Institute and Faculty of Actuaries

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Risk aggregation: comparing the covariance method with simulation methods

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IFoA GIRO Conference 2024

AFIR / ASTIN Working Party

- I'm presenting work of the AFIR / ASTIN working party on "**Risk Aggregation with Correlation Matrices**"
- The work was presented at the IAA Joint Colloquium in Brussels on 25th September

Introduction: risk aggregation in standard formulas

- Risk categories $i = 1, \ldots, n$
- Capital for category i is K_i
- Correlation between categories is ρ_{ij}

	Life	Non-Life	Catastrophe	Market	Credit
Life	100%	0%	25%	25%	25%
Non-Life	0%	100%	25%	25%	25%
Catastrophe	25%	25%	100%	25%	25%
Market	25%	25%	25%	100%	25%
Credit	25%	25%	25%	25%	100%

- Total capital is then calculated using:

$$\mathrm{K}_{Total} = \sqrt{\sum_{i,j=1}^{n} \mathrm{K}_i \mathrm{K}_j \rho_{ij}}$$

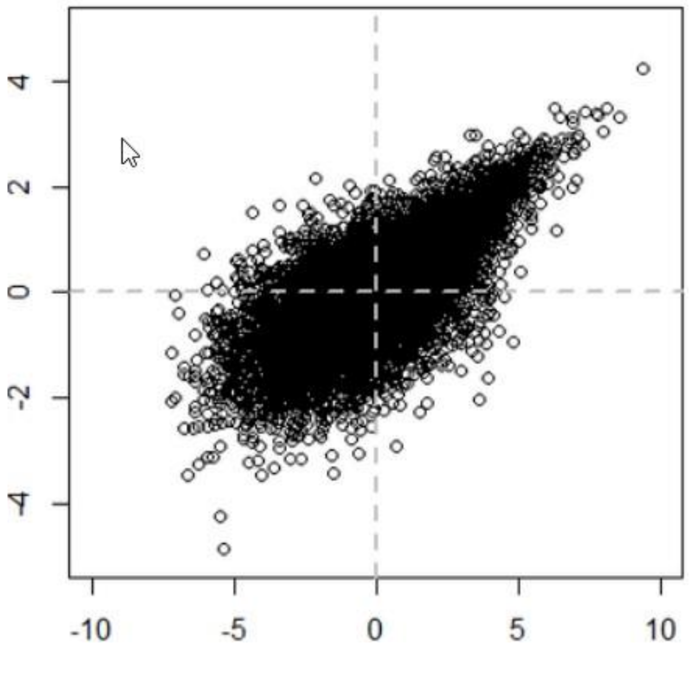
This is just the formula for standard deviation re-purposed

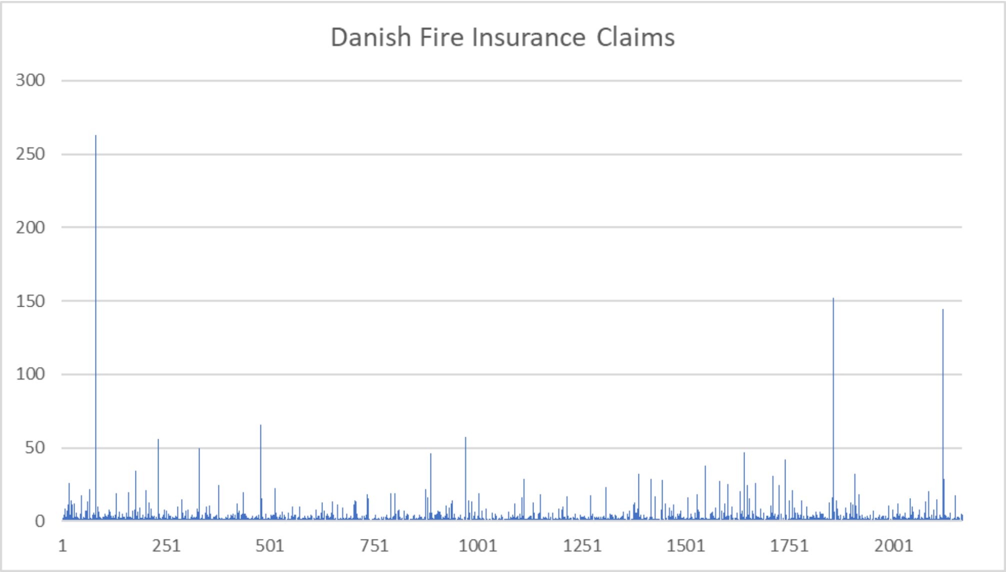
Just exactly how wrong is this?

