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IFoA Actuarial Research Centre and ABI research project

Equity Release Mortgages: No Negative Equity Guarantee

LIFE Conference Dublin 22 November 2019

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Research project results

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Disclaimer

Findings from reviewing the literature

- GBM used before 2000 in NNEG literature to model house prices
- Over the last decade new models were proposed
- Better econometric models at the cost of risk-neutralisation difficulty
- ARMA-EGARCH with conditional Esscher transform a viable solution
- Other risk-neutralisation solutions are available
- Parameter estimation risk is real and sensitivity analysis would be useful

Here are the main critiques and reservations about the GBM-rn/Black 76 methodology.

- GBM as a data generating process for house prices is not the best because it ignores serial correlation and stickiness of prices, as well as clustered volatility and downward jumps.
- GBM may forecast inflated values of the house price. This can be very dangerous for real-world valuations, making the NNEG valuations very small because of the overshooting in house prices.
- The assumptions needed to apply the GBM-rn or Black 76 (computationally identical) are not satisfied in financial economics terms.
- The application of GBM-rn or Black 76 require short-selling of house prices, which is not possible currently.
- NNEG risk-neutral valuation is driven directionally by $r - g$. Estimation of rental yield becomes crucial.
- While we agree with the principle of risk-neutral valuation this should not be confused or assimilated with the GBM model as a data generating process.
- A constant rental yield parameter from one year ahead to a long maturity (45 years) may be unrealistic.

ARMA-EGARCH Model I

As in Li et al. (2010), we identify an ARMA(m,M)-EGARCH(P,Q) model by comparing log-likelihood, AIC criterion and other goodness-of-fit analysis.
It follows then that, under the real-world measure $\mathbb{P}_t$

$$\boxed{Y_t|\mathcal{F}_{t-1} \sim N(\mu_t, h_t)} \tag{1}$$

where $\mu_t = c + \sum_{i=1}^{m} \phi_i Y_{t-i} + \sum_{j=1}^{M} \theta_j \epsilon_{t-j}$.

Risk-neutralisation with conditional Esscher transform

Following Buhlman et al. (1996), Siu et al. (2004) and Li et al. (2010), for a given sequence of constants $\lambda_1, \lambda_2, \ldots, \lambda_t, \ldots$ the sequence $\{Z_t\}_{t\geq 0}$ starting at $Z_0 = 1$ and defined by

$$Z_t = \prod_{k=1}^{t} \frac{e^{\lambda_k Y_k}}{E(e^{\lambda_k Y_k} | \mathcal{F}_{t-1})} \tag{2}$$

is a martingale. The conditional Esscher distribution $\widetilde{\mathbb{P}}_t$ emerges from $d\widetilde{\mathbb{P}}_t = Z_t d\mathbb{P}_t$ and $\widetilde{\mathbb{P}}_t = \widetilde{\mathbb{P}}_{t+1} | \mathcal{F}_t$ and it is defined computationally through

$$E_{\widetilde{\mathbb{P}}_t}(e^{zY_t}; \lambda_t | \mathcal{F}_{t-1}) = e^{(\mu_t + h_t \lambda_t)z + \frac{1}{2} h_t z^2} \tag{3}$$

The risk-neutral-measure is identified by finding those λ_t^q such that

$$E_{\widetilde{\mathbb{P}}_t}(e^{Y_t}; \lambda_t^q | \mathcal{F}_{t-1}) = e^{r-g} \tag{4}$$

$$\lambda_t^q = \frac{r - g - \mu_t - \frac{1}{2} h_t}{h_t} \tag{5}$$

$$\boxed{Y_t|\mathcal{F}_{t-1} \sim N(r - g - \frac{1}{2}h_t, h_t)} \tag{6}$$

$$H_\tau = H_0 \exp(\sum_{i=1}^{i=\tau} Y_i)$$

We shall refer to this Monte Carlo simulation approach as the ARMA-EGARCH risk neutral (ARMA-EGARCH-rn) for the former and the ARMA-EGARCH real world (ARMA-EGARCH-rw).

