Efficient Sensitivity Analysis via Scenario Weighting

http://openaccess.city.ac.uk/18896/

Silvana Pesenti & Andreas Tsanakas
joint work with Pietro Millossovich

Silvana.Pesenti@cass.city.ac.uk;
A.Tsanakas.1@city.ac.uk

Technical Issues in General Insurance, 25. April 2019

Motivation

Complex quantitative models

- Capital modelling and beyond
- Granularity v opaqueness

Motivation

Complex quantitative models

- Capital modelling and beyond
- Granularity v opaqueness

Questions

- Which parts of the portfolio drive performance?
- Where do model-risk vulnerabilities lie?

Motivation

Complex quantitative models

- Capital modelling and beyond
- Granularity v opaqueness

Questions

- Which parts of the portfolio drive performance?
- Where do model-risk vulnerabilities lie?

Sensitivity analysis

- Repeated model runs
- What to do with the results?

Motivation

Complex quantitative models

- Capital modelling and beyond
- Granularity v opaqueness

Questions

- Which parts of the portfolio drive performance?
- Where do model-risk vulnerabilities lie?

Sensitivity analysis

- ~~Repeated model runs~~ **Single model run**
- ~~What to do with the results?~~ **Consistent sensitivity measurement**

Example

A non-linear insurance portfolio

Portfolio consisting of

- Two lines of business
- Same multiplicative factor, e.g. inflation
- Reinsurance layer on the portfolio
- Reinsurance company can default

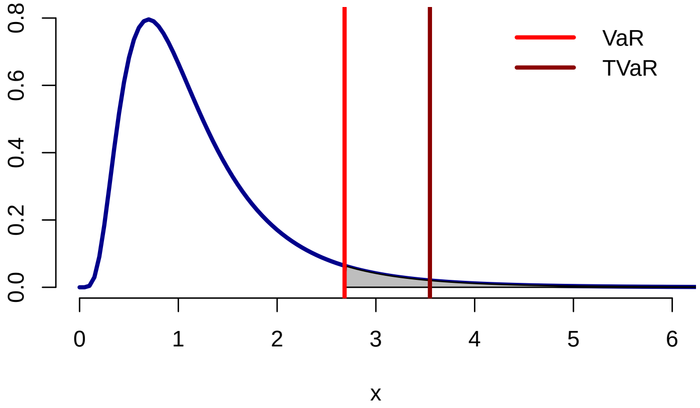
A non-linear insurance portfolio

Portfolio consisting of

- Two lines of business
- Same multiplicative factor, e.g. inflation
- Reinsurance layer on the portfolio
- Reinsurance company can default

	Input risk factors		**Output**
X_1	Claims from 1st LoB	Y	Portfolio loss
X_2	Claims from 2nd LoB		
X_3	Multiplicative factor		
X_4	% of RI recovery lost		