What might make the formula wrong?

- Tail dependence
- Heavy tails
- Skewed distributions

The formula works for normal distributions as all common risk measures are constant multiples of standard deviation





Typical marginals and dependence used in non-life insurance models

Loss distributions

- Lognormal
- Gamma
- Poisson
- Negative Binomial

- For nat-cat use ELTs or YLTs
- For market risk use ESG

Dependencies

- Copulas
  - Gaussian
  - t-copula
  - Gumbel

- Non-copula methods
  - Driver based

Measuring the covariance method error

Capital is calculated using a risk measure – typically VaR or TVaR

Ratio of risk measure to standard deviation

- This is constant for normal marginals with a Gaussian copula

Ratio of “true” value of risk measure to value calculated using the covariance method

- This is 100% for normal marginals with a Gaussian copula

Investigate how these ratios vary with other combinations of marginals and copulas

Simulate a ‘typical’ insurance portfolio

- ‘Typical’ insurance portfolio:
  - Life risk — Normal distribution
  - Non-Life risk — Weibull, positive skew
  - Nat-Cat risk — Poisson / Pareto
  - Market risk — Student’s t, 5 dof
  - Credit risk — Weibull, positive skew
- Aggregate using the ICS correlation matrix (top right)

	Life	Non-Life	Catastrophe	Market	Credit
Life	100%	0%	25%	25%	25%
Non-Life	0%	100%	25%	25%	25%
Catastrophe	25%	25%	100%	25%	25%
Market	25%	25%	25%	100%	25%
Credit	25%	25%	25%	25%	100%

- Ratio of “true” 99.5% VaR to covariance VaR shown in table at right
  - Covariance method overestimates the total VaR
- If, in the correlation matrix, we replace 25% with 15% then the total covariance VaR is similar to the “true” VaR using the normal or t-copula shown in the table

Copula	**Ratio**
Normal	89%
t-copula (9 DoF)	93%
Fréchet-Mardia	81%

Specific case of simulation study: Lognormal marginals with t-copula

1. Simulate lognormal distributions $X_1, \dots, X_5$ each with same mean, μ and standard deviation σ
2. Apply a t-copula between the X_i with degrees-of-freedom d and correlation ρ between all pairs
3. Calculate the totals $T_i = X_1 + \cdots + X_i$ for $i = 2, 3, 4, 5$
4. Calculate VaR **from the simulations** for all of $X_1, \dots, X_5$ and $T_2, \dots, T_5$
5. Calculate VaR for $T_2, \dots, T_5$ **using the covariance method** from the VaRs calculated in step 4, and the correlation ρ
6. Calculate the following ratios:
    1. The “true” VaR from the simulations (step 4) to the standard deviation
    2. The “true” VaR from the simulations (step 4) to the VaR from covariance method (step 5)

