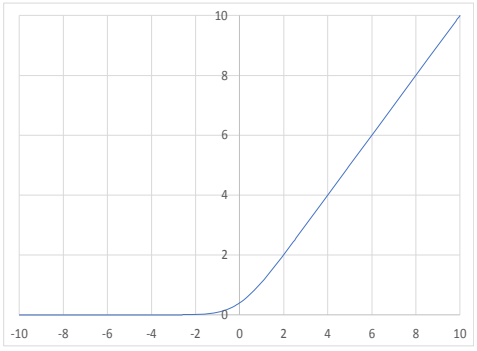
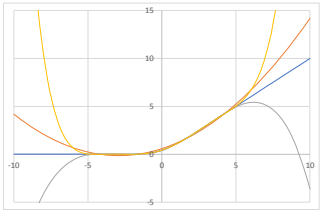
Example: Option Pricing with Bachelier's Formula

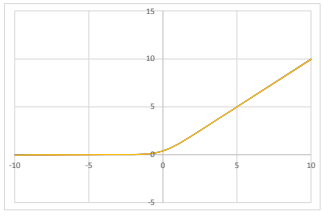
Least squares fits on [-5,5]



Option formula with σ = 1



Polys of order 2,4,6



Splines with 3, 5, 7 knots

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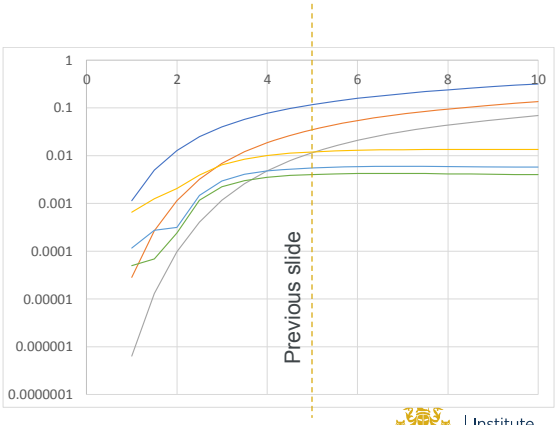
Calibration Point Placement = Fitting Bandwidth

Relationship: Bandwidth and RMSE

- Larger bandwidth = larger RMSE
- For small bandwidth, polynomials win
  - Because we are fitting an analytic function
  - Approximating Taylor expansion
- For large bandwidth, splines win
  - Non-exploding behaviour for large moneyness



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What's Going On? Smoothing Gamma

- Everyone agrees fitting straight line is easy.
- Difficulty comes because of the second derivative, convexity (in finance this is gamma; in physical applications this is acceleration)
  - Cubic splines approximate gamma with a piecewise linear function,
  - while for polynomial approximations, the fitted gamma is still a polynomial
- This distinction underpins the theory of how good the fit can be.
- For most financial (and physical) applications, Gamma is close to zero except in a particularly interesting range where options flip in or out of the money.
- Could robotics automate the search for significant gamma?

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Trade-off between spanning and sampling errors

Spanning Error: RMSE

- For polynomials and smooth functions (with finite or infinite Taylor radius of convergence), the RMSE falls exponentially with the order of the polynomial
- For functions with jumps or kinks, RMSE convergence is inverse polynomial in order.
- For splines, RMSE convergence is generally inverse polynomial in the number of knots.

Sampling Error (Stochastic models)

- Sampling error is generally proportional to $N^{-1/2}$
- How to increase number of parameters in model fit as number of simulations increased?
- To balance sampling and spanning error, number of parameters generally looks like a power of N between 0 and 1
- Unless you are within the Taylor radius in which case number of parameters grows like log N.
- Akaike Information Criteriod generally adds too many parameters compared to this, but firms are overriding this.

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Conclusions

Conclusions

- Convergent market practice in proxy models
  - Polynomial basis functions, fitted by least squares
  - No dominant approach to parsimony / over-fitting
  - Number of out-of-sample validation stresses
- Experience from other industries suggests more lessons can be learned
  - Polynomials may not be the best choice; splines popular elsewhere
  - Articulating the difficult points: high gamma regions
  - Choice of stress placement (outer fitting distribution) is important
  - Better estimates of spanning and sampling error.

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Questions

Comments

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