

International Mortality and Longevity Webinar

# Longevity trend risk over limited time horizons

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Institute  
and Faculty  
of Actuaries

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## 1. Theory v. practice

1. Theory v. practice
2. About Longevitas

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2. About Longevity
3. The longevity-risk problem

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4. Multi-year view

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5. Deferred annuities
6. VaR v. CTE
7. Managing longevity risk
8. Conclusions

# 1 Theory v. practice

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*“practice suggests problems essentially new for science and thus challenges one to seek quite new methods. And if theory gains much when new applications or new developments of old methods occur, the gain is still greater when new methods are discovered”*

**Chebyshev [1856]**

# 2 About Longevity

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- Research partnership with Heriot-Watt University.



Actuarial  
Research Centre

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- Experience analysis and mis-estimation:



- Experience analysis and mis-estimation:



- Stochastic mortality projections and capital:



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- Stochastic mortality projections and capital:



- Rating pension schemes:



# 3 The longevity-risk problem

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*“Whereas a catastrophe can occur in an instant, longevity risk takes decades to unfold”*

**The Economist [2012]**



- Longevity trends emerge slowly over many years. . .



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- How do you reconcile the two?



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. . .but insurance regulations view risks as  
single-year catastrophes.
- How do you reconcile the two?
- How do you fit a long-term risk into a short-term  
view?

# 3 Trend v. one-year view

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- Create new models just for this specific task, e.g Plat [2011] and Börger [2010], or

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- Create a framework for existing projection models like Lee and Carter [1992], Cairns et al. [2006].

# 3 Trend v. one-year view

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Solution from Richards et al. [2014]:

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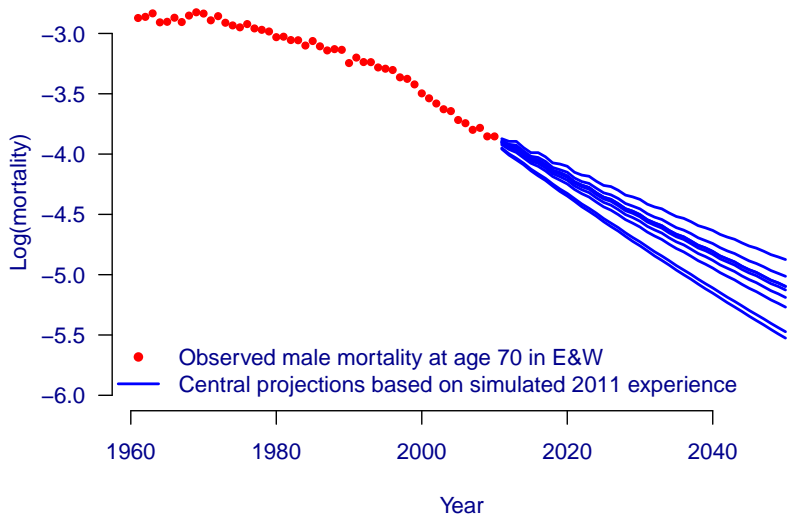
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2. Use model to simulate next year's experience data.

Solution from Richards et al. [2014]:

1. Pick a model and fit it to real data.
2. Use model to simulate next year's experience data.
3. Refit the model using real and simulated data.



# 3 Sensitivity of forecast



Source: Lee-Carter example from Richards et al. [2014].

Solution from Richards et al. [2014]:

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6. Repeat (2)–(5) a few thousand times.

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Sample of liability values  $\{x_1, x_2, \dots, x_m\}$ .

- Our unknown liability is  $X$  (say).

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- VaR-style solvency capital:

$$\left( \frac{Q_\alpha}{\mathbb{E}[X]} - 1 \right) * 100\%$$

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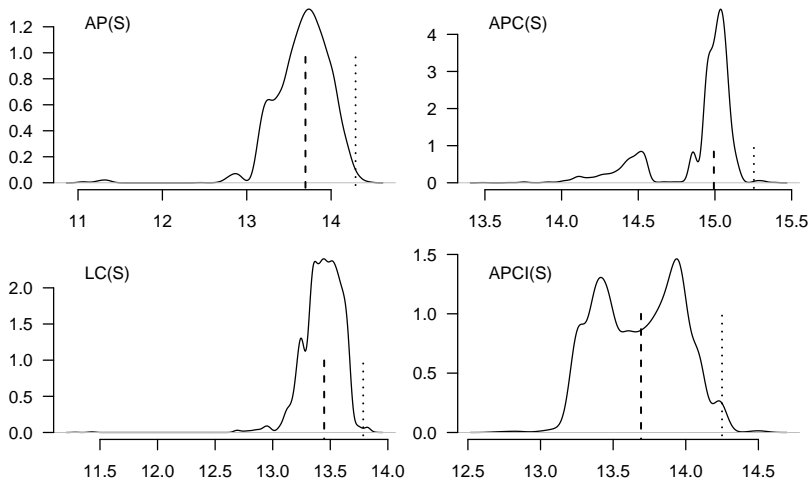
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- Estimate  $\mathbb{E}[X]$  from mean of sample.
- Estimate  $Q_\alpha$  from sample using Harrell and Davis [1982].