Outline of simulation investigations: general case

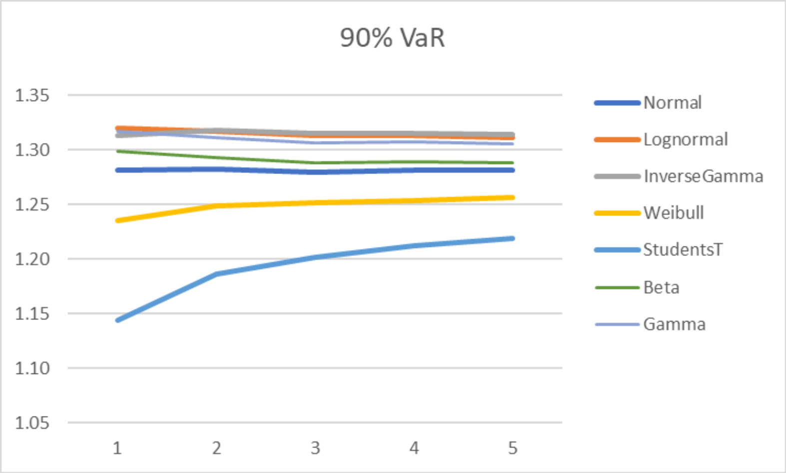
1. Simulate sets of marginals, varying the distributions between sets
2. Apply a variety of copulas between the marginals
3. Calculate the sum of 2, 3, 4, … of the marginals
4. Calculate a variety of risk measures for the marginals and the aggregates directly from the simulations – call this the “true” value of the risk measure
5. Calculate the aggregate risk measures using the covariance method
6. Calculate the following ratios:
    1. The “true” value of the risk measure to the standard deviation
    2. The “true” value of the risk measure to the value calculated using the covariance method

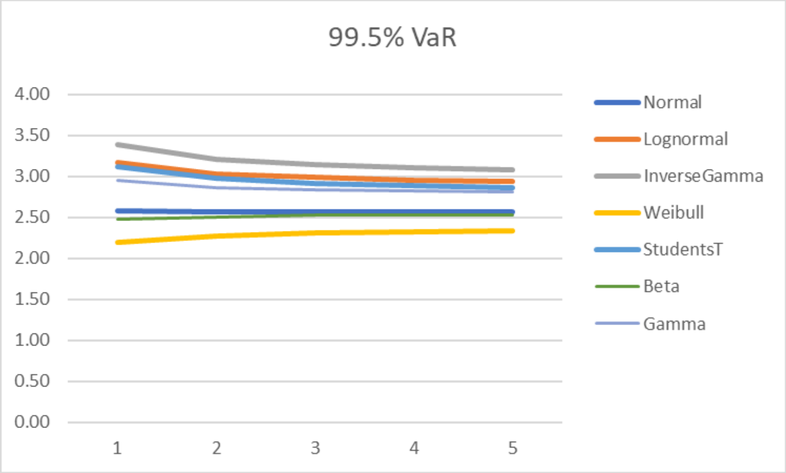
Copulas, marginals, and risk measures investigated

Marginals	Copulas	Risk measures
Normal Lognormal Student's t Gamma Inverse Gamma Weibull Beta	Normal t-copula Gumbel Clayton Frank	VaR TVaR

Some specific results in detail: normal copula

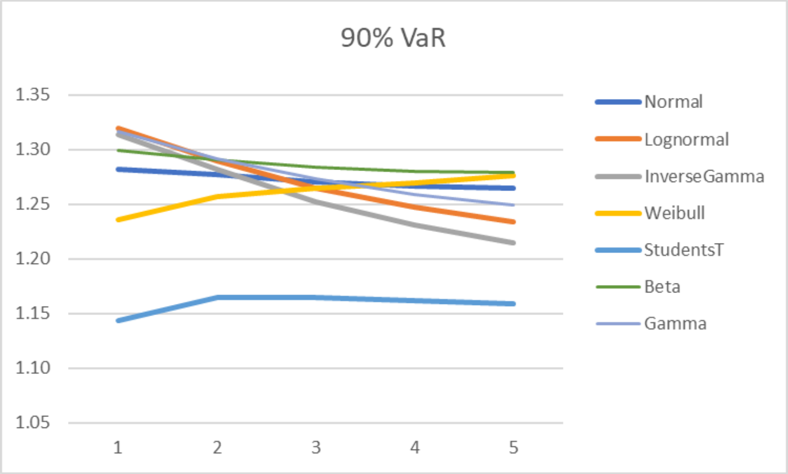
- Sets of 5 marginals with normal copula
- Calculated mean-centred VaR and TVaR
- Graphs show ratio of risk measure to standard deviation for VaR (TVaR similar)
- For normal marginals this is constant
- More skewed distributions *tend* to show a higher ratio
- The difference for more skewed distributions is higher at higher percentiles
- The ratios converge towards the normal distribution as the number of marginals increases

