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Capital Allocation – Should we open Pandora's Jar?

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Appendix

Literature review: bibliography and notes

References

1	Bodoff, N., 2008. Capital Allocation by Percentile Layer. Casualty Actuarial Society E-Forum, Winter 2008.	Introduces the percentile layering method of capital allocation, which aims to capture the contribution of risks across the whole distribution, not just in the tail.
2	Boonen, T.J., Tsanakas, A. and Wüthrich, M.V., 2017. Capital allocation for portfolios with non-linear risk aggregation. Insurance: Mathematics and Economics, 72, pp.95-106.	Consider allocations in the presence of a non-linear portfolio aggregation. Whilst the Euler rule is not applicaple for non-homogeneous fuzzy games, the authors study risks under a Levy process model and introduce an auxiliary linearised game to which the Euler rule can be applied and which satisifes the core property, providing a plausible capital allocation.
3	Buch, A. and Dorfleitner, G. (2008) Coherent risk measures, coherent capital allocations and the gradient allocation principle. Insurance: Mathematics and Economics 42: 235–242	Analyses the coherence of the Euler method, and for that principle shows equivalences of pairs of Denault's coherence axioms and shows that symmetry holds iff the risk measure is linear.
4	Correia, R.S., 2017. Methods of capital allocation in a Solvency II environment (Doctoral dissertation, Instituto Superior de Economia e Gestão).	Accessible review of methods. Analysis of Euler Allocation applied to the Solvency II standard formula.
5	Czernuszewicz, A. (2015) GIRO Conference	A talk on the future of modelling. Recoginises the importance of different stakeholders and that modelling of risk is not just about capital. Discusses the use of risk surfaces to handle the treatment of time.
6	Denault, M., 2001. Coherent allocation of risk capital. Journal of risk, 4, pp.1-34.	Builds the formal framework for the capital allocation problem. Sets out the properties for a coherent allocation and frames these in a coalitional game theory setting, for both atomic and fuzzy games and shows that the Aumann-Shapley value is coherent.
7	Dhaene, J., Tsanakas, A., Valdez, E.A. and Vanduffel, S., 2012. Optimal capital allocation principles. Journal of Risk and Insurance, 79(1), pp.1-28.	Defines a capital allocation as a solution of a generalised optimisation problem, and show that this framework contains many of the extant methods from the literature. The authors point out that this can both aid interpretation of methods that have been proposed in an ad-hoc fashion and provide a way of defining new methods.

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References

8	Francesca Centrone, Emanuela Rosazza Gianin, Capital allocation à la Aumann–Shapley for non-differentiable risk measures, European Journal of Operational Research, Volume 267, Issue 2, 2018, Pages 667-675, ISSN 0377-2217,	Extends the theory developed by e.g. Denault (2001) to consider Auman-Shapley allocations for non-differentiable risk measures.
9	Furman, E., Kuznetsov, A. and Zitikis, R., 2018. Weighted risk capital allocations in the presence of systematic risk. Insurance: Mathematics and Economics, 79, pp.75-81.	Analyse the Weighted Insurance Pricing Model under multiplicative and additive systematic-risk frameworks. Quite “useful” distributions allowed for. Dependency through explicit effects – analogous to CAPM.
10	Kaye, P., 2005. Risk Measurement in Insurance A Guide To Risk Measurement, Capital Allocation And Related Decision Support Issues. Casualty Actuarial Society Discussion Paper.	Accessible summary aimed at practitioners, which considers several methods within the literature at the time of publication and gives a technical and practical evaluation of each.
11	Major, J.A., 2018. Distortion measures and homogeneous financial derivatives. Insurance: Mathematics and Economics, 79, pp.82-91.	Derives results using distortion measures for allocation of financial derivatives of some underlying e.g. reinsurance recoveries. Relies on homogeneity of the operator.
12	Mango, D. and Cox, A., 2013. Capital (Cost) Allocation Leading Practices - A brief tour. GIRO Conference 2013.	Presentation focused on the process of capital allocation. Theroetical treatment/opinion piece on of what would make a good process and what certain frameworks look like in practice, rather than giving explicit case studies.
13	Mango, D., 2005. Insurance Capital as a Shared Asset. ASTIN Bulletin: The Journal of the IAA, 35(2), pp.471-486.	Considers capital as a shared asset, distinguishing between consumptive and non-consumptive usage. The author offers this as an alternative to capital allocation.
14	Maume-Deschamps, V., Rullière, D. and Said, K., 2015. A risk management approach to capital allocation. arXiv preprint arXiv:1506.04125.	Studies allocation via minimisation of multivariate risk indicators in the context of an insurance group and shows this to be coherent.