# 3 One-year liability densities



Annuities payable to male aged 70. Means marked with dashed line and  $Q_{99.5\%}$  marked with dotted line.

Source: Richards et al. [2017, Table 4].



- Wide variety of density shapes.



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  - ⇒ need to use multiple models...
  - ... and exercise *actuarial judgement*.

# 4 Multi-year view

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- Richards et al. [2014] was for one-year insurer solvency.

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- The same methodology has other applications. . .

Medium-term business planning:

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- Ten-year “glide path” to buy-out for pension schemes.

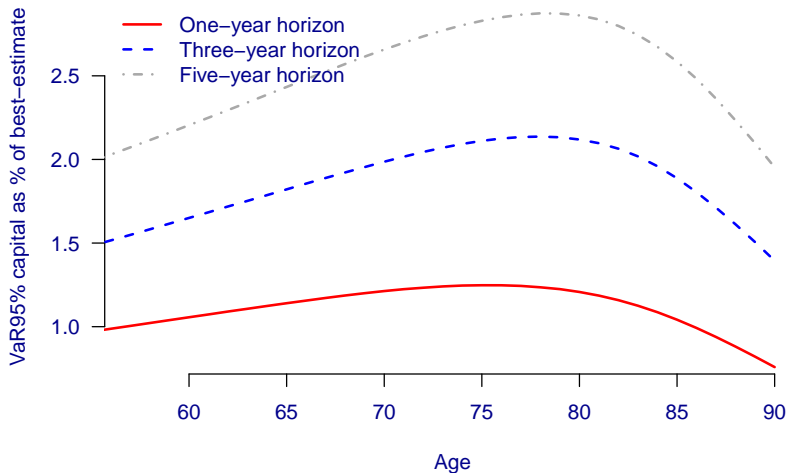


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- Extend time horizon to 3–5 years.

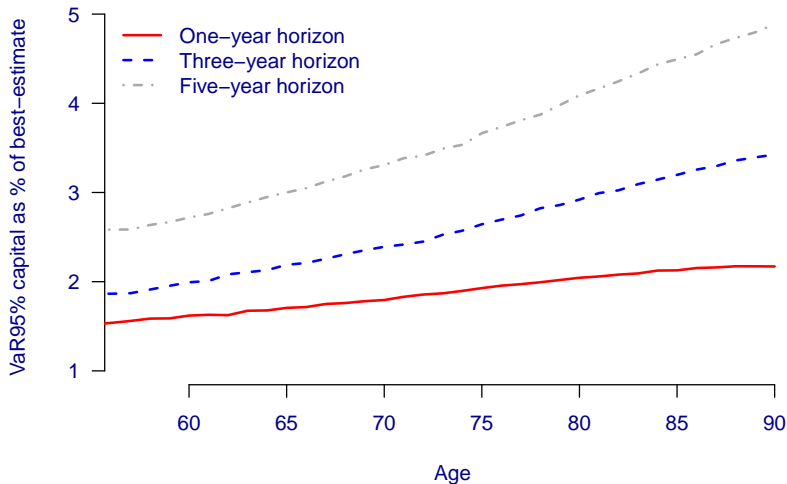
- Take one-year framework from Richards et al. [2014].
- Extend time horizon to 3–5 years.
- Reduce p-value to, say, 95%...