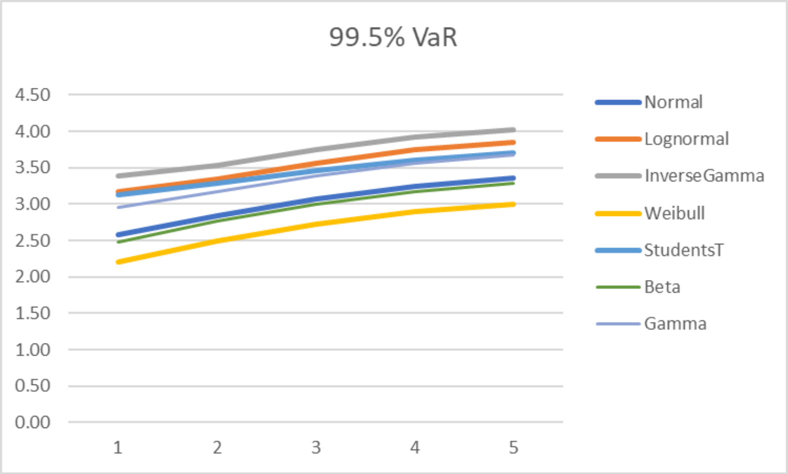




Some specific results in detail: Gumbel copula

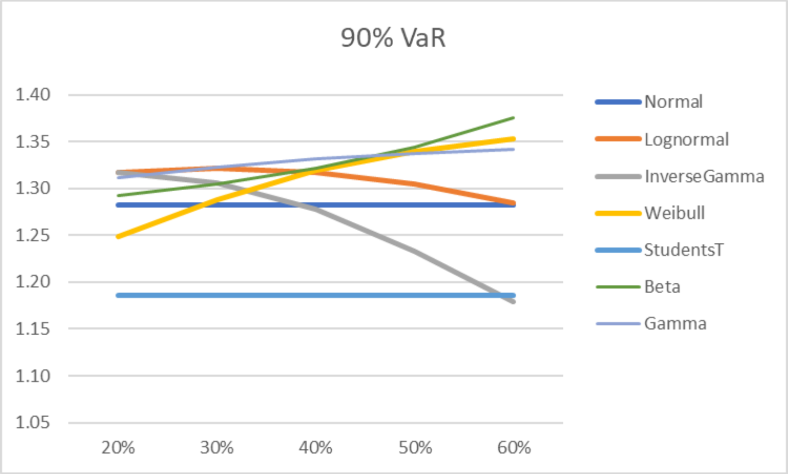
- Same basis as previous slide, but with a Gumbel copula
- 90% VaR graph looks quite different
- More skewed distributions *tend* to show a higher ratio
- The difference for more skewed distributions is higher at higher percentiles
- Convergence towards normal is much slower, and not clear from the graph

