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Institute and Faculty of Actuaries

Actuarial Research Centre (ARC)

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The Actuarial Research Centre (ARC) is the Institute and Faculty of Actuaries' network of actuarial researchers around the world.
The ARC seeks to deliver research programmes that bridge academic rigour with practitioner needs by working collaboratively with academics, industry and other actuarial bodies.
The ARC supports actuarial researchers around the world in the delivery of cutting-edge research programmes that aim to address some of the significant challenges in actuarial science.

Multi-population mortality models

Fitting, Forecasting, Comparison and Applications

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Joint work with Andrew J.G. Cairns and Torsten Kleinow

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HERIOT WATT UNIVERSITY

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Motivation and purpose

- Forecasting joint mortality rates
  - The projections are correlated
  - Learn from other populations

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- Multi-population risk assessment
  - Annuities
  - Reserving
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  - Reserving
  - Diversifying portfolio risk
  - SCR estimates
- Hedging risk
  - Q-forward contracts
  - S-forward contracts

Data specifics

Six populations considered:

		Exposure to risk at the age of 60 in 2010	
i	Country	Male	Female
1	Austria	47023	49526
2	Belgium	65344	66434
3	Czech Republic	71575	71575
4	Denmark	34420	35132
5	Sweden	59759	59742
6	Switzerland	46527	47078

Data range: 30 ages (from 60 to 89 years old) and 50 calendar years (from 1961 up to 2010)

Importance of data chosen

- Closely related countries
- Similar size and features

Multi-population mortality models

Model 0 : *"Li and Lee (2005)"*

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- Model 1 and Model 2 - special cases of Model 0

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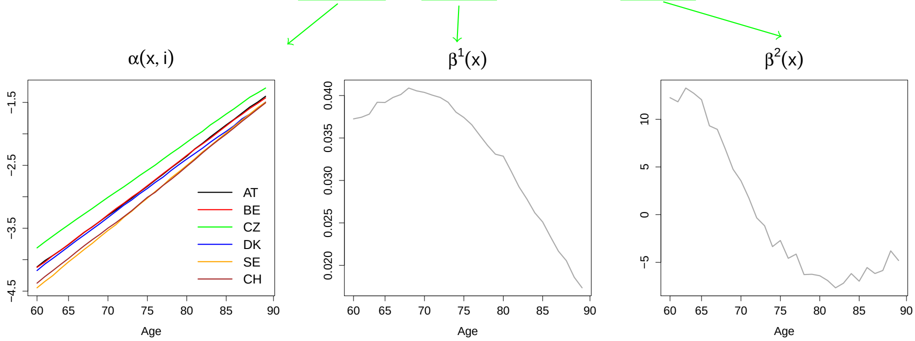
- Model 1 and Model 2 - special cases of Model 0
- Identifiability issue - *Constraints implementation*
- Parameter estimation - *Maximum likelihood estimation*
- Model selection criterion - *BIC*

	BIC value	Rank
Model 0	100043.08	(2)
Model 1	101628.38	(4)
Model 2	101525.12	(3)
Model 3	**99805.38**	**(1)**

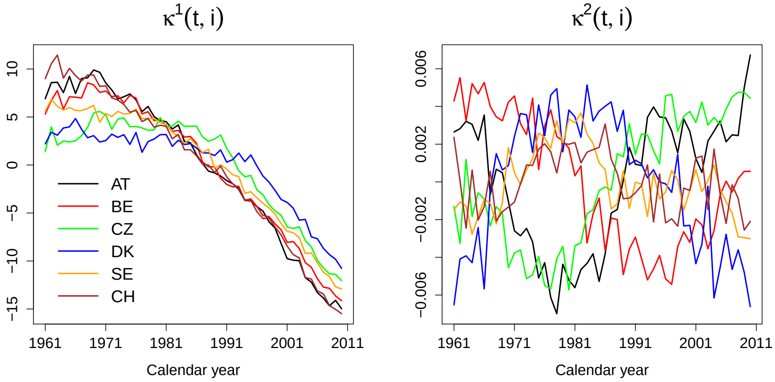
Estimated age dependent parameters in Model 3

$$\log m(x,t,i) = \boxed{\alpha(x,i)} + \boxed{\beta^1(x)}\kappa^1(t,i) + \boxed{\beta^2(x)}\kappa^2(t,i)$$



Estimated time dependent parameters in Model 3

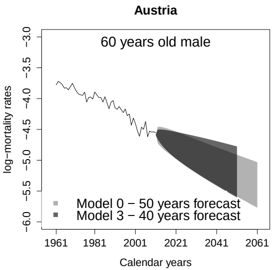
$$\log m(x,t,i) = \alpha(x,i) + \beta^1(x)\boxed{\kappa^1(t,i)} + \beta^2(x)\boxed{\kappa^2(t,i)}$$

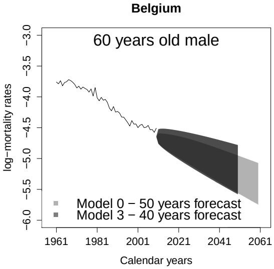


Forecasting joint mortality rates

Points to consider

- Shape of the estimated parameters
- Common time dependent parameters
  - Single variate time series processes
- Country specific time dependent parameters
  - Multivariate time series processes
- Projected joint mortality forecasts