# 4 Females, Lee-Carter model

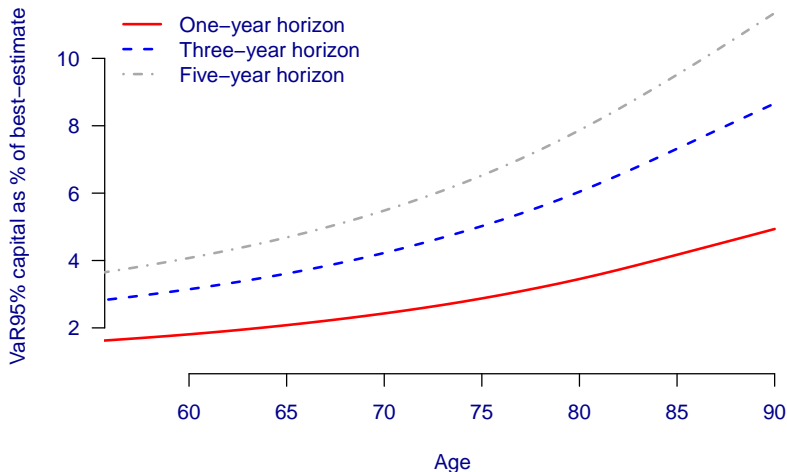


Immediate annuities under Lee-Carter model. UK data ages 50–104, 1971–2016

# 4 Females, APC model

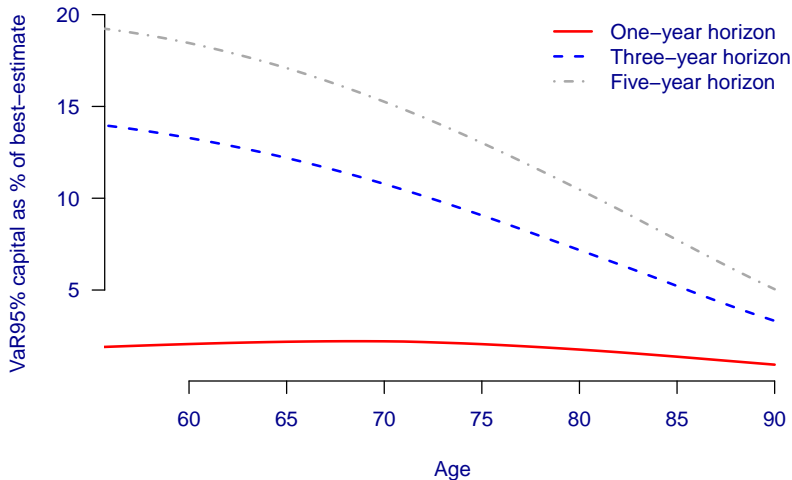


Immediate annuities under APC(S) model. UK data ages 50–104, 1971–2016



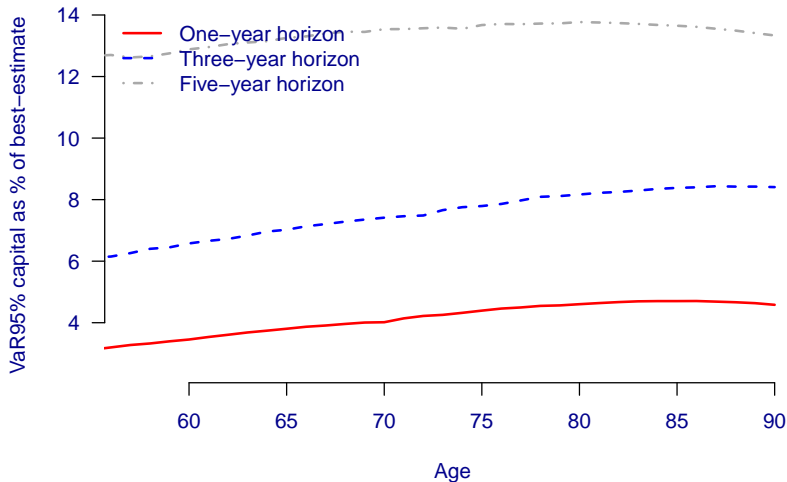
Immediate annuities under M5(S) model. UK data ages 50–104, 1971–2016

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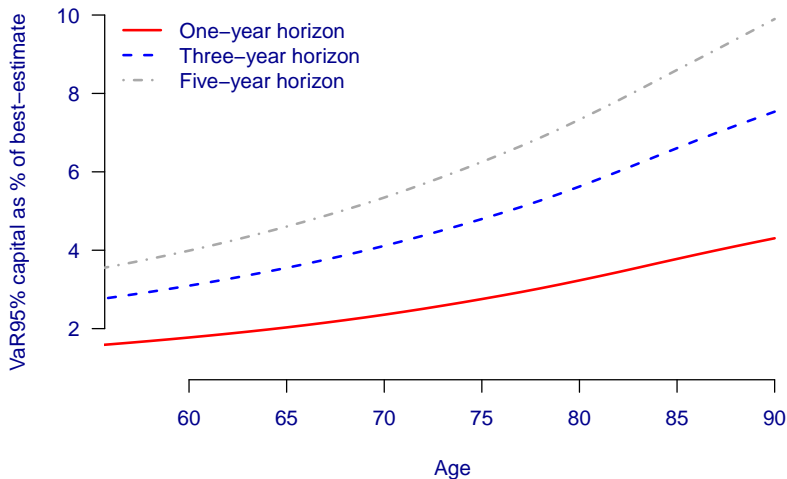
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Immediate annuities under APC(S) model. UK data ages 50–104, 1971–2016





