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Investment risk-sharing

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The ‘**Minimising Longevity and Investment Risk while Optimising Future Pension Plans**’ research programme is being funded by the Actuarial Research Centre.

What has been studied on investment risk-sharing?

- Investigated what has been studied on inter-generational investment risk-sharing.
- Interested in scientific foundations of risk-sharing.
- For more details and references, see Investment risk-sharing: a State of the Art Report, *by R. Chehab and C. Donnelly (2018)*

(download at either www.risk-insight-lab.com/outputs/ or IFoA ARC website).

- Self-selection and risk sharing in a modern world of life-long annuities, *by R. Gerrard, M. Hiabu, I. Kyriakou, J.P. Nielsen,* proposes an intra-generational risk-sharing method.

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What has been studied on investment risk-sharing?

- Discovered:
  - no established definition of ``what does risk-sharing look like''?
  - no established, non-utility-based framework for investment risk-sharing
  - existing frameworks are based on utility theory.
- Existence of a buffer does not mean there is investment risk-sharing.
- Many things can be mixed up in buffers, e.g.
  - Inter-generational investment risk-sharing, and
  - Investment return smoothing.

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Conclusions to inform our research

- Some contracts look like they have risk-sharing, but there isn't any.
- Establish a scientific, practically implementable foundation for investment risk-sharing.
  - Allow for inter-generational fairness or equitability.
  - Understand who is gaining and losing from risk-sharing.
  - Optimal strategy, e.g. replicating portfolio/long-term strategic allocation or one at different times?

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Outline

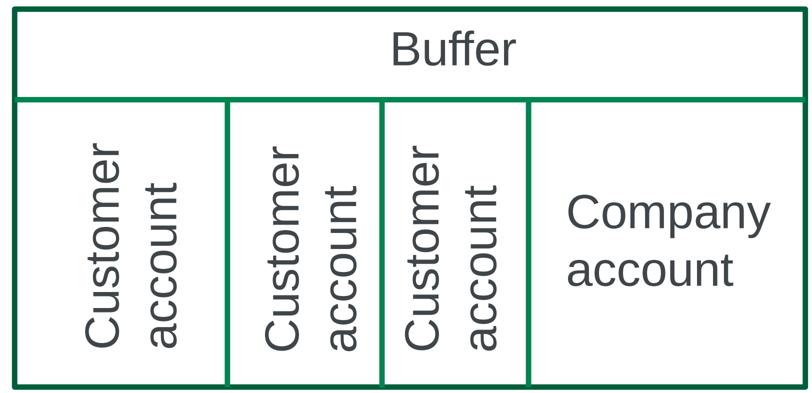
- With-profits contracts
- Theoretical risk-sharing frameworks
- DB/CDC pension plans
- Our ongoing work

With-profits contracts – typical features

- Individual customer accounts,

  a buffer and

  possibly a company account.



- Investment returns proportional to a Reference Portfolio return.
- Minimum guaranteed interest rate.
- Possibly a percentage of buffer granted at contract maturity date.

With-profits contracts – simple example

- Italian contract studied by Bacinello (2001).
- Customer account value at time *n* is

$$P(n) = \left(1 + \max\{\underbrace{r_G}_{\text{Guaranteed minimum interest rate}}, \underbrace{\alpha}_{\text{Participation constant}} \underbrace{\frac{A(n) - A(n-1)}{A(n-1)}}_{\text{Return on reference portfolio from time } n\text{-}1 \text{ to time } n.}\}\right) \times \underbrace{P(n-1)}_{\text{Customer account value at time } n\text{-}1.}.$$

- Financial fairness fixes α and also a replicating portfolio.

With-profits contracts – simple example

- UK contracts studied by Haberman, Ballota & Wang (2003).
- E.g. Bacinello's Italian contract with a terminal bonus.
  - Customer payoff at maturity time T is

$$P(T) + \gamma \times \max\{A(T) - P(T), 0\}.$$

Customer account value

Participation constant #2

Reference portfolio value

  - Financial fairness fixes (α, γ).
- They also look at arithmetic averaging over >1 year, geometric averaging and smoothed asset share.

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With-profits contracts – intermediate conclusions

- Financial fairness means "you get what you pay for":

Contract premium = Expected discounted value of maturity value,

under a risk-neutral measure.

- Where is the opportunity for investment risk-sharing?

- There is no buffer account in these contracts.

With-profits contracts – with buffer

- Danish contract *TidsPension*, studied by Guillén et al (2006).
- Customer account value at time *n* is

$$P(n) = \alpha \times A(n) + (1-\alpha) \times \left(1 + r_p\right) \times P(n-1).$$

Participation constant

Reference portfolio value at time *n-1.*

Policy interest rate

Customer account value at time *n-1.*

- Buffer account value = $A(n) - P(n)$.
- There exists a replicating portfolio, which is not the reference portfolio.

Fig 3 from Guillén et al (2006), Bull market scenario, $r_P = 0.045, \alpha = 0.20$.

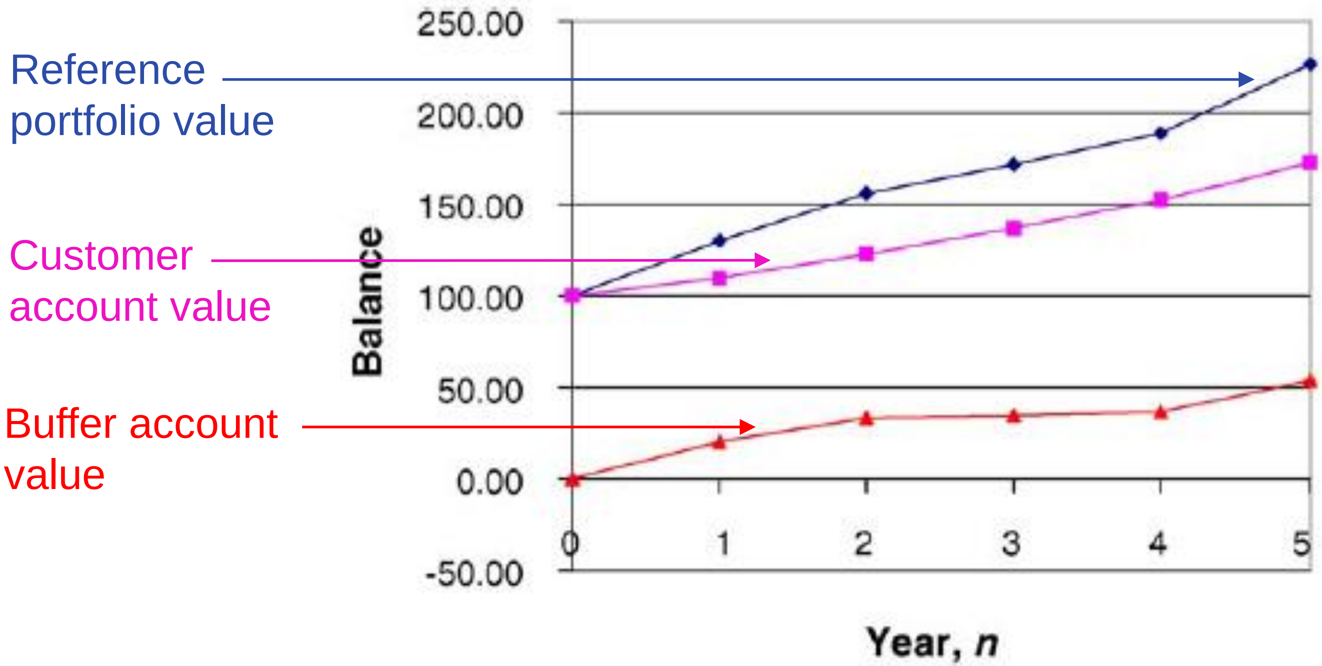


Fig 3 from Guillén et al (2006), Bull market scenario, $r_P = 0.045, \alpha = 0.20$.