Assumptions

- Our scenarios are selected based on discussions with experts working on ERMs and using public available tables from Legal & General, Just Group and Equity Release Council, as of November/December 2018.
- The assumptions made for the inputs of our analysis reflect current market conditions on the ERM market in the UK.
- The essential inputs:
  - a vector of LTV loadings for the vector of age group,
  - risk-free rate r or a term structure of risked-free rates $\{r_t\}_{t\geq 0}$;
  - fixed roll-up rate R;
  - rental yield g,
  - mortality/morbidity/prepayment rates,
  - house price volatility σ.

Roll-up rate

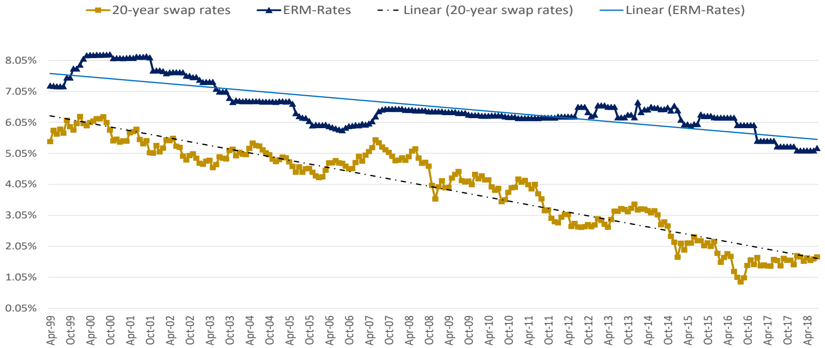


Figure: Average of top ERM customer rates against 20 year swap rates: Source Hosty et al. (2008) until 2006, Bloomberg and other combined by us using monthly interpolation.

For the roll-up rate we take the following two baseline rates: $R_1 = 4.15\%$, L&G and $R_2 = 5.25\%$ Equity Release Council (2018). For sensitivity $R \in \{6.15\%, 7.15\%; 3.5\%, 2.5\%\}$.

Rental Yield

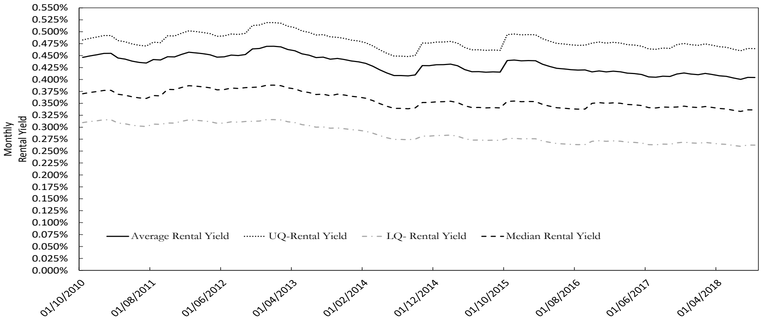


Figure: monthly series, average, proxy median and proxy lower and upper quartiles for England between Oct 2010 and Sep 2018. Source: Office for National Statistics.

- There is a split roughly 80% houses owned for consumption and the remaining 20% involved in some way with renting. According to the Office for National Statistics there were about 26.4 million households in the UK in the 2012 (following 2011 census) out of which approximately 5 million are rented out properties.
- The Office for National Statistics has been gathering data on rental yields for a sample of 10% of all properties rented out.
- This means that a rough calculation would give a total rental yield, weighted by the 20% representing the actual renting market, of 1.03% ($5.1776\% \times 20\%$) per annum.

- *Net* rental yield taking into account the voids, letting agent's fees in the range 10-15% of the rental income plus VAT (12%-18% including VAT)at the current rate of 20%, and maintenance costs that are typically around 15% of the gross rental income, inclusive of any VAT, gives an annualised net rental yield of 0.66%.
- Another feasible solution if NNEG(1) represents the NNEG valuation calculated with $g = 0$ and NNEG(2) represents the NNEG valuation (under same model) calculated with $g = 5\%$ then

$$NNEG = 0.8 \times NNEG(1) + 0.2 \times NNEG(2). \tag{7}$$

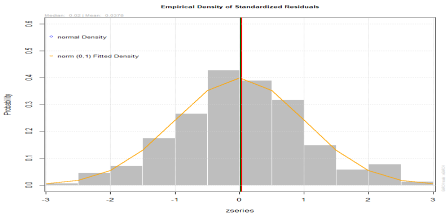
This value is evidently different from the value of the NNEG calculated with a weighted rental yield average.