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... and exercise actuarial judgement (*again!*).

# 5 Deferred annuities

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- What about deferred annuities and pensions?



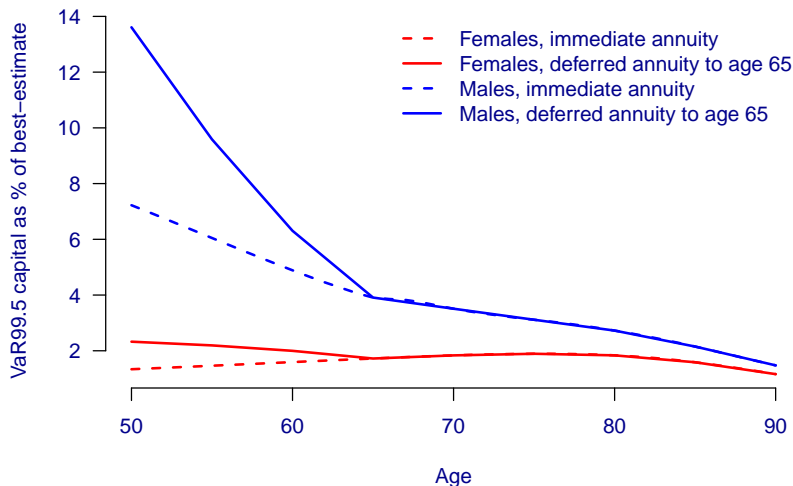


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- What about deferred annuities and pensions?
- Assume payment from age 65.



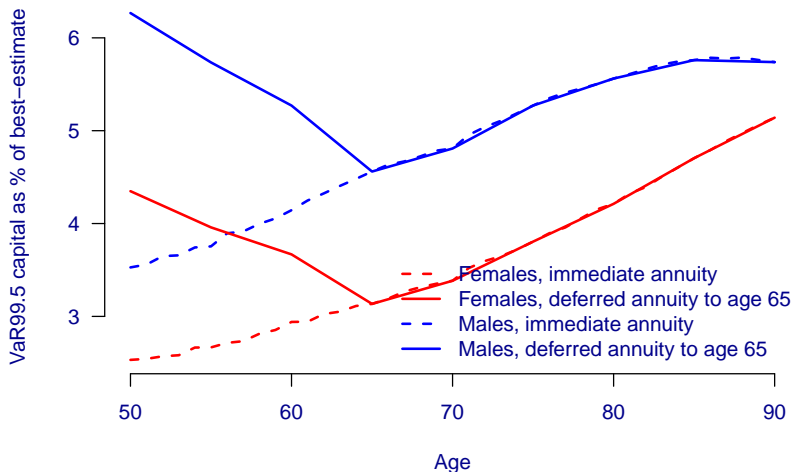
- Most published work concerns immediate annuities and pensions in payment.
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- Compare VaR99.5% solvency capital for immediate and deferred annuities.

# 5 Solvency capital, Lee-Carter

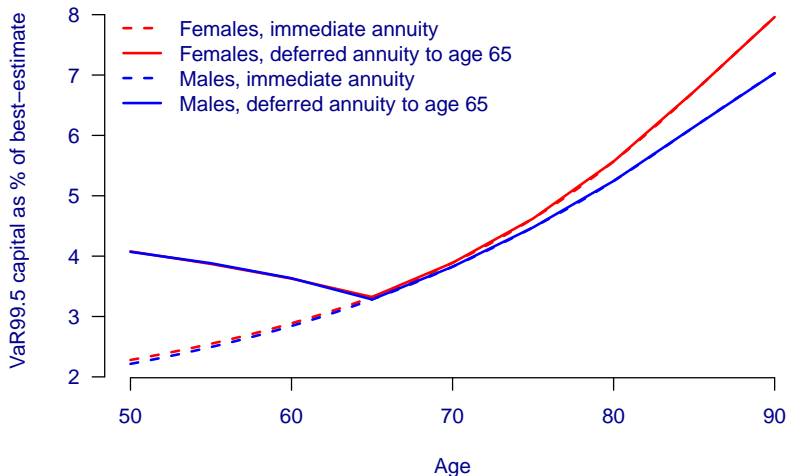


Deferred and immediate annuities under Lee-Carter model. UK data ages 50–104, 1971–2016