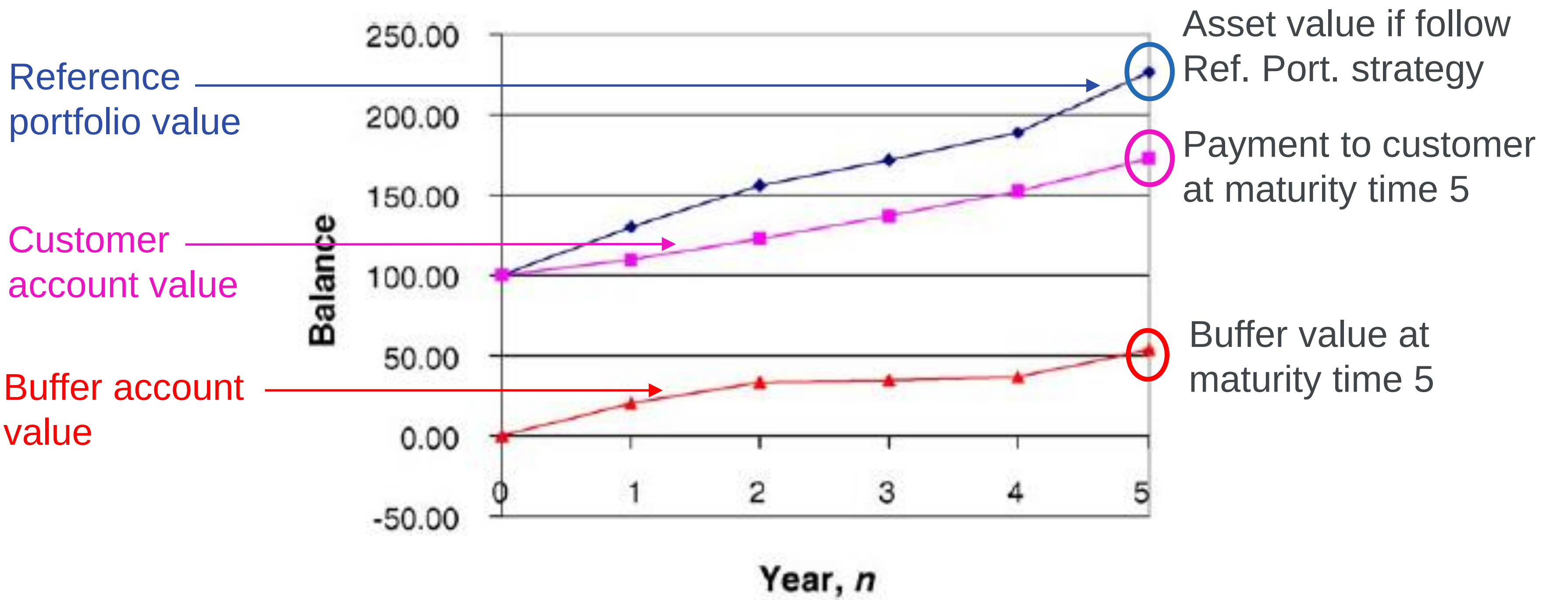
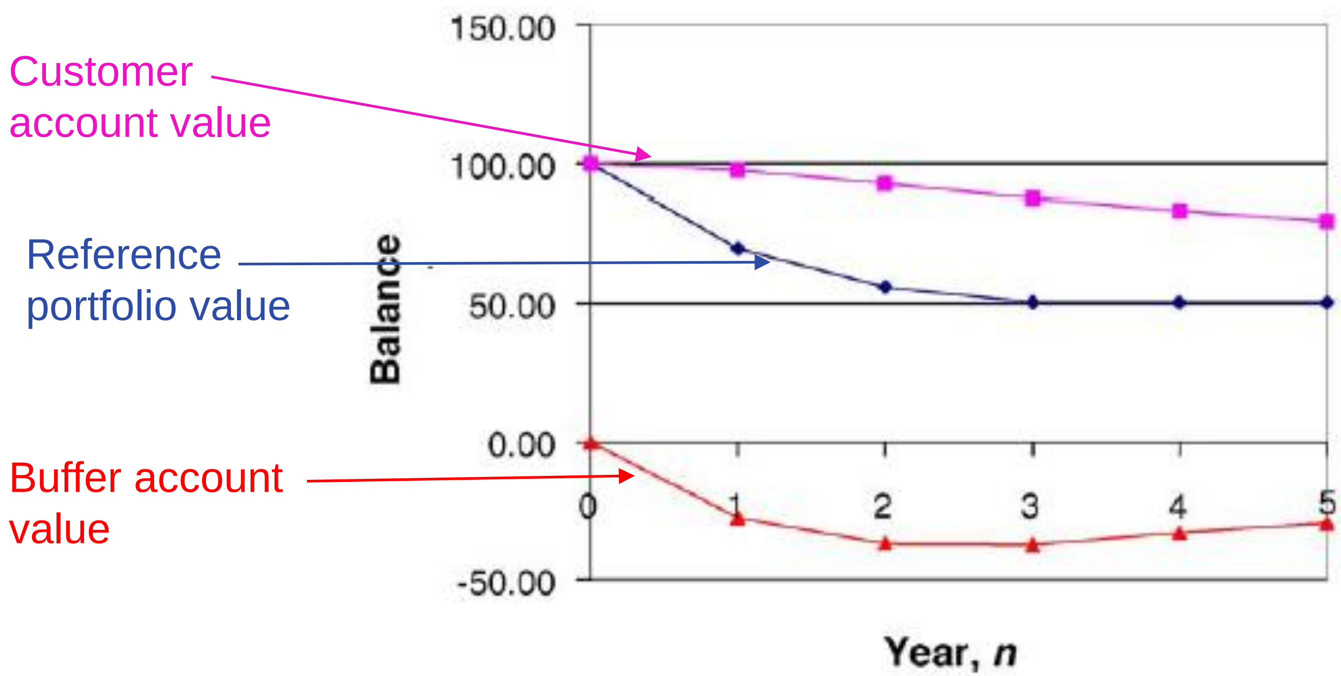


Fig 4 from Guillén et al (2006), Bear market scenario, $r_P = 0.045, \alpha = 0.20$.