Robustness of the estimated parameters

- Full data set: 1961 - 2010
- Reduced data set: 1961 - 2000
    - One step MLE
    - Two step MLE

Robustness of the estimated parameters

- Full data set: 1961 - 2010
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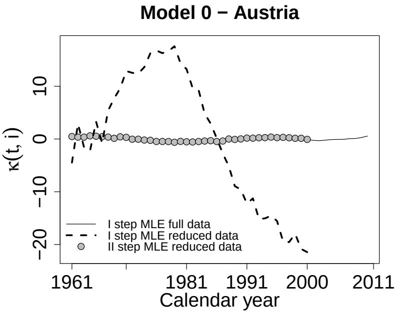
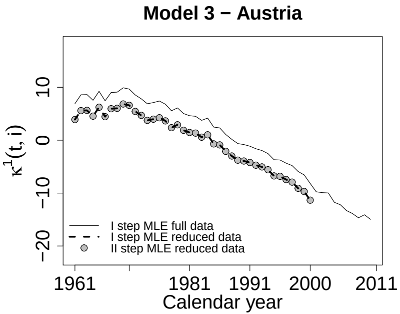
- Models robustness
  - Model 0 suffers from multi maxima problems
  - Model 3 is very robust
- Conclusion
  - Model 3 ranks as the best model

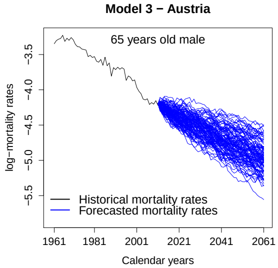


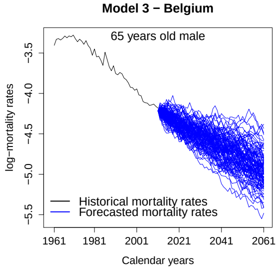


Multi-population mortality models applications

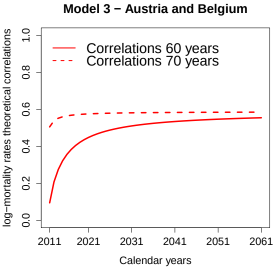
$$\log m(x,t,i) = \alpha(x,i) + \beta^1(x)\kappa^1(t,i) + \beta^2(x)\kappa^2(t,i)$$

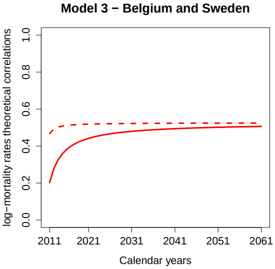
Joint forecasts





Theoretical correlations





Multi-population mortality models applications

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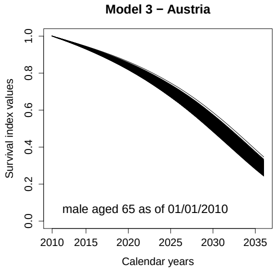
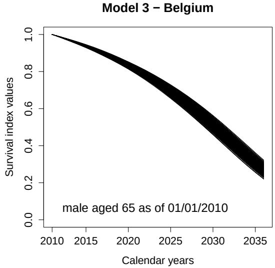
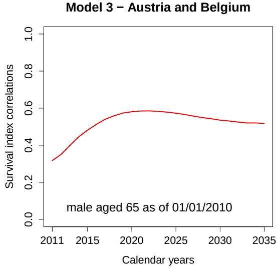
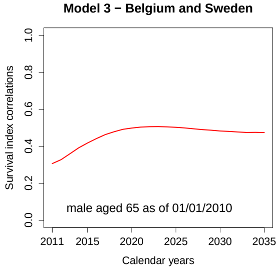
Survival index values





Empirical correlations





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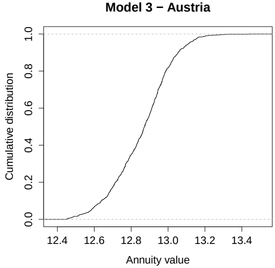
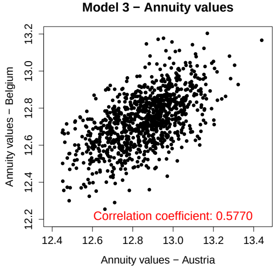
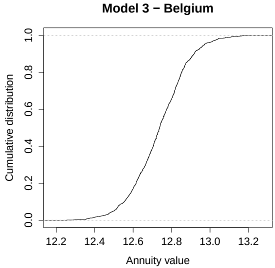
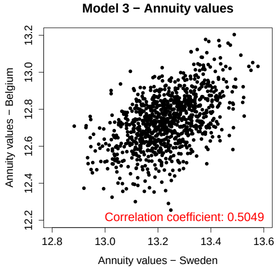
Annuity ECDF

Annuity scatter plots









Multi-population mortality models applications

Other applications of multi-population mortality models include

- Reserving
- Assess benefits of diversification across countries
- SCR values: single and multi country
- Hedging portfolio risk
  - Q-forward contracts
  - S-forward contracts

http://www.macs.hw.ac.uk/ andrewc/papers/Enchev2015.pdf

▸ Paper link