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15	Myers, S.C. and Read Jr, J.A., 2001. Capital allocation for insurance companies. Journal of Risk and Insurance, pp.545-580.	Considers the default option of an insurance company and the marginal contributions to the default value from the underlying risks. Applies option pricing techniques to derive the allocations (this method has later been shown to be an application of Aumann Shapley)
16	Pesenti S.M., Tsanakas A., Millossovich P., Euler allocations in the presence of non-linear reinsurance: Comment on Major (2018). Insurance: Mathematics and Economics (2018), https://doi.org/10.1016/j.insmatheco.2018.09.001	Preprint. Response to Major (2018). Considers allocation of net rather than gross total risk. A different direction is taken and the resultant allocation is noted to be a corollary of work already done on risk measure sensitivity.
17	Powers, M.R., 2007. Using Aumann-Shapley values to allocate insurance risk: the case of inhomogeneous losses. North American Actuarial Journal, 11(3), pp.113-127.	Paper focuses on Aumann-Shapley for cases where homogeneity does not apply (so the general form of AS rather than the Euler method)
18	Ruhm, D., Mango, D. and Total, R., 2003. A Risk Charge Calculation Based on Conditional Probability. Risk, 200(50), pp.1-268.	Describes a conditional risk charge method that can be used to extend a portfolio risk measure down to the level of individual risks and their derivatives, such as excess loss layers. The algorithm described was independently proposed by Kreps and is now generally referred to as Ruhm-Mango-Kreps (RMK)
19	Tasche, D., 1999. Risk contributions and performance measurement. Report of the Lehrstuhl für mathematische Statistik, TU München.	Defines a framework for performance measurement in terms of returns and introduces the concept of suitability of a risk measure for performance measurement. Shows within this framework that the Euler (gradient) allocation principle is the unique suitable allocation principle.
20	Tasche, D., 2007. Capital allocation to business units and sub-portfolios: the Euler principle. arXiv preprint arXiv:0708.2542.	Detailed paper focused on the Euler allocation method.
21	Tsanakas, A., 2004. Dynamic capital allocation with distortion risk measures. Insurance: Mathematics and Economics, 35(2), pp.223-243.	Generalises previously derived results for allocations using distortion measures. Considers nonlinear portfolios and analyses the impact of correlation order on risk loadings in a pricing context, and further considers implications for equilibrium market prices. Extends the allocation methodology studied to a dynamic setting.
22	Vaughn, T.R., 2007. Comparison of Risk Allocation Methods-Bohra-Weist DFAIC Distributions. In CAS Forum (pp. 329-337).	Comparison of several methods on a known example "realistic" dataset. Accompanying excel workbook provides a handy quick reference for example implementations of the methods.

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Disclaimer

- The informal notes herein were made during the course of a **high-level** review of the literature, from a practitioner's rather than an academic's perspective
- This is being shared in the hope that (i) it provides a useful resource for anyone wishing to look into the literature to get started and (ii) it may stimulate further research within the profession
- It is likely that we will have missed key points or summarised some papers incorrectly or unfairly; time has not permitted us to follow every derivation and numerical example
- Whilst an attempt has been made to provide a reasonably comprehensive overview, undoubtedly some important contributions will have been missed
- Please feel free to get in touch to comment, ask questions, and indeed to point out omissions or errors: andrew.mcguinness@willistowerswatson.com

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