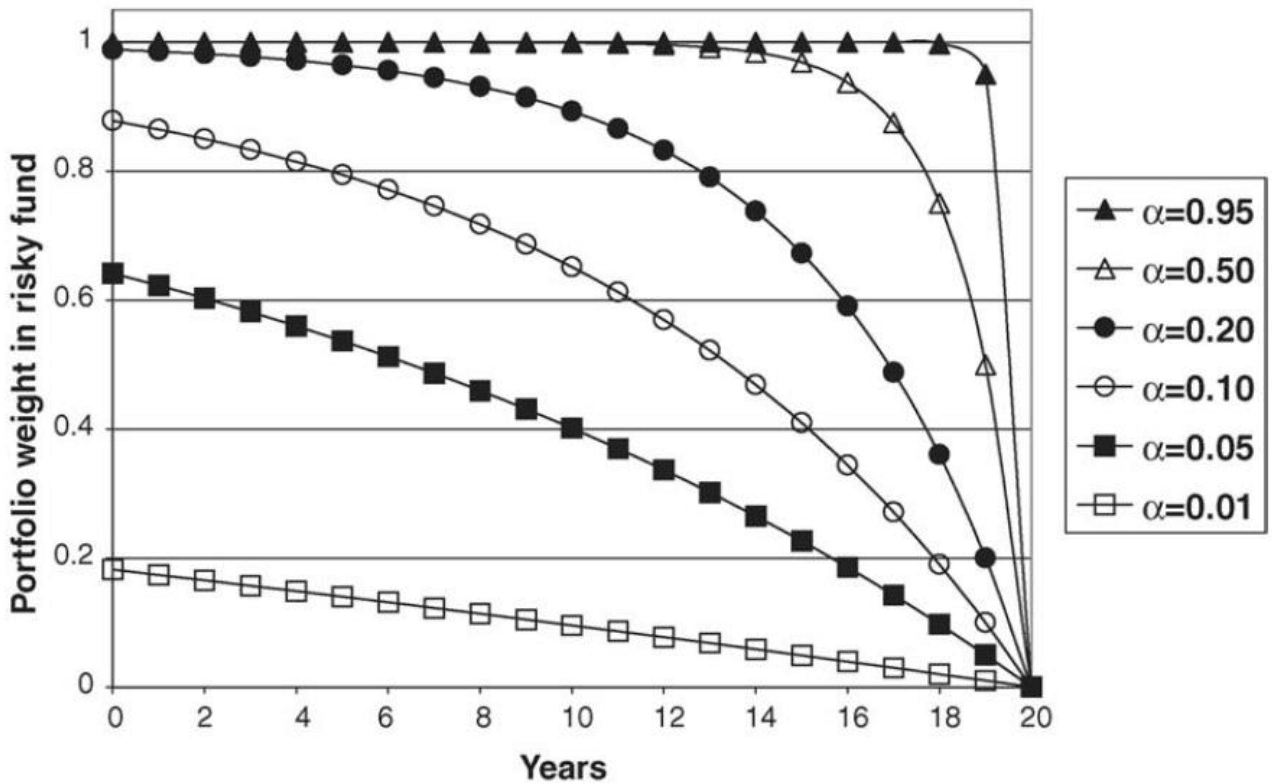
With-profits contracts – with buffer

- There exists a replicating portfolio, which is not the reference portfolio.
- Buffer is really a `hedging error' account in *TidsPension*.
- Properly, it is the responsibility of the company.

[19] In practice, the issuer of *TidsPension* contracts has so far not specifically hedged its obligations apart from actually investing in the underlying fund. The arguments have been two-fold: first, in the introductory phase of this product when reserves were still relatively small, Codan simply did not find that specific hedging considerations were worthwhile. Second, Codan wanted to use their taking part in the risky fund investment as a sales argument and as a quality signal. The implementation of a perfect hedging strategy would clearly destroy this argument. However, as *TidsPension* has become a success and as volume and risk exposure has grown, Codan are now revising their view on hedging, and some first steps have been taken towards the implementation of a risk management program.

Footnote 19 in Guillén et al (2006).

With-profits contracts – with buffer
Figure 8 from Guillén et al (2006)



With-profits contracts – a different contract with a buffer

- Danish/Dutch-inspired contract, studied by Døskeland & Nordahl (2008).
- Aims pay an annual, fixed target return on customer accounts.
- Payment from the buffer to customers only if return granted on all customer accounts and on the company account.
- First-come, first-served model:
  - older generations are granted increases before newer generations,
  - older generations' contract values are secured in full before newer generations if there are insufficient assets.

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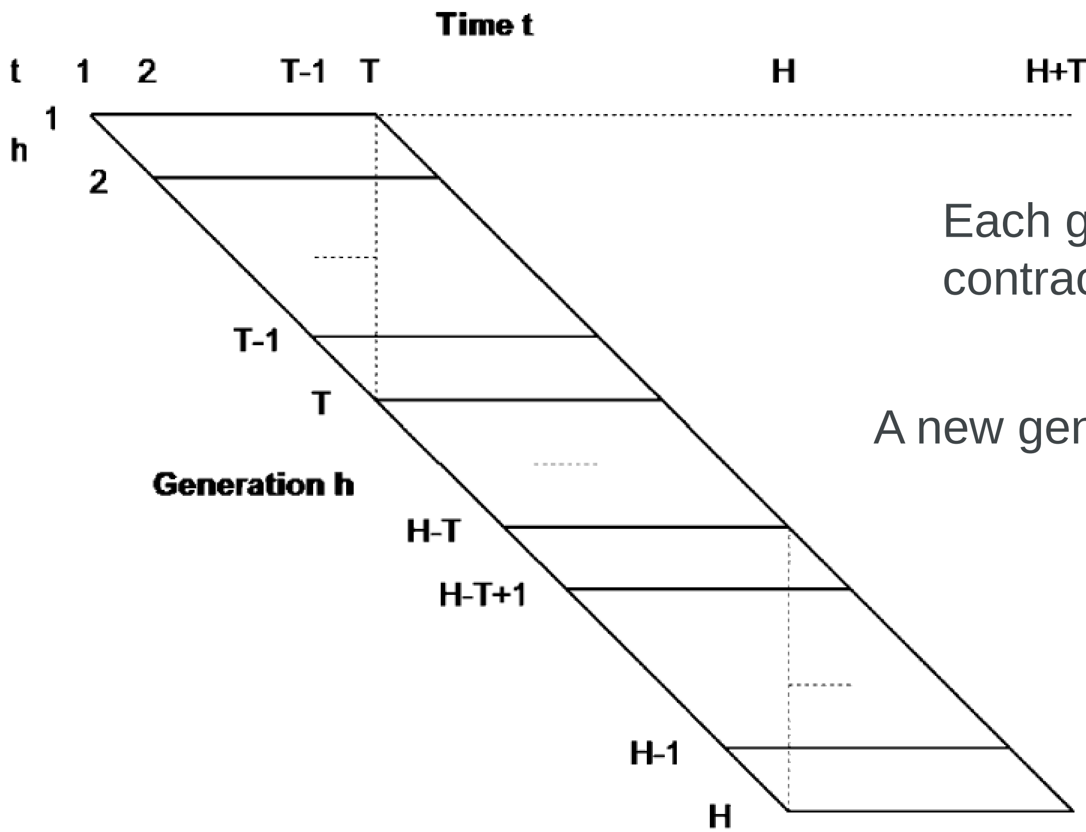
With-profits contracts – with buffer
Figure 1 from Døskeland & Nordahl (2008)