Risk assessment of the portfolio loss

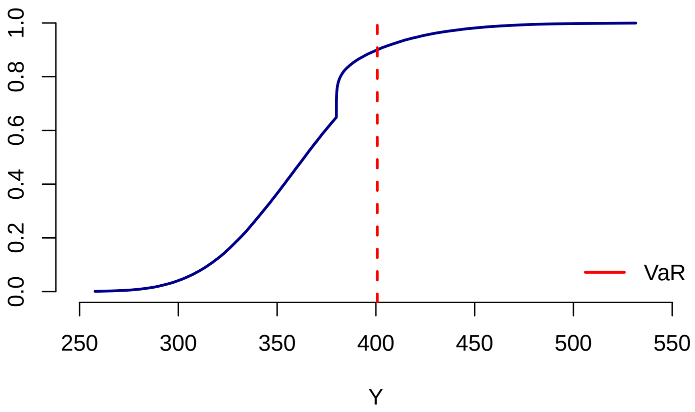


1. Which risk factor is most important?

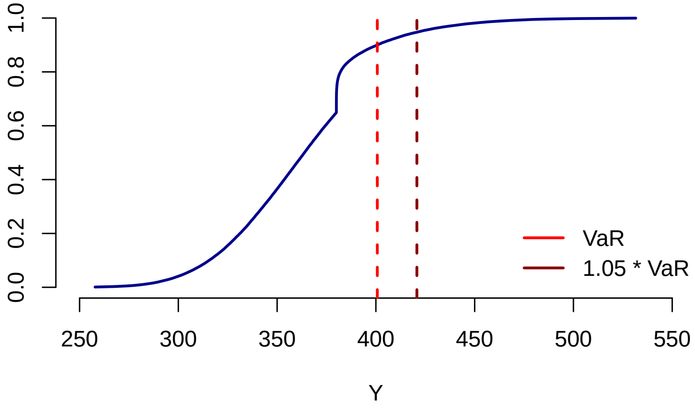
1. Which risk factor is most important?