GBM data generating process

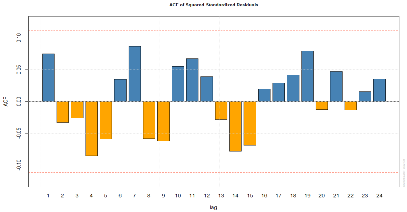
Parameter estimates (annualised) for the GBM process applied to the monthly Nationwide index, between Jan 1991 and Sep 2016 and Halifax Monthly Jan 1983- Dec 2014

	Nationwide		Halifax	
Method of Estimation	μ	σ	μ	σ
Maximum Likelihood (MLE)	5.36%	3.94%	5.80%	3.96%
Generalized Method of Moments (GMM)	3.33%	3.84%	6.45%	2.27%
Method of Moment (MM)	5.36%	3.94%	5.88%	3.96%

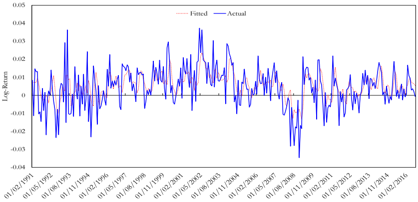
Region	MLE		Method of Moments		GMM	
	μ	σ	μ	σ	μ	σ
Period 1974-2018						
North	6.48%	6.48%	6.69%	6.50%	4.62%	5.23%
YorksHside	6.54%	6.32%	6.74%	6.34%	5.18%	5.65%
NorthWest	6.92%	5.41%	7.06%	5.42%	5.26%	4.64%
EastMids	7.06%	5.91%	7.24%	5.93%	5.76%	4.98%
WestMids	6.92%	5.88%	7.10%	5.89%	5.50%	4.80%
East Anglia	7.27%	6.50%	7.48%	6.52%	6.25%	6.20%
Outer Seast	7.47%	5.89%	7.65%	5.91%	6.87%	5.81%
Outer Met	7.69%	5.63%	7.84%	5.64%	7.49%	5.46%
London	8.29%	6.23%	8.49%	6.25%	7.96%	6.00%
South West	7.45%	5.69%	7.61%	5.70%	5.94%	5.12%
Wales	6.60%	6.44%	6.81%	6.45%	5.27%	5.67%
UK	7.07%	4.78%	7.19%	4.79%	6.03%	4.70%
Period covered in Hosty et al. (2008) paper 1974-2006						
North	8.87%	6.83%	9.10%	6.86%	7.23%	5.82%
YorksHside	8.68%	6.66%	8.91%	6.68%	7.23%	6.08%
NorthWest	9.20%	5.48%	9.35%	5.51%	7.24%	4.60%
EastMids	8.96%	6.27%	9.16%	6.29%	7.36%	5.17%
WestMids	8.84%	6.26%	9.03%	6.28%	7.13%	5.10%
East Anglia	9.00%	6.87%	9.24%	6.89%	7.92%	6.52%
Outer Seast	9.11%	6.15%	9.29%	6.17%	8.35%	6.07%
Outer Met	9.12%	5.75%	9.29%	5.78%	8.66%	5.63%
London	9.51%	6.30%	9.71%	6.32%	9.05%	6.10%
South West	9.33%	5.91%	9.51%	5.94%	7.29%	5.16%
Wales	8.82%	6.58%	9.04%	6.60%	7.03%	5.53%
UK	8.91%	4.78%	9.02%	4.80%	6.97%	4.41%



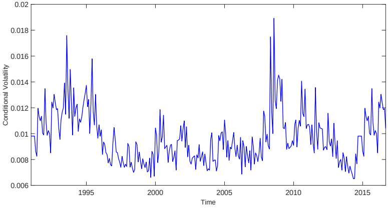
(a) histogram of standardized residuals



(b) autocorr. of standardized res.



(c) in sample fitted log-returns



(d) in sample fitted conditional vols

Figure: Goodness-of-fit for the ARMA(4,3)-EGARCH(1,1) model for Nationwide house price time-series monthly Jan 1991 to Sep 2016.