Overlapping generations model



Each generation buys a contract of duration *T* years

A new generation joins each year

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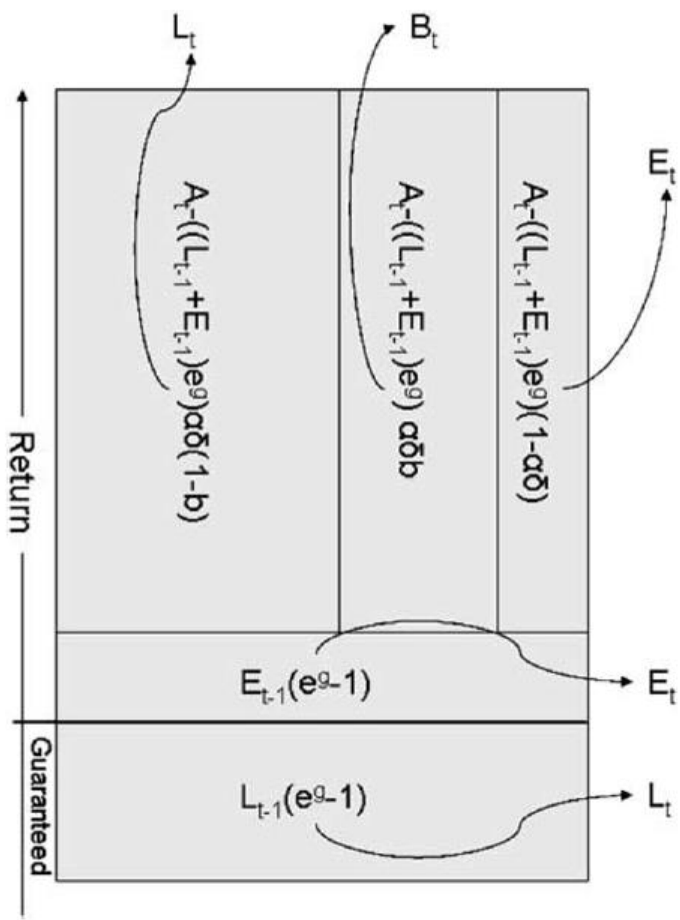
With-profits contracts – with buffer
Figure 2 from Døskeland & Nordahl (2008)



Customers also get a proportion of the buffer account upon contract maturity.

3. Bonus account *B*, company account *E* and customer accounts *L* share in any surplus returns once Steps 1 and 2 are fulfilled.

2. Company account *E* is credited at rate *g* if enough money left over.

1. Customer accounts *L* credited at rate *g*, with longest in-force getting first dibs.

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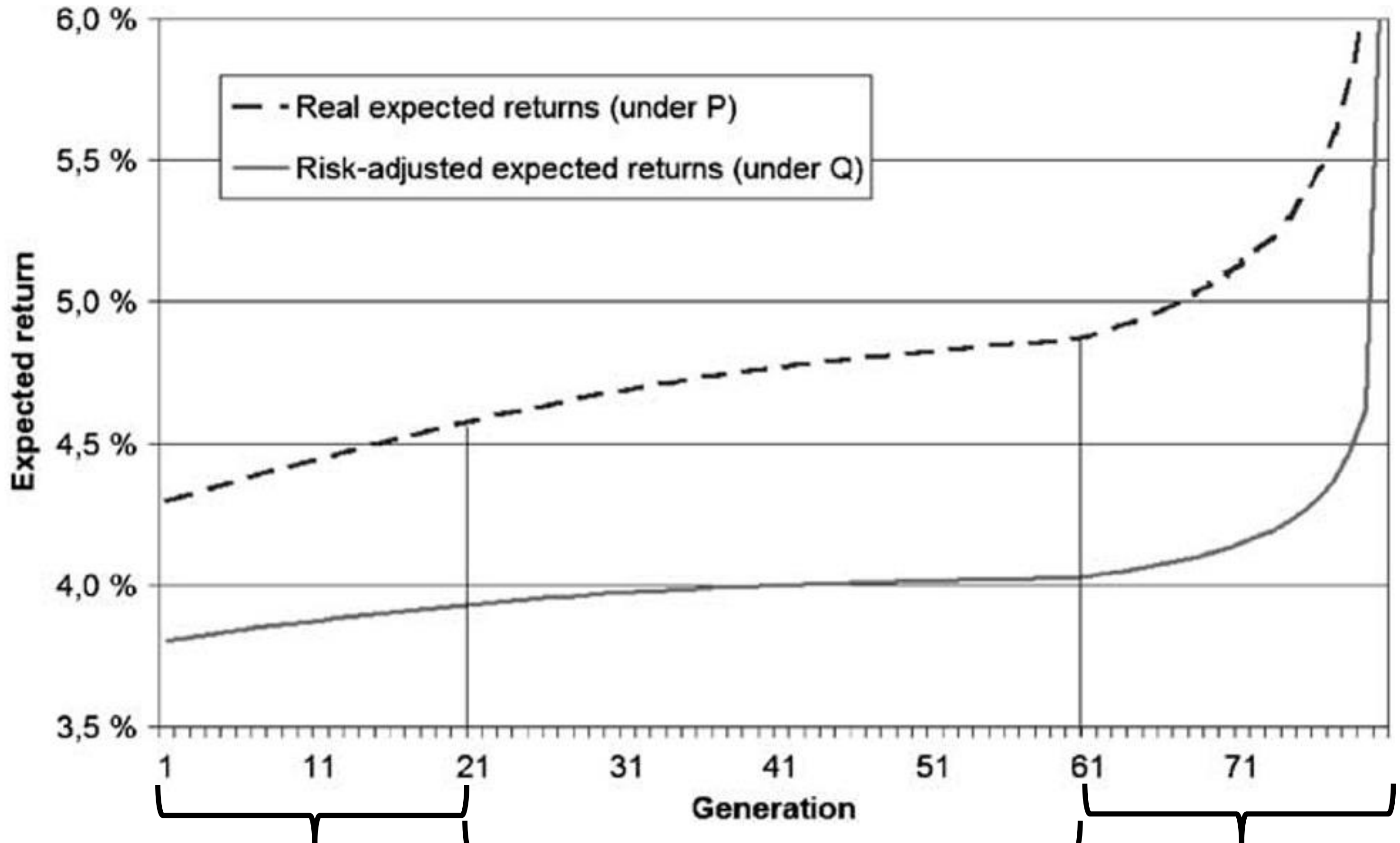
With-profits contracts – with buffer
Figure 10 from Døskeland & Nordahl (2008)



Building up the buffer

Going-concern phase

Running down the buffer

Cf. risk-free rate 4% p.a.

For 200 generations, risk-adjusted expected return converges to 4.06% p.a.