2. Which is the most plausible model that gives a 5% higher portfolio VaR?

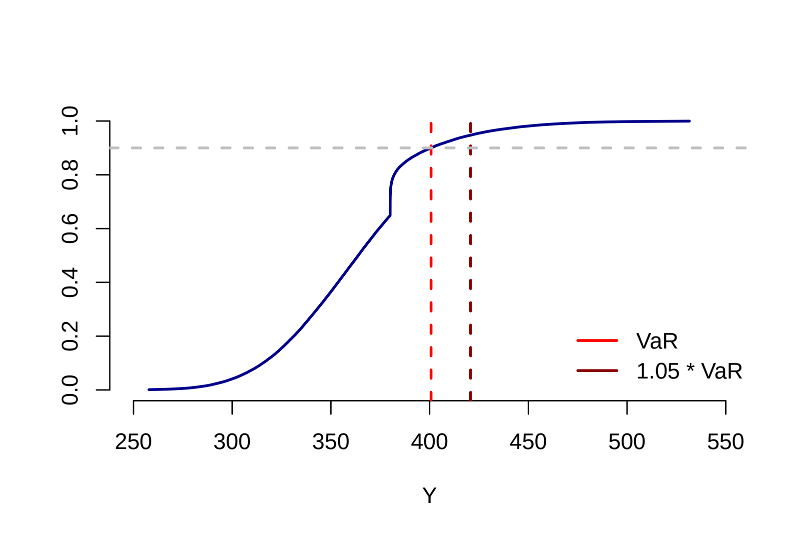
Distribution of portfolio loss



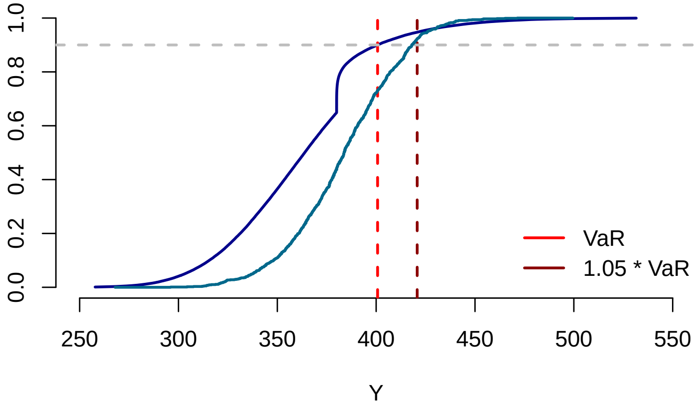
Distribution of portfolio loss



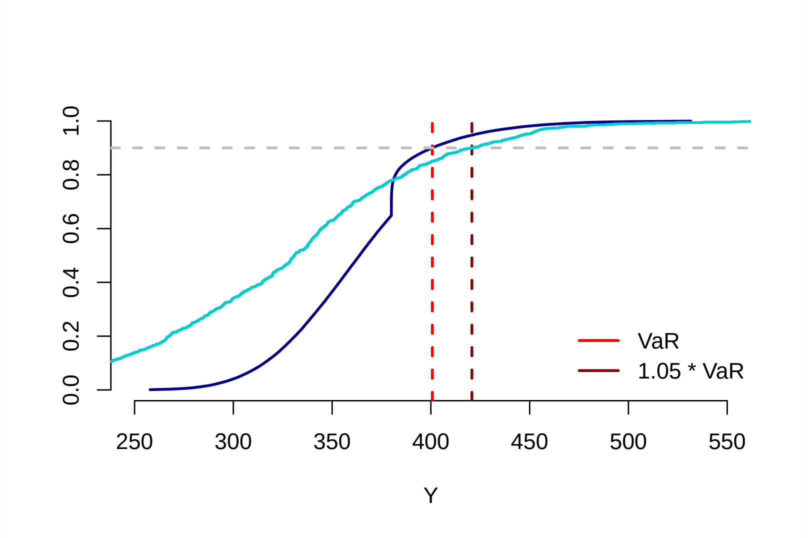
Distribution of portfolio loss



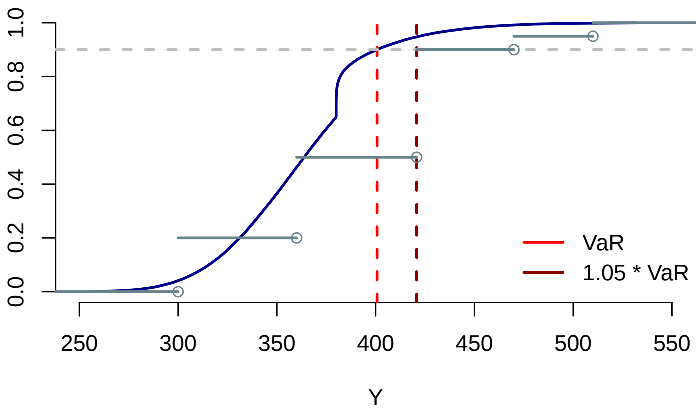
Distribution of portfolio loss



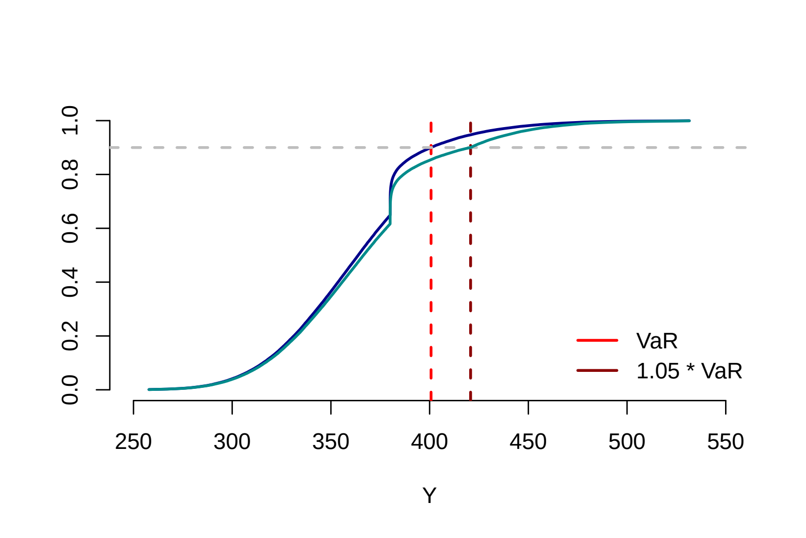
Distribution of portfolio loss



Distribution of portfolio loss



Distribution of portfolio loss



How to chose a model stress?

Scenario weighting!