# 5 Solvency capital, APC model



Deferred and immediate annuities under APC(S) model. UK data ages 50–104, 1971–2016



Deferred and immediate annuities under M5(S) model. UK data ages 50–104, 1971–2016

- Depending on age, solvency capital for deferred annuities can be double that of annuities in payment.

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- Sharp differences in solvency capital by gender.





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- VaR-style solvency capital:

$$\left( \frac{Q_\alpha}{\mathbb{E}[X]} - 1 \right) * 100\%$$

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- CTE-style solvency capital:

$$\left( \frac{\mathbb{E}[X|X \geq Q_\alpha]}{\mathbb{E}[X]} - 1 \right) * 100\%$$

where  $Q_\alpha$  is  $\alpha$ -quantile of  $X$ , i.e.  $\Pr(X < Q_\alpha) = \alpha$ .



- How does VaR capital compare to CTE capital?



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- $CTE_{\alpha} > VaR_{\alpha}$  (obviously!)

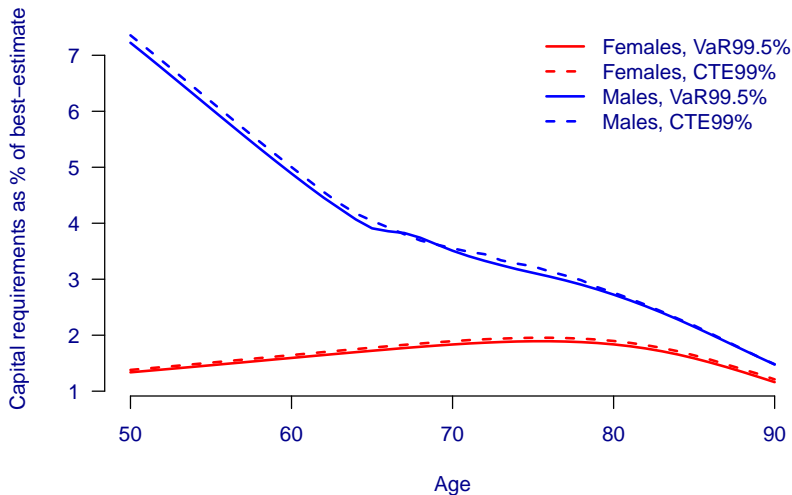
- How does VaR capital compare to CTE capital?
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- But how does  $VaR_{99.5\%}$  compare to  $CTE_{99\%}$ ?



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- But how does VaR99.5% compare to CTE99%?
- Can calculate both from same sample...