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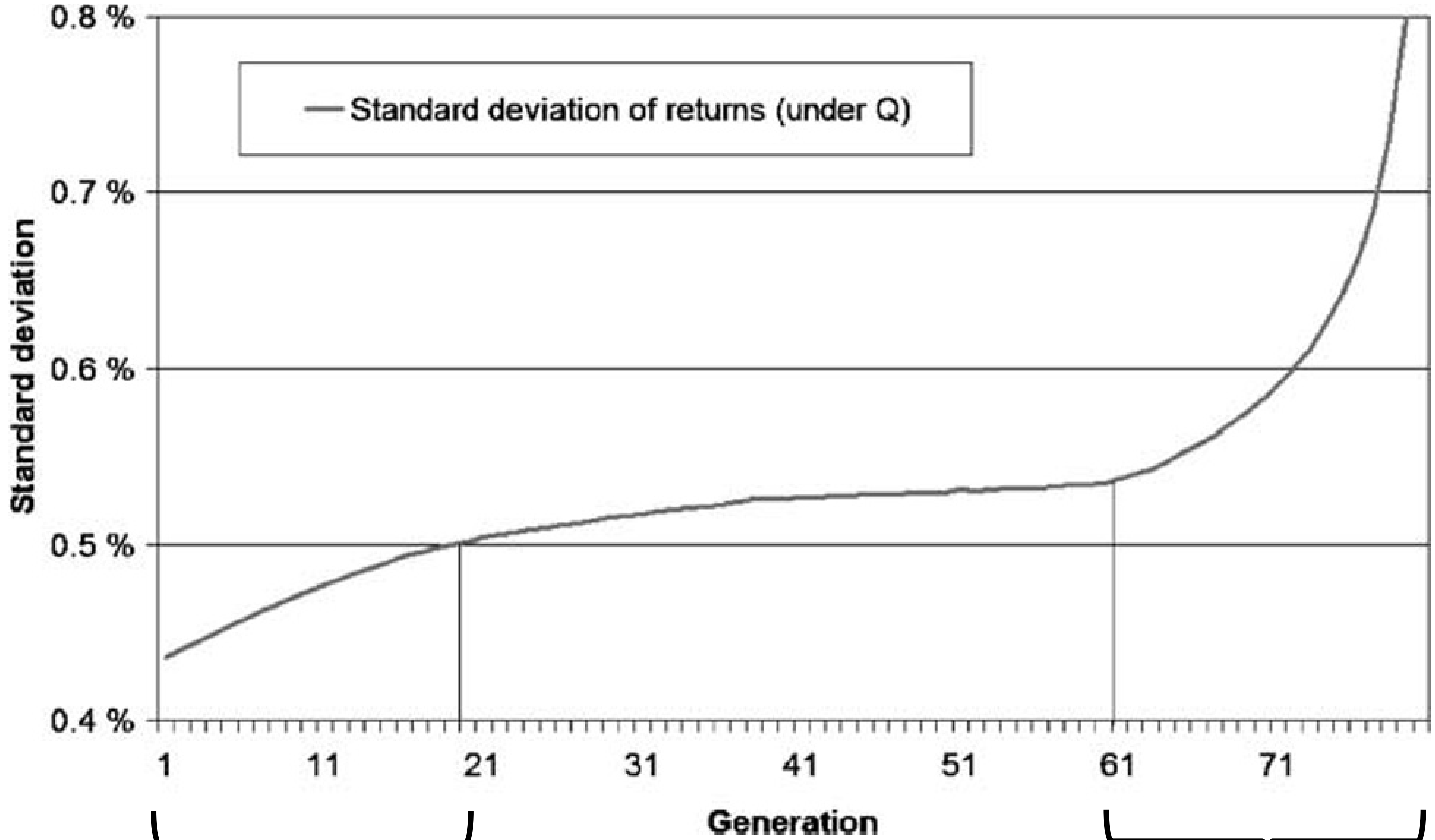
With-profits contracts – with buffer
Figure 4 from Døskeland & Nordahl (2008)

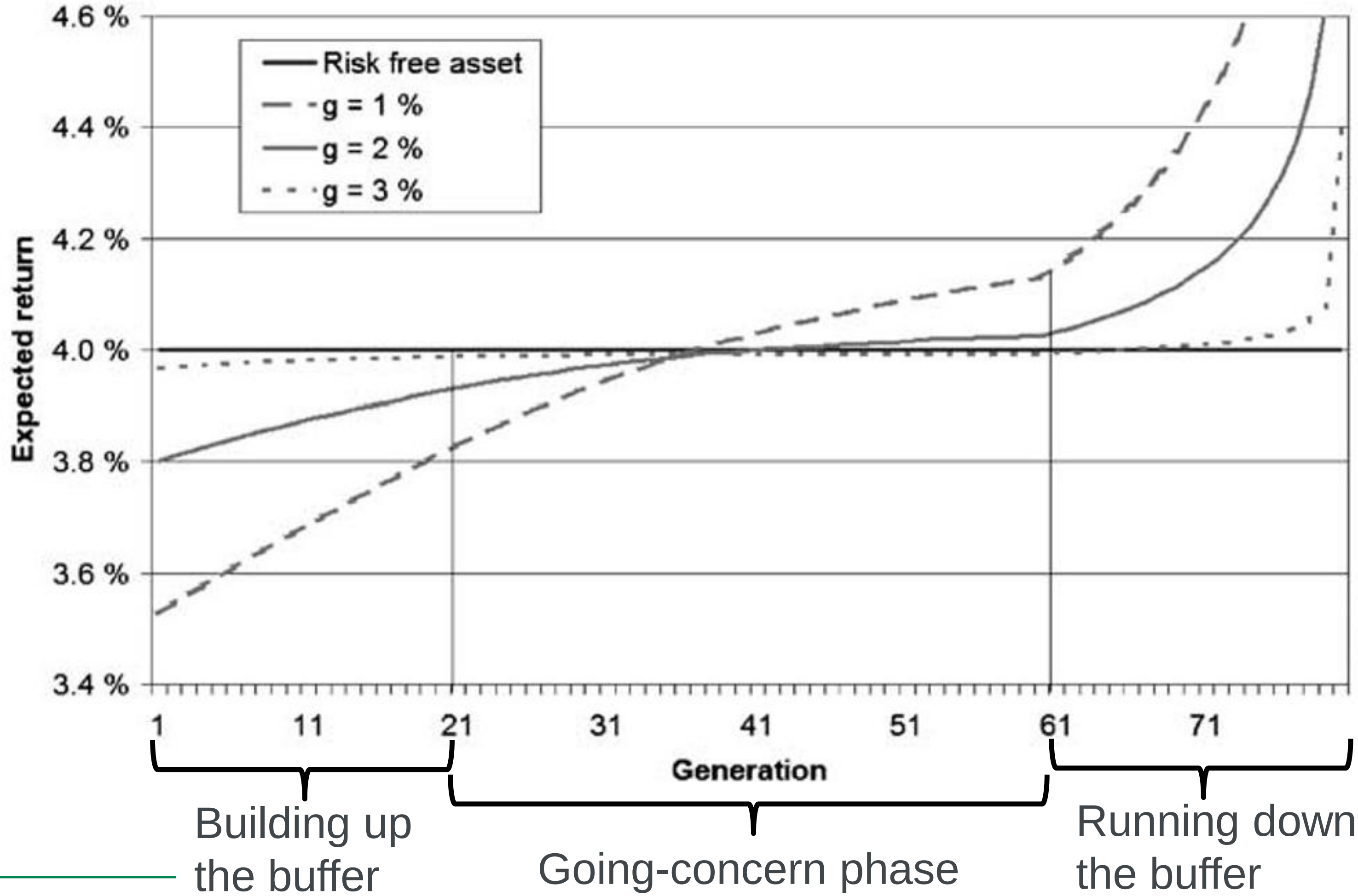


Building up the buffer

Going-concern phase

Running down the buffer

With-profits contracts – with buffer
Figure 6 from Døskeland & Nordahl (2008)