Scenario Weights

Constructing scenario weights

1. Define a **stress** on the output as an increase of VaR or/and TVaR

2. Derive **scenario weights** (change of measure) such that

   - re-weighted output fulfils the required stress
   - **most plausible / least distorting** (minimal entropy)
   - mathematically consistent

Constructing scenario weights

1. Define a **stress** on the output as an increase of VaR or/and TVaR

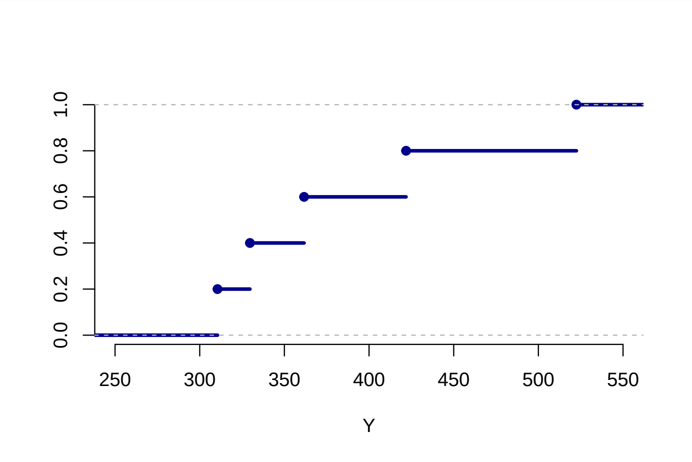
2. Derive **scenario weights** (change of measure) such that

   - re-weighted output fulfils the required stress
   - **most plausible / least distorting** (minimal entropy)
   - mathematically consistent

▷ Typically we have a Monte Carlo sample and work with the empirical distribution.

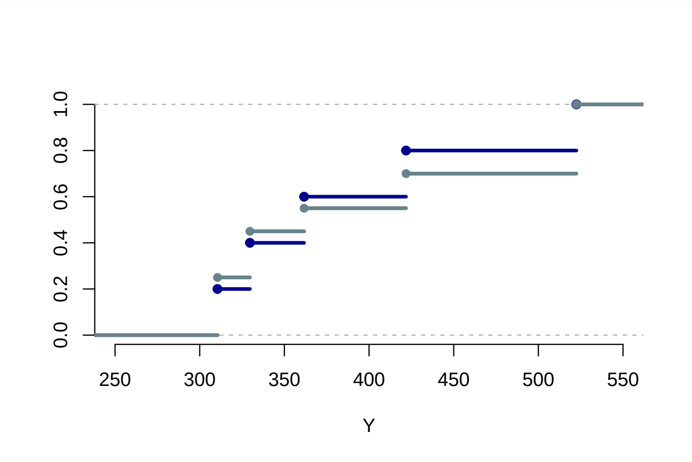
Constructing scenario weights

Monte Carlo sample: $Y = \{311, 330, 362, 422, 522\}$ with equal probability $= \{\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}\}$, that is equal weights



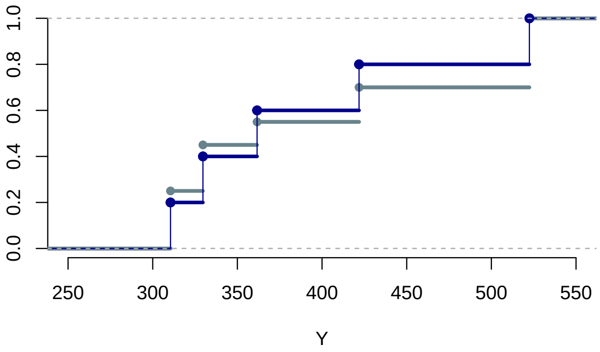
Constructing scenario weights

Re-weighting subject to constraints (e.g. increase in VaR)



Constructing scenario weights

- ▷ do NOT change the data points: $Y = \{311, 330, 362, 422, 522\}$
- ▷ change height between points: **scenario probabilities** $= \{0.25, 0.2, 0.1, 0.15, 0.3\}$, that is different weights