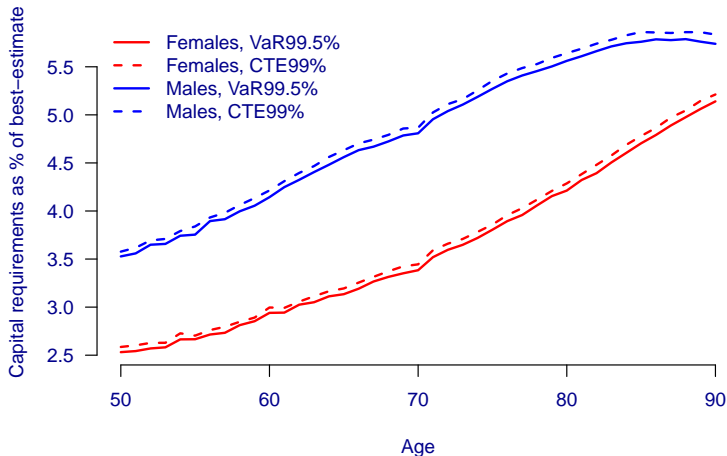


# 6 UK, Lee-Carter model

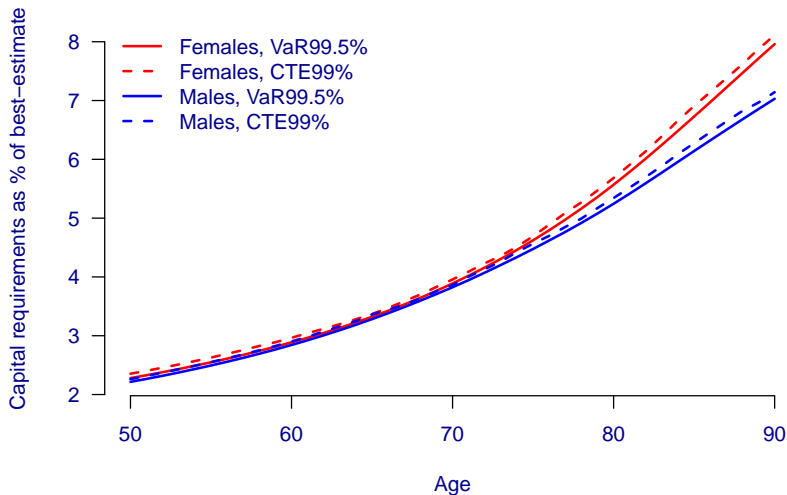


Annuities in payment under Lee-Carter model. UK data ages 50–104, 1971–2016

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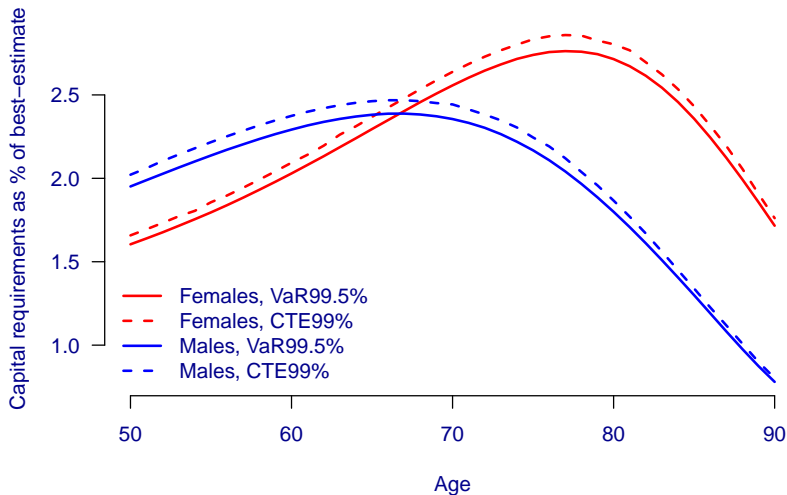


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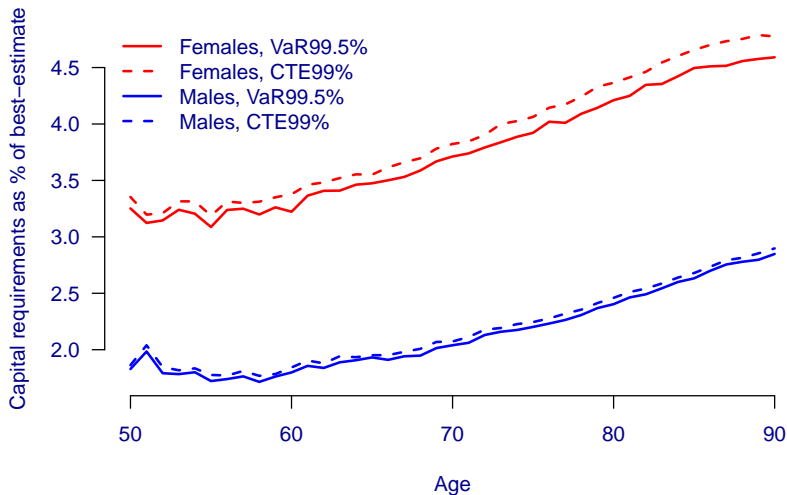
Annuities in payment under M5(S) model. UK data ages 50–104, 1971–2016