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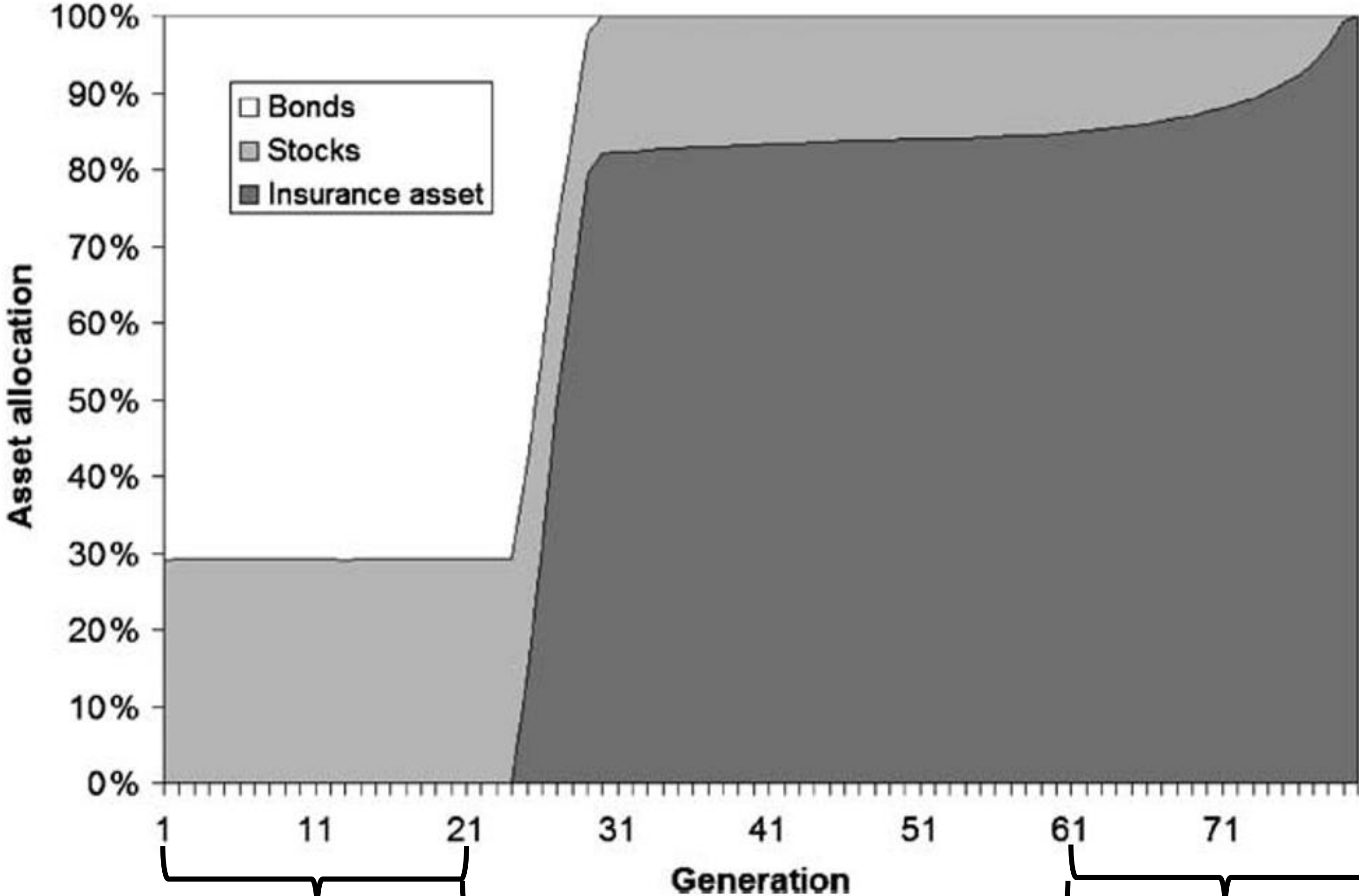
With-profits contracts – with buffer
Figure 11 from Døskeland & Nordahl (2008)



Building up the buffer

Going-concern phase

Running down the buffer

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With-profits contracts – conclusions

- Financial fairness means "you get what you pay for":

Contract premium = Expected discounted value of maturity value,

under a risk-neutral measure.

- For contracts where financial fairness is imposed, a replicating strategy exists.
- Buffer account existence does not imply risk-sharing.
- Find one genuine investment risk-sharing, but it is difficult to understand the underlying mechanism.

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Theoretical risk-sharing frameworks

- Various theoretical risk-sharing frameworks.
- Few are easily interpretable in a pension scheme context.
- Aim is to find a risk-sharing scheme: who should get what and when.
- Difficult to see how currently they would be implemented in practice.