Scenario weights

Before re-weighting

- ▷ Every scenario has equal probability of occurring

After re-weighting

- ▷ data points stay the same
- ▷ we change the probability that a scenario occurs
- ▷ such that the constraints (e.g. increase in VaR) are fulfilled
- ▷ scenarios are re-weighted in the most plausible way

Scenario weights

Before re-weighting

- ▷ Every scenario has equal probability of occurring

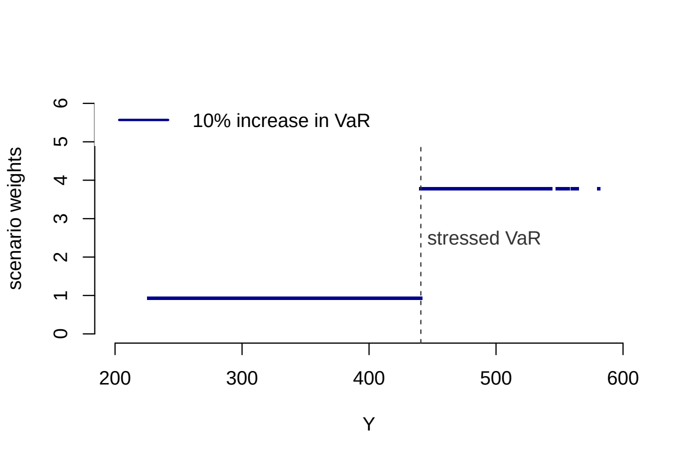
After re-weighting

- ▷ data points stay the same
- ▷ we change the probability that a scenario occurs
- ▷ such that the constraints (e.g. increase in VaR) are fulfilled
- ▷ scenarios are re-weighted in the most plausible way

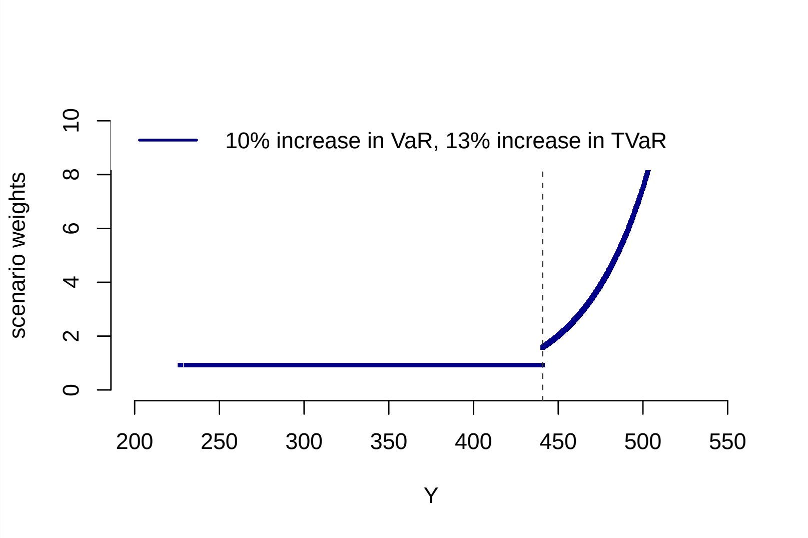
An increase in VaR means that scenarios where portfolio loss is high are given more weight: they are now more likely to occur.

Scenario weights for a stress on VaR

Scenario probabilities $= 0.92 * \frac{1}{10^6}$, for low Y
Scenario probabilities $= 3.77 * \frac{1}{10^6}$, for high Y



Scenario weights for a stress on VaR and TVaR



Back to the example

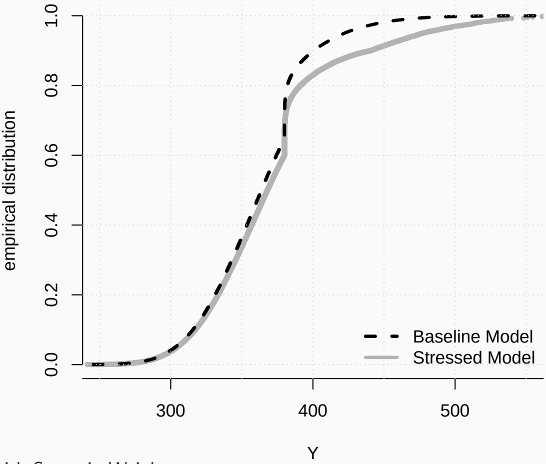
Insurance portfolio

Recall:

- X_1, X_2 are claims from different LoB
- X_3 is positive multiplicative factor
- X_4 is % of RI lost to default

Insurance portfolio - Output

Stress VaR by 10% and TVaR by 13%, at level 0.95

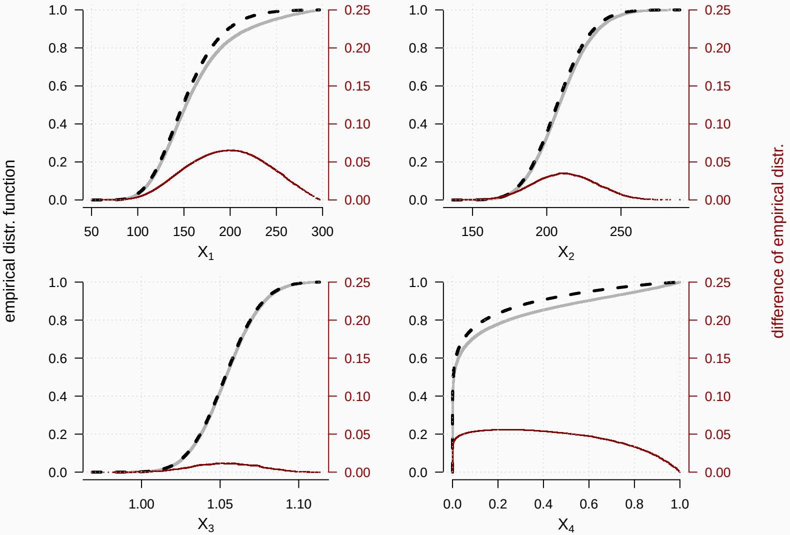


Which input factor is most important?

Which input factor is most important?

Weighting applies to simulated scenarios, including inputs!

Insurance portfolio - Input



Insurance portfolio

	X_1	X_2	X_3	X_4	Y
Mean	150	200	1.05	0.10	362
Mean, stressed	157	202	1.05	0.14	371
Relative increase	5%	1%	0%	44%	3%
Standard deviation	35	20	0.02	0.20	36
Standard deviation, stressed	43	21	0.02	0.26	50
Relative increase	25%	5%	1%	30%	38%

Stressing the inputs

Stressing the inputs

Stress input risk factor by a 10% increase of its VaR, at level 0.9.

Stress on input	Change in output		
	mean	VaR	TVaR
1st LoB	1.3%	3.9%	4.2%
2nd LoB	1.2%	2.8%	3.0%
Multiplicative factor (3% VaR stress)	0.4%	0.6%	0.6%
Loss to RI default	0.1%	0.4%	0.4%

Sensitivity measures

Sensitivity measure

Sensitivity measure for input risk factor X_i

$$\Gamma_i = \frac{E^{\text{stressed}}(X_i) - E(X_i)}{\text{normalised}}$$

- depends on the output through the scenario weights.

Real-data example

Proprietary model of a London insurance market portfolio

$$Y = \sum_{i=1}^{72} a_i X_i$$

with exposures $a_1, \ldots, a_{72}$.

Facts

- 500,000 Monte Carlo simulations of input and output
- no knowledge about distributional assumptions

Real-data example

Proprietary model of a London insurance market portfolio

$$Y = \sum_{i=1}^{72} a_i X_i$$

with exposures $a_1, \ldots, a_{72}$.