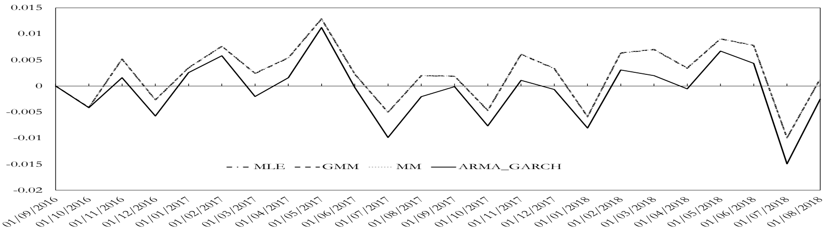


Figure: Comparison of out-of-sample forecasting error (actual minus forecast) for Nationwide House Price Index Monthly for ARMA(4,3)-EGARCH(1,1) and GBM model specifications, over the out-of-sample period Oct 2016 to Sep 2018.

MODEL	RMSE	MAE	
GBM-MLE	0.00579	0.0004982	
GBM-GMM	0.00581	0.0050080	
GBM-MM	0.00563	0.0041140	
ARMA(4,3)-EGARCH(1,1)	0.0151	0.0126	

Diebold-Mariano Forecast Accuracy Testing

MODEL 1	MODEL 2	STATISTIC	P-VALUE
GBM-MLE	GBM-GMM	-2.3477	0.0278
GBM-MLE	GBM-MM	-2.1684	0.0407
GBM-MLE	ARMA(4,3)-EGARCH(1,1)	0.2327	0.8180
GBM-GMM	GBM-MM	0.4038	0.6900
GBM-GMM	ARMA(4,3)-EGARCH(1,1)	0.2649	0.7934
GBM-MM	ARMA(4,3)-EGARCH(1,1)	0.2637	0.7943

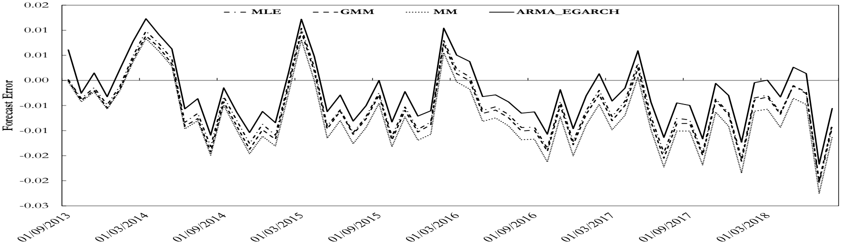


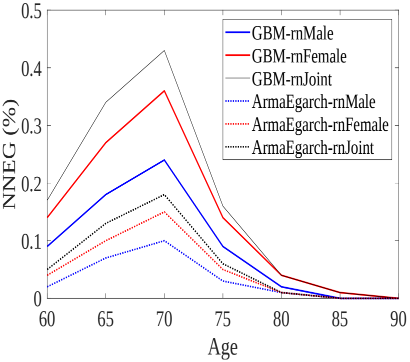
Figure: Comparison of out-of-sample forecasting error (actual minus forecast) for Nationwide House Price Index Monthly for ARMA(4,3)-EGARCH(1,1) and GBM model specifications, over the out-of-sample period Oct 2013 to Sep 2018.

MODEL	RMSE	MAE	
GBM-MLE	0.0079	0.0067	
GBM-GMM	0.0081	0.0069	
GBM-MM	0.0090	0.0078	
ARMA(4,3)-EGARCH(1,1)	0.0063	0.0051	

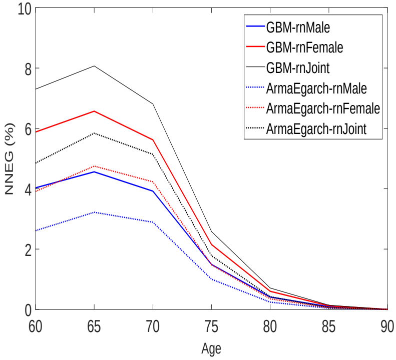
Diebold-Mariano Forecast Accuracy Testing

MODEL 1	MODEL 2	STATISTIC	P-VALUE
GBM-MLE	GBM-GMM	-3.9838	0.0002
GBM-MLE	GBM-MM	-6.7823	0.0000
GBM-MLE	**ARMA(4,3)-EGARCH(1,1)**	3.7681	0.0004
GBM-GMM	GBM-MM	-6.0371	0.0000
GBM-GMM	**ARMA(4,3)-EGARCH(1,1)**	3.9739	0.0002
GBM-MM	**ARMA(4,3)-EGARCH(1,1)**	4.8545	0.0000