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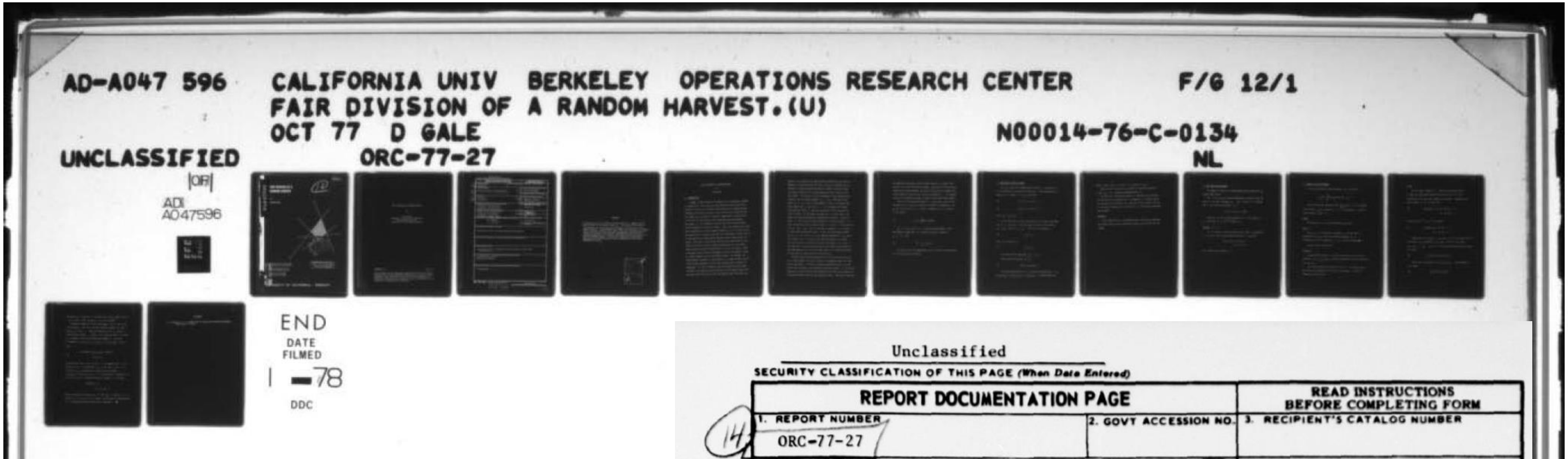
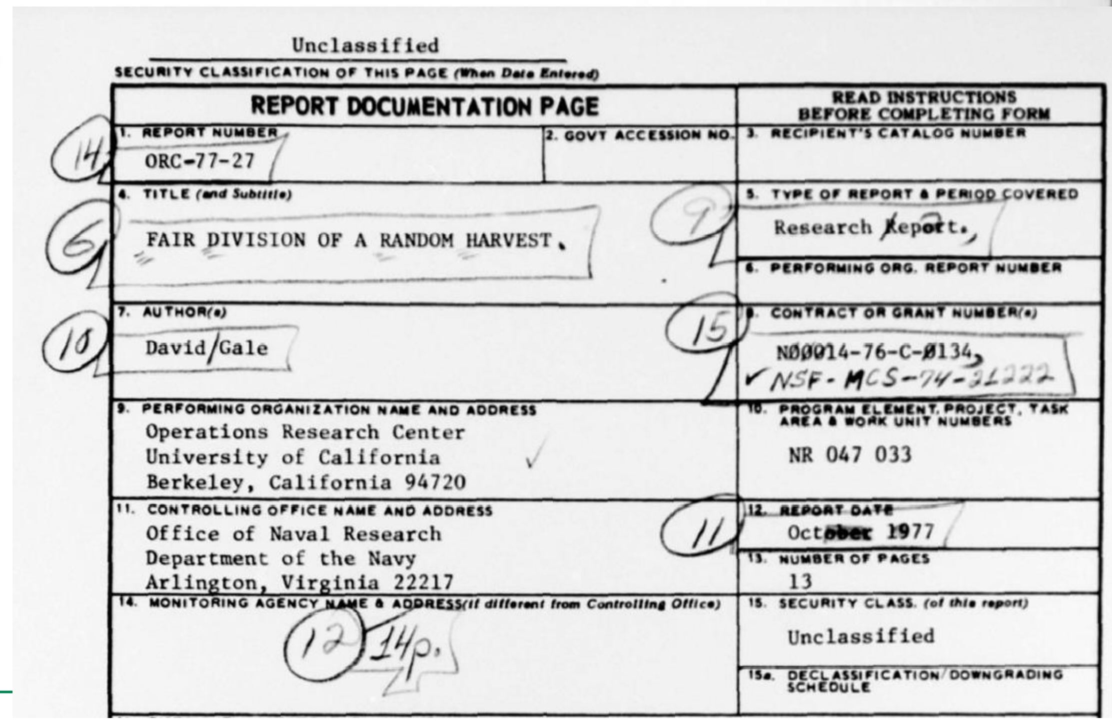
Theoretical risk-sharing frameworks

- Set-up: finite group of people.
- Everyone contributes money and in return receives a random amount back.
- Two main features:
  - Financial fairness: you get what you pay for, and
  - Pareto optimality/efficiency: your utility of wealth cannot be increased without reducing another's utility.
- These give existence and uniqueness of a risk-sharing scheme.

Gale (1977)

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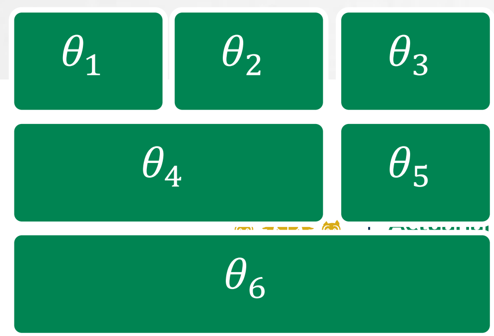
Gale (1977)

ABSTRACT

A resource such as an orchard is owned jointly by m agents, the i^{th} agent's *share* of the resource being θ_i. The yield of the resource, (the *harvest*) and the utilities of each agent are functions of the state of nature. A *fair distribution scheme* is one which is (1) (Pareto) optimal and which (2) gives each agent an expected consumption proportional to his share of the resource. ~~We show that~~ with the usual concavity assumptions on utilities there always exists one and only one fair distribution scheme. The proof is achieved by constructing a suitable social welfare function which is maximized at the desired distribution scheme.



Pazdera et al (2016), $r = 0$

1. Collective investment decision, X
2. Allocation rule among agents, $\boldsymbol{y}$

Investment outcome X=x

Time

w_1	w_2	w_3
w_4		w_5
w_6		

Agents' initial wealth

$y_1(x)$	$y_2(x)$	$y_3(x)$
$y_4(x)$		$y_5(x)$
$y_6(x)$		

Allocation of investment outcome

Financially fair if $\mathbb{E}^{\mathbb{Q}}\big(y_i(X)\big) = w_i$

Pareto efficient $(\boldsymbol{y}, X)$ if there does **not** exist $(\tilde{\boldsymbol{y}}, \tilde{X})$ such that
$$\left(\mathbb{E}U_1\left(\tilde{y}_1(\tilde{X})\right), \dots, \mathbb{E}U_6\left(\tilde{y}_6(\tilde{X})\right)\right) >_{\neq} \left(\mathbb{E}U_1\big(y_1(X)\big), \dots, \mathbb{E}U_6\big(y_6(X)\big)\right).$$

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Pazdera et al (2016)

- Prove existence of a pair $(\boldsymbol{y}, X)$.
- Worked examples give sensible answers,
  - e.g. for certain utility functions, $y_i(X) = \frac{w_i}{\sum_{k=1}^{6} w_k} X$.
- It is a one-period model.
- Intra-generational risk-sharing.

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Bao et al (2017)

- Multi-period model.
- Single payment in and single payment out in each time period.
- No group of agents individually expecting a payment at each time period.
- Buffer carried over between time periods.
- Investment returns earned on the buffer.
- System must be in balance at all times.