# 6 Netherlands, Lee-Carter

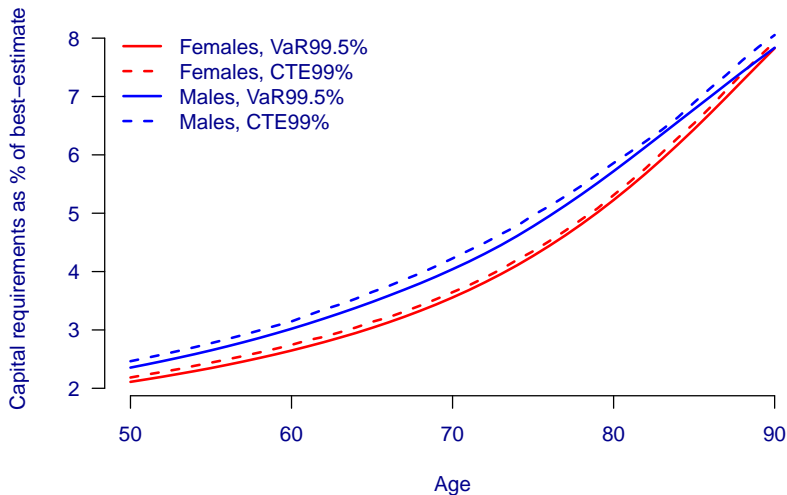


Annuities in payment under LC(S) model. Netherlands data ages 50–104, 1971–2016

# 6 Netherlands, APC model



Annuities in payment under APC(S) model. Netherlands data ages 50–104, 1971–2016



Annuities in payment under M5(S) model. Netherlands data ages 50–104, 1971–2016



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- Longevity trend-risk capital very comparable between VaR99.5% and CTE99%.
- CTE99% usually slightly more prudent than VaR99.5%.
- Difference usually under 0.1%.

# 7 Managing longevity risk

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- Keep risk, or

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# 7 Managing longevity risk

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- Keep risk, or
- Transfer risk, or
- Hedge risk.

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...although there is a risk that the reinsurer might fail (counterparty risk).



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- The value of a hedging contract is supposed to move in line with the liabilities.
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...although there is a risk that the hedge is imperfect (basis risk).
- How big is this risk?

- Define contract using population mortality.

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- Use a function of  $X$  to close out the contract.  
⇒ This is just another multi-year VaR calculation.

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- Standardise payoff,  $h$ , as:

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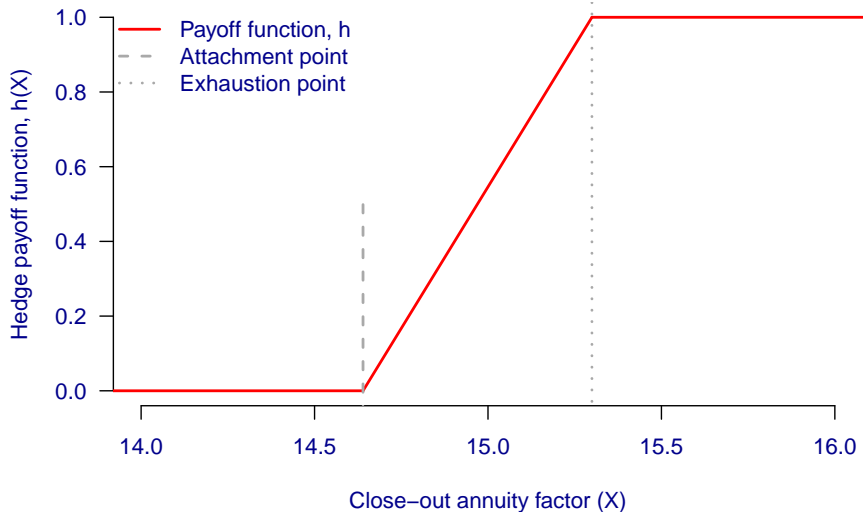
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- See Cairns and El Boukfaoui [2017] for detailed discussion.



# 7 Hedge payoff function



- Set  $AP = Q_{\alpha_1}$  and  $EP = Q_{\alpha_2}$  ( $\alpha_1 < \alpha_2$ ).

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- Mean payoff can be estimated from VaR results.

# 7 Example hedge contract

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- Probability of a payoff is 0.4.
- Average payoff is 0.375 (from 5,000 simulations).

- Lee-Carter model used for both sample paths over  $n$  years **and** for payoff calculation.

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- What happens if the sample paths follow a *different* model?

Impact of different sample-path models on payoff:

| Model | Payoff<br>prob. | Mean<br>payoff |
|-------|-----------------|----------------|
| LC(S) | 0.40            | 0.375          |
| M5(S) | 0.53            | 0.592          |
| 2DAC  | 0.80            | 0.434          |
| M6    | 0.82            | 0.710          |

Source: own calculations using population data for males in Netherlands, ages 50–104, 1971–2016.  
Annuity values discounted at 2% p.a.