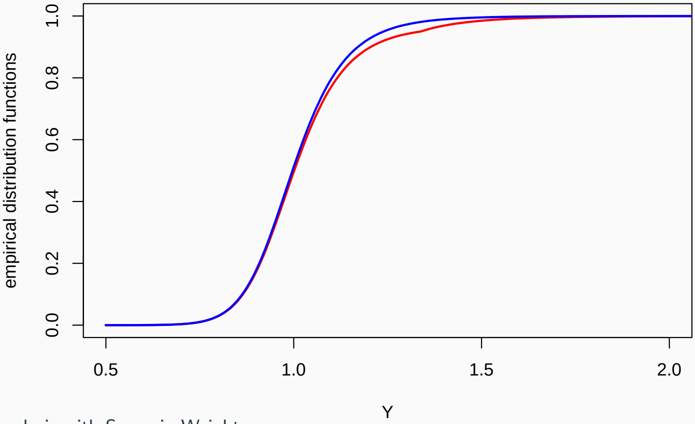
Facts

- 500,000 Monte Carlo simulations of input and output
- no knowledge about distributional assumptions

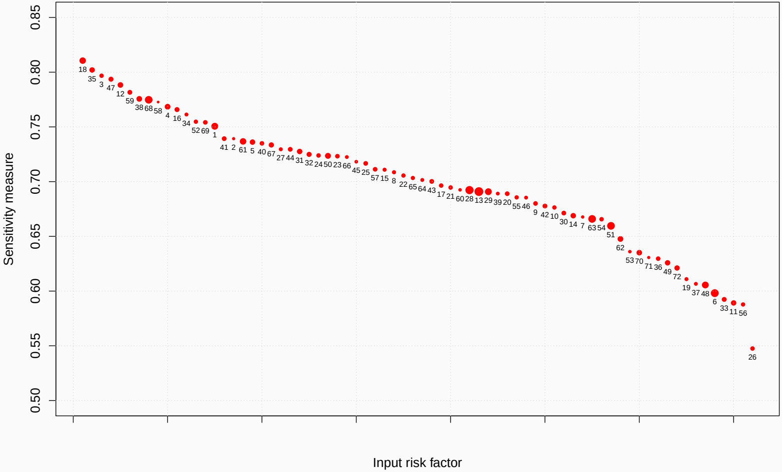
Stress: increase VaR by 8% and TVaR by 10%, at level 0.95

Real-data example

Distribution of the portfolio loss (blue) and after re-weighting (red).



Real-data example



Summary

Sensitivity Analysis with Scenario Weights:

1. Define a stress on the output
2. Calculate the scenario weights
3. Compare the distribution before and after re-weighting

Variations:

- Stressing output or inputs
- Different stresses: VaR, TVaR, mean, standard deviation, higher moments
- Decrease or increase of VaR, TVaR

Outlook and discussion

- Coming soon: the *SWIM* package in **R**
- Applicability and business benefits
- Academia $\leftrightarrow$ industry feedback loop
- If you are interested in using our approach, let us know!

Thank you!

Appendix

Non - linear insurance portfolio

Non-linear insurance portfolio

$$Y = L - (1 - X_4) \min\left\{(L - d)_+, l\right\}$$
$$L = X_3(X_1 + X_2),$$

where

- X_1, X_2 different lines of business
- X_3 positive multiplicative risk factor, e.g. inflation
- X_4 percentage lost due to default of the reinsurance company
- reinsurance limit l and deductible d

Insurance portfolio - Assumptions

Assumptions:

- $X_1 \sim$ (truncated) $LogNormal$ with mean 150 and sd 35.
- $X_2 \sim Gamma$ with mean 200 and sd 20.
- $X_3 \sim$ (truncated) $LogNormal$ with mean 1.05 and sd 0.02.
- $X_4 \sim Beta$ with mean 0.1 and sd 0.2.
- X_1, X_2, X_3 are independent.
- X_4 dependent on L through a Gaussian copula with correlation 0.6.
- $d = 380$, $l = 30$.