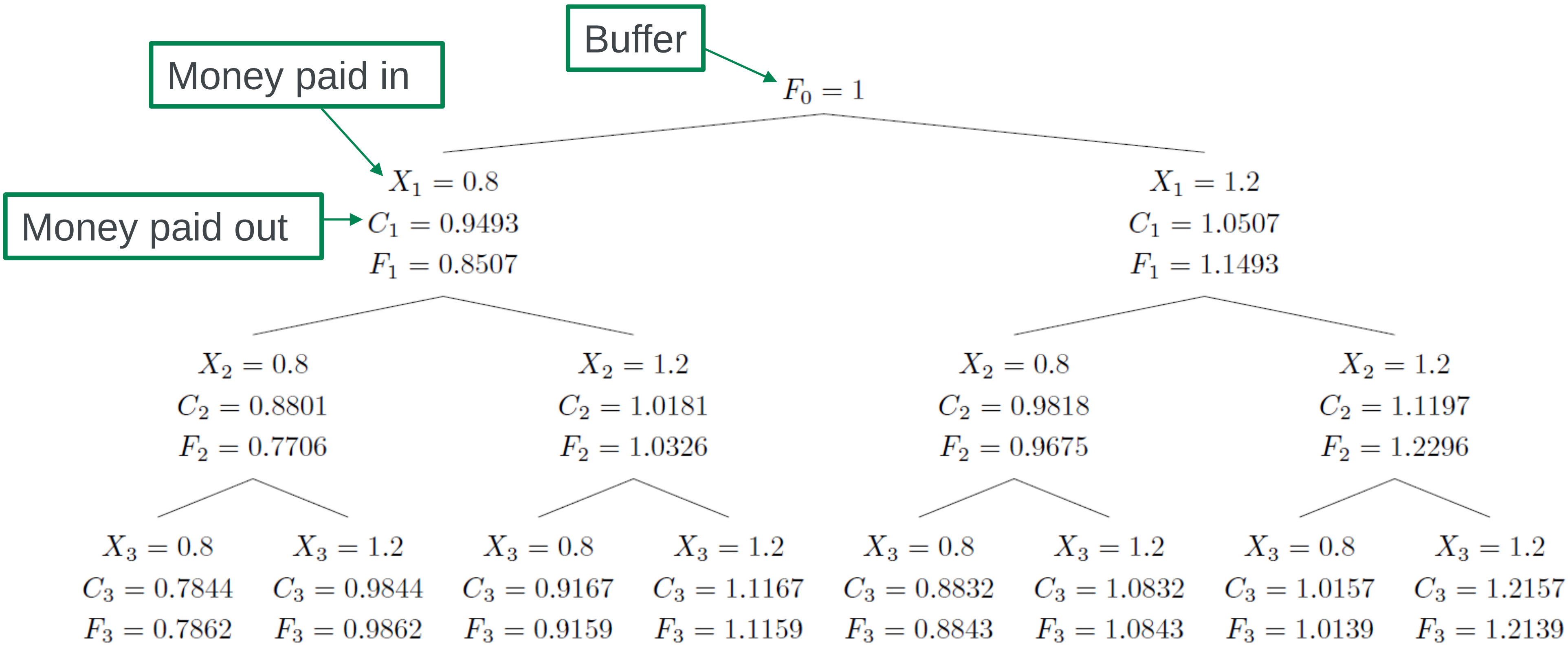
Bao et al (2017)

$$\overbrace{B_{n-1}}^{\text{Last time's buffer}} \overbrace{R_n}^{\text{Investment return on buffer}} + \overbrace{X_n}^{\text{Payment into system}} = \underbrace{P_n}_{\text{Payment out of system}} + \underbrace{B_n}_{\text{New buffer}}$$

Pareto efficient $(\boldsymbol{P}, B_N)$ if there does not exist $\left(\widetilde{\boldsymbol{P}}, \widetilde{B}_N\right)$ such that $\left(\mathbb{E}U_1(\tilde{P}_1), \dots, \mathbb{E}U_N(\tilde{P}_N), \mathbb{E}U_b(\tilde{B}_N)\right) >_{\neq} \left(\mathbb{E}U_1(P_1), \dots, \mathbb{E}U_N(P_N), \mathbb{E}U_b(B_N)\right)$.

Given constants $(v_1, \dots, v_N, v_b)$, $(\boldsymbol{P}, B_N)$ is financially fair if $\mathbb{E}_{\mathbb{Q}}(P_k) = v_k$ for k=1,...,N and $\mathbb{E}_{\mathbb{Q}}(B_N) = v_b$.

Bao et al (2017), no return on buffer



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Bao et al (2017)

- Prove existence of a pair $(\boldsymbol{P}, B_N)$.
- Algorithm to solve for values of $(\boldsymbol{P}, B_N)$, in each future state of the world
- It is a multi-period model.
- Inter-generational risk-sharing.

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CDC/DB schemes

- CDC/DB schemes involve collective investment risk-sharing.
- DB schemes have the backstop of the sponsoring employer.
- CDC schemes can adjust benefits instead of requiring more contributions.

CDC/DB schemes

- What is meant by inter-generational fairness?
  - Same contributions?
  - Same benefits?
- Relevant to inter-generational risk-sharing.
- Collective fund facilitates more than inter-generational risk-sharing, e.g. intra-generational return smoothing.
- Gollier (2008) and Cui et al (2011) find that collective schemes are welfare-increasing.

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Cui et al (2011) – four schemes

- Standard DB scheme in which only the contributions are adjusted in response to funding risks. The sponsoring employer and the active members bear the investment risk.
- CDC scheme in which only the benefits-in-payment are adjusted in response to funding risks. The retirees bear the investment risk.
- A CDC scheme in which both the benefits in payment and the contributions are adjusted in response to funding risks. The sponsoring employer, active members and retirees bear the investment risk.
- Individual defined contribution (DC) scheme.

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Cui et al (2011) – set up

- DB/CDC: adjustments to benefits and contributions given by mathematical rules
- Optimal investment and benefit strategy determined for first cohort, and kept thereafter (adjusted if required)
  - Maximise lifetime benefit payment
- Welfare measured by certainty equivalent consumption.

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Cui et al (2011) – best performing

- Standard DB scheme in which only the contributions are adjusted in response to funding risks. The sponsoring employer and the active members bear the investment risk.
- CDC scheme in which only the benefits-in-payment are adjusted in response to funding risks. The retirees bear the investment risk.
- **A CDC scheme in which both the benefits in payment and the contributions are adjusted in response to funding risks. The sponsoring employer, active members and retirees bear the investment risk.**
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Cui et al (2011) - worst performing

- Standard DB scheme in which only the contributions are adjusted in response to funding risks. The sponsoring employer and the active members bear the investment risk.
- **CDC scheme in which only the benefits-in-payment are adjusted in response to funding risks. The retirees bear the investment risk.**
- A CDC scheme in which both the benefits in payment and the contributions are adjusted in response to funding risks. The sponsoring employer, active members and retirees bear the investment risk.
- Individual defined contribution (DC) scheme.

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Summary

- Investment risk-sharing mechanisms are not well understood.
- Many things mixed up in a buffer, including hedging error.
- Difficult to put risk-sharing into a framework.
- Best attempts are the Pareto Efficient/Financially Fair approaches.
  - Require utility functions,
  - Solutions calculated via algorithm.

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The '**Minimising Longevity and Investment Risk while Optimising Future Pension Plans**' research programme is being funded by the ARC.

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Questions

Comments

The views expressed in this presentation are those of the presenter.

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