Baseline Scenarios

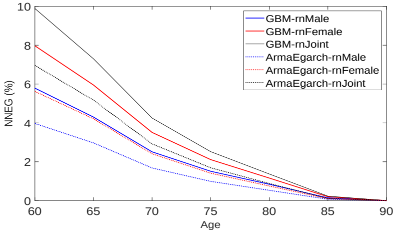


(a) $R = 4.15\%$

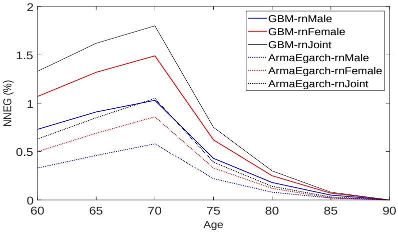


(b) $R = 5.25\%$

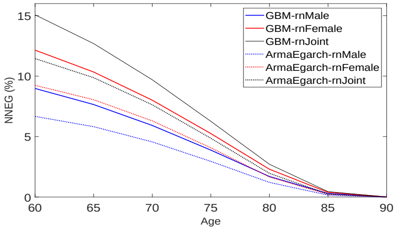
Figure: NNEG valuations as percentage of lump sum for GBM-rn and Arma-Egarch-rn , under multiple decrement rates for the two baseline scenario with $r = 1.75\%, g = 1\%, \sigma = 3.90\%$ and standard Flexible LTV vector valuations



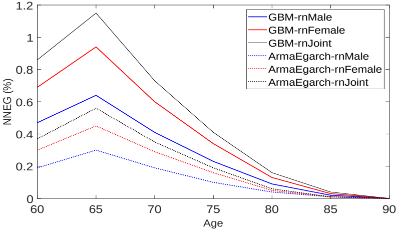
(a) FlexibleMax



(b) Flexible Plus

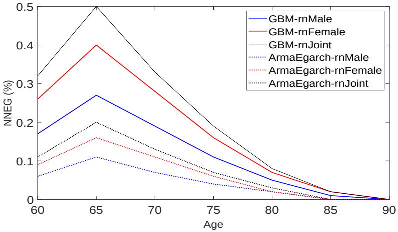


(c) Flexible Max Plus

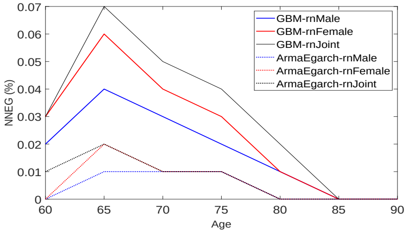


(d) ERC

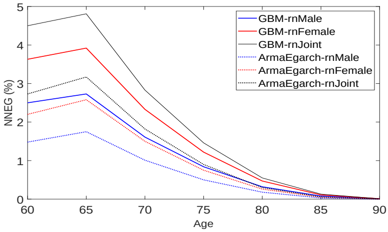
Figure: Sensitivity Analysis of NNEG valuation for baseline scenarios w.r.t.different LTV loadings and $r = 1.75\%, R = 4.15\%, g = 1\%, \sigma = 3.90\%$