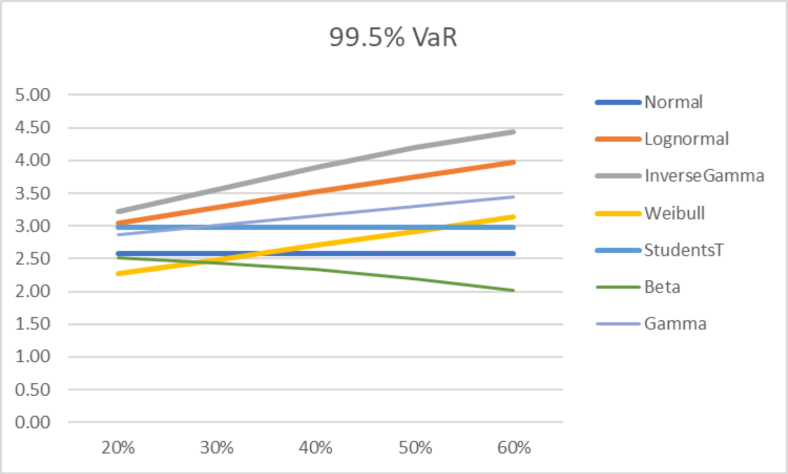




Some specific results in detail: varying the CoV

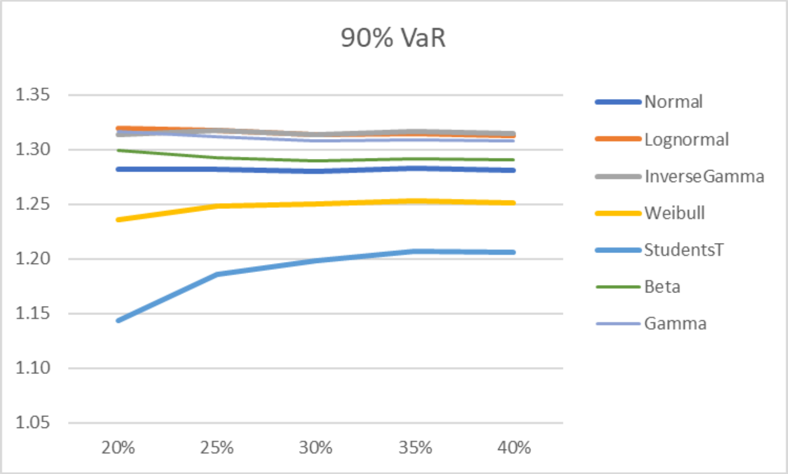
- Same basis as previous two slides, but with two summands, and varying the marginal CoV
    - Gaussian copula
- For normal and student t marginals the ratio is constant
- Very different behaviour from previous slides
- No clear pattern across the distributions

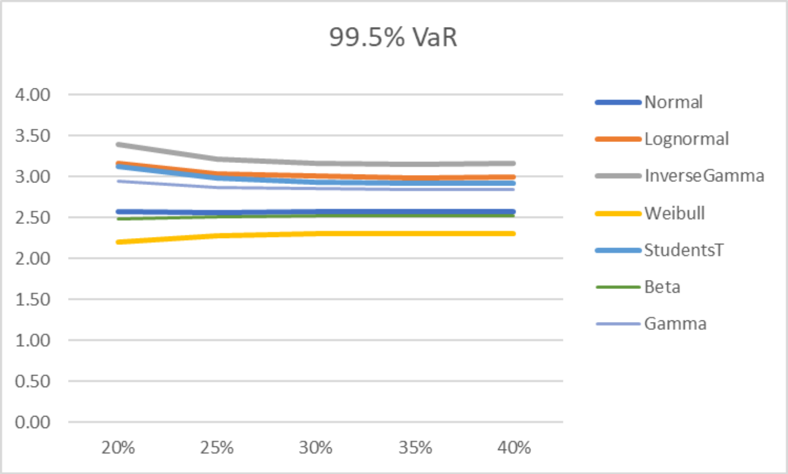




Some specific results in detail: varying the correlation

- Same basis as previous slide, but varying the correlation between marginals
  - 20% CoV
- Ratio does not vary a lot with correlation for most distributions
  - although note Student's t at 90% VaR





Tentative classification of results

- **Overestimation**: ratios < 100%
  - Clayton, Frank, Gaussian (ex Beta, Weibull) copulas
- **Underestimation**: ratios > 100%
  - Gaussian with Beta, and Weibull
  - t-copula (ex Student's t, Inverse Gamma)
  - Gumbel
- **Correct**: ratios = 100%
  - t-copula with Student's t, and Inverse Gamma
  - Gaussian with Normal
  - Frank with Weibull

		Copula				
		Clayton	Frank	Gaussian	t-copula	Gumbel
Marginals	StudentsT	88%	91%	95%	100%	106%
	InverseGamma	87%	90%	95%	100%	106%
	Lognormal	88%	92%	96%	102%	107%
	Gamma	90%	93%	97%	103%	108%
	Normal	93%	96%	100%	106%	110%
	Beta	95%	98%	101%	108%	112%
	Weibull	97%	100%	103%	109%	113%

The table shows the ratio

$$\frac{\text{"true"}\ VaR}{Covariance\ VaR}$$

What might explain this?