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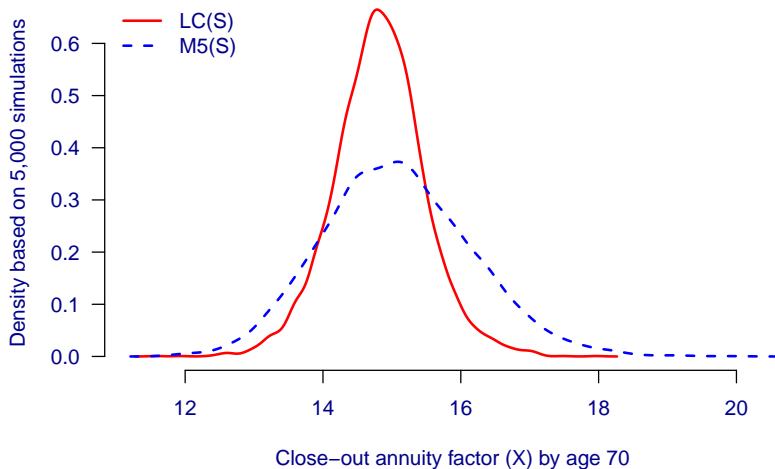
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    - ▶ What value should the hedge contract have on the balance sheet?
    - ▶ What solvency capital relief should be given?
- ⇒ Actuarial judgement required on both counts.



- How different can the answers get?

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- Consider the spread at various ages under CBD model (M5)...

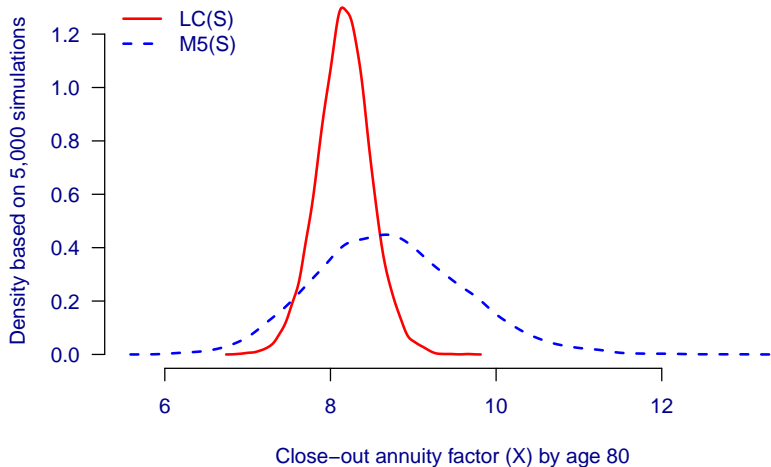
# 7 M5 sample paths — age 70



VaR annuity factors valued under Lee-Carter model after 15 years of mortality following the M5(S) model.

Netherlands data for males aged 50–104, 1971–2016. Annuity cashflows discounted at 2% p.a.

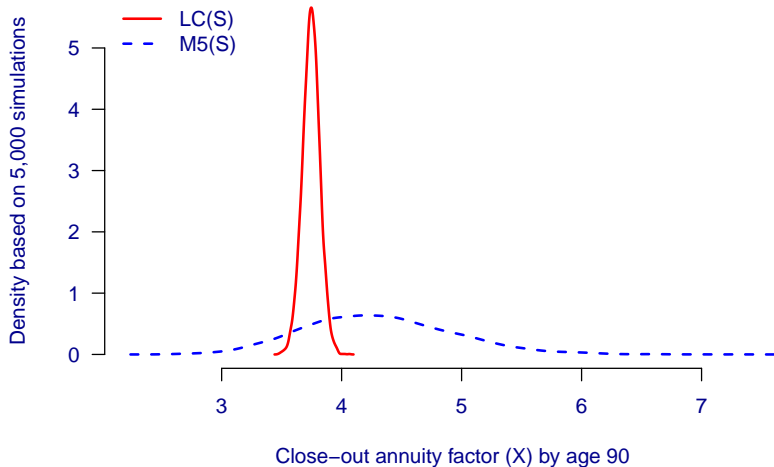
# 7 M5 sample paths — age 80



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# 7 M5 sample paths — age 90



VaR annuity factors valued under Lee-Carter model after 15 years of mortality following the M5(S) model.

Netherlands data for males aged 50–104, 1971–2016. Annuity cashflows discounted at 2% p.a.



# 8 Conclusions

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- Same outputs can be used for both VaR- and CTE-style solvency regimes.
- Framework extends to ORSA for insurers...  
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