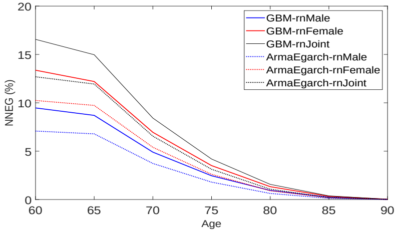
(a) $r = 2\%$



(b) $r = 2.5\%$

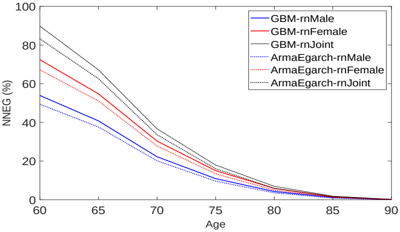


(c) $r = 1.25\%$

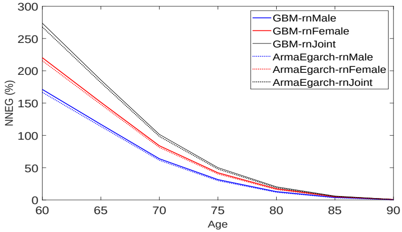


(d) $r = 0.75\%$

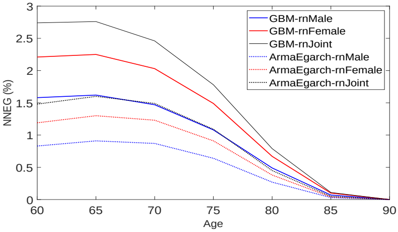
Figure: Sensitivity Analysis of NNEG valuation w.r.t. r under ERC LTV and loading and $R = 4.15\%, g = 1\%, \sigma = 3.90\%$



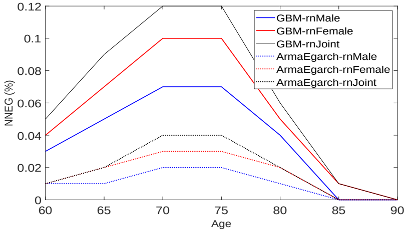
(a) ERC LTV $R = 6.15\%$



(b) ERC LTV $R = 7.15\%$

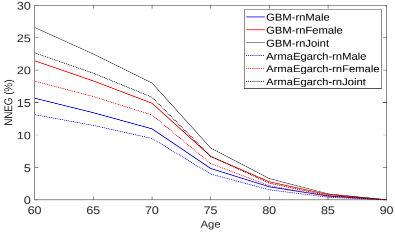


(c) Flexible MaxPlus LTV $R = 3.5\%$

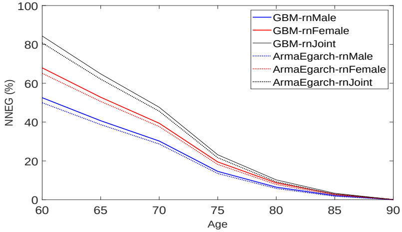


(d) Flexible MaxPlus LTV $R = 2.5\%$

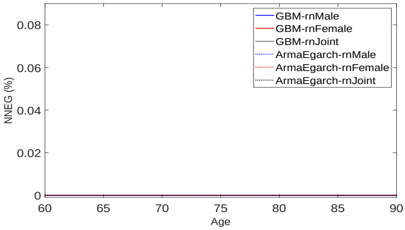
Figure: Sensitivity Analysis of NNEG valuation w.r.t. R under ERC and Flexible MaxPlus LTV loading and $r = 1.75\%, g = 1\%, \sigma = 3.90\%$



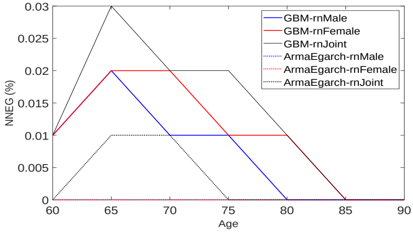
(a) Flexible Plus $g = 2\%$



(b) Flexible Plus $g = 3\%$

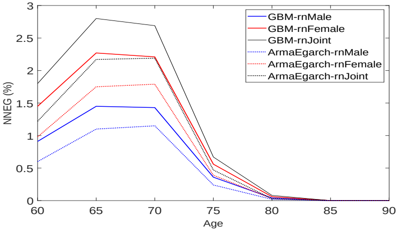


(c) ERC $g = -0.5\%$

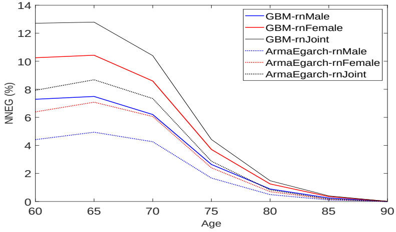


(d) ERC $g = 0\%$

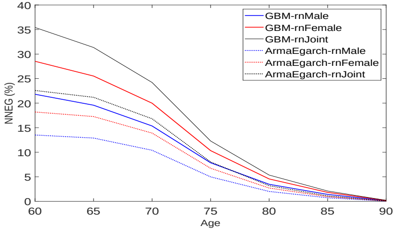
Figure: Sensitivity Analysis of NNEG valuation w.r.t. g under ERC with $R = 4.15\%$,and Flexible Plus LTV loading with $R_{fp} = 4.43\%$,, and $r = 1.75\%$, $R_{fp} = 4.43\%$, $\sigma = 3.90\%$



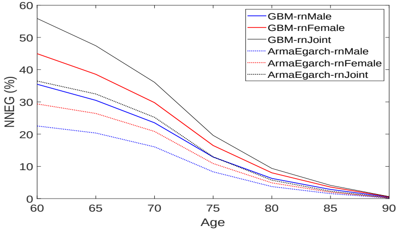
(a) $\sigma = 2\%$



(b) $\sigma = 5\%$



(c) $\sigma = 8\%$



(d) $\sigma = 10\%$

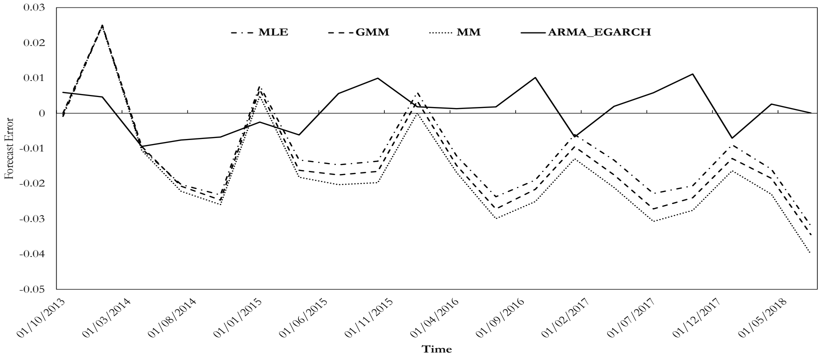


Figure: Comparison of out-of-sample forecasting error (actual minus forecast) for Nationwide Average House Price Quarterly (non-seasonally adjusted) for ARMA(4,3)-EGARCH(1,1) and GBM model specifications, over the out-of-sample period Q4 2013 to Q3 2018.

The analysis is for a five year out-of-sample period, with quarterly data going back to 1952.

Using quarterly data 1952-2018 calibrated models for Joint Life NNEG

Age	60	65	70	75	80	85	90
LTV	17.0%	22.5%	28.5%	32.4%	36.5%	41.5%	41.5%
InitialLoan	65,100	82,150	102,300	114,700	130,200	145,700	145,700
GBM- ARMA/EGARCH							
Baseline							
g=1%,σ=4.88% R=5.25%, r=1.75%	11.84	11.28	8.76	4.13	1.38	0.37	0.01
Rental yield							
g = 0.5% ($\downarrow$0.5%)	4.22	4.98	4.50	1.86	0.60	0.16	0.00
g = 2.5% ($\uparrow$1.5%)	23.72	17.33	12.61	8.43	4.69	2.20	0.08
g = 4.0% ($\uparrow$3%)	12.45	9.97	8.43	7.06	5.02	3.10	0.58
House price volatility							
σ = 2% ($\downarrow$2.88%)	1.62	1.88	1.49	0.61	0.08	0.00	0.00
σ = 8% ($\uparrow$3.12%)	35.24	30.40	23.01	12.23	5.35	2.10	0.20
σ =13% ($\uparrow$8.12%)	91.88	74.40	55.52	32.97	17.29	8.42	1.88
Risk-free rate							
r = 0.75% ($\downarrow$1.00%)	33.65	23.86	16.24	9.61	4.61	1.66	0.04
r = 1.25% ($\downarrow$0.5%)	23.89	18.73	13.30	7.09	2.95	0.82	0.01
r = 2.50% ($\uparrow$0.75%)	1.81	2.41	2.39	1.02	0.33	0.09	0.00
Roll-up rate							
R = 3.50% ($\downarrow$1.75%)	0.07	0.14	0.21	0.10	0.03	0.01	0.00
R = 6.15% ($\uparrow$0.90%)	31.43	22.72	15.78	8.88	4.18	1.38	0.03

Deferment Rate

- If H_0 is the house price today the deferment price to get the house at future time T is denoted by $\overleftarrow{F}(T)$ and the deferment rate q is defined by

$$\overleftarrow{F}(T) = H_0 e^{-qT} \tag{8}$$

- $\overleftarrow{F}(T)$ is the prepaid forward price of the house as the underlying asset.

$$F_0(T) = e^{rT} \overleftarrow{F}(T) \tag{9}$$

$$F_0(T) = H_0 e^{(r-q)T} \tag{10}$$

- Computationally, if $r - q < 0$ then $\{F_0(T)\}_{T \geq 0}$ decreases with time to maturity so the forward house price curve will be in backwardation. Vice versa, if $r - q > 0$ then $\{F_0(T)\}_{T \geq 0}$ increases with time to maturity so the forward house price curve will be in contango.

- The PRA condition is requiring $\overleftarrow{F}(T) < H_0$ which from (8) is equivalent to ask that $q > 0$.
- At the same time, the same condition is equivalent to

$$F_0(T) < H_0 e^{rT} \tag{11}$$

- The condition states that the forward curve on house prices will be bounded by the current house price inflated at the risk-free rate.
- The identity (10) can be rearranged as

$$\frac{F_0(T)}{H_0} e^{-rT} = e^{-qT} \tag{12}$$

so to test whether $q > 0$ we can use data on the left side quantities and see if

$$\frac{F_0(T)}{H_0} e^{-rT} < 1 \tag{13}$$

- If on the contrary $\frac{F_0(T)}{H_0} e^{-rT} > 1$ then this is evidence that $q < 0$.
- We shall call $\boxed{\frac{F_0(T)}{H_0} e^{-rT}}$ the deferment condition term (DCT). Hence $DCT > 1$ is equivalent to $q < 0$.

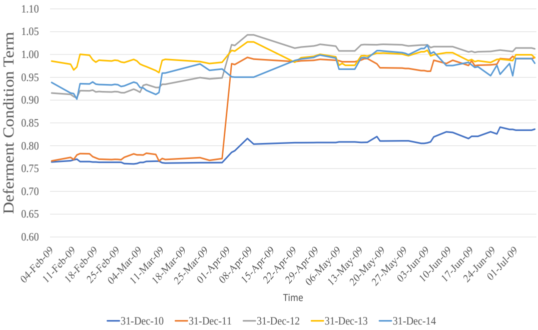


Figure: Deferment ratio condition for EUREX futures contracts for the period 4 Feb 2009 to 7 Jul 2009, for all existent five IPD futures contracts.

Conclusions

- A positive rental yield larger than a low risk-free rate leads to low level projections of house prices under a risk-neutral valuation approach, implying high NNEG values.
- The GBM-risk-neutral/Black76 may inflate under current market conditions the NNEGs values through higher than necessary volatility at long horizons. This effect may impact on the availability of ERMs and the final cost carried by the borrowers.
- The same models may swing the opposite way when the risk-free rates are larger than the rental yield, overestimating house prices and implying low NNEGs.
- The GBM-risk-neutral/Black 76 has theoretical deficiencies, but this does not imply that the risk-neutral valuation approach is inadequate.

- For the valuation of NNEG:
  - select an appropriate model for house price dynamics,
  - risk-neutralize the process
  - use Monte Carlo simulations to get house prices in this risk-neutralized world
  - value the NNEG.
- It is preferable to select as the data generating model for house prices a model that can forecast well house prices out-of-sample, for at least few years ahead.
- The ARMA-EGARCH family of models are suitable for time-series with serial correlation and volatility clustering such as Nationwide house price index series. The ARMA(4,3)-EGARCH(1,1) outperforms the GBM model under real-world measure in terms of forecasting house prices in the UK.

- The volatility estimates may vary with the data generating process, estimation method and the period of estimation. From Nationwide index data, values between 2.5% to 6% seem representative for volatility and 10% is more of a stressed scenario.
- The NNEG values are very sensitive to LTV assumptions so the design of the ERM product is important for risk management purposes. The NNEG valuations are also very sensitive to the roll-up rate R. Since this is a fixed-rate in the UK, an important risk-management control can be obtained at the outset, when the loan is issued.
- It is possible, in the high-risk NNEG region, for the GBMrn/Black 76 and the ARMA-EGARCH model to give almost the identical NNEG valuations.

THANK YOU!

QUESTIONS in Q&A, PLS HIT THE APPLE!

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