		Copula						
		Clayton	Frank	Gaussian	t-copula	Gumbel	Skewness	Kurtosis
Marginals	StudentsT	88%	91%	95%	100%	106%	0.0	8.1
	InverseGamma	87%	90%	95%	100%	106%	0.8	4.3
	Lognormal	88%	92%	96%	102%	107%	0.6	3.7
	Gamma	90%	93%	97%	103%	108%	0.4	3.2
	Normal	93%	96%	100%	106%	110%	0.0	3.0
	Beta	95%	98%	101%	108%	112%	0.0	2.8
	Weibull	97%	100%	103%	109%	113%	-0.4	3.0
	Normalised JEP	0%	0%	2%	13%	22%		

Overestimation increases with:

- Heavier tails
- Lower tail correlation

Underestimation increases with:

- Lighter tails
- Greater tail correlation

Skewness and Kurtosis both matter

The relationship with Skewness and Kurtosis is not simple

		Copula						
		Clayton	Frank	Gaussian	t-copula	Gumbel	Skewness	Kurtosis
Marginals	InverseGamma	85%	88%	92%	97%	102%	3.7	42.1
	Lognormal	84%	87%	91%	97%	103%	2.0	11.0
	StudentsT	88%	91%	95%	100%	106%	0.0	8.1
	Gamma	86%	89%	93%	100%	105%	1.2	5.2
	Weibull	88%	92%	96%	102%	107%	0.8	3.7
	Normal	93%	96%	100%	106%	110%	0.0	3.0
	Beta	119%	120%	121%	124%	124%	0.0	1.7
	Normalised JEP	0%	0%	2%	13%	22%		

- Increase CoV from 20% to 60%
- General relationships still in place, but details change
- Order of marginal distributions changes to reflect changes in skewness and kurtosis
  - Both still matter – compare Student's t, Gamma, and Weibull

Summary

- Aggregating any risk measure using the covariance method works when all marginal distributions are normal, and the copula is Gaussian
- For arbitrary marginals and copulas it still works for aggregating standard deviations
- We carried out simulation studies to see whether the method over- or under-estimated the total risk for the risk measures VaR and TVaR
- We observed that in these cases it can either over- or under-estimate the total risk
- Overestimation increases with heavier tails and lower tail correlation
- Underestimation increases with lighter tails and greater tail correlation

Questions

Comments

Expressions of individual views by members of the Institute and Faculty of Actuaries and its staff are encouraged.

The views expressed in this presentation are those of the presenter.

Parameters used (unless otherwise stated)

Distribution	Mean	CoV	Other parameters
Normal	1000	20%	
Lognormal	1000	20%	$\mu = 6.75$, $\sigma = 0.555$
Student's t	1000	20%	Degrees of freedom = 5
Gamma	1000	20%	$\alpha = 2.78$, $\beta = 0.00278$
Inverse Gamma	1000	20%	$\alpha = 4.78$, $\beta = 3778$
Weibull	1000	20%	$\lambda = 1122$, $k = 1.72$
Beta	1000	20%	$\alpha = 0.889$, $\beta = 0.889$ Lower bound = 0 Upper bound = 2000

Parameters used (unless otherwise stated)

Copula	Correlation	Other parameters
Gaussian	25%	
t-copula	25%	Degrees of freedom = 5
Gumbel	25%	$\theta = 1.192$
Clayton	25%	$\theta = 0.383$
Frank	25%	$\theta